

Introduction to Derivative Instruments

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Slides on book: John C. Hull, "Options, Futures, and Other Derivatives", Pearson ed.

LEDa

Chapter 7

Motivation

- The first swap contracts were negotiated in the early 1980s.
 - ▶ Swaps now occupy a position of central importance in derivatives markets.
- A **swap** is an OTC agreement between two counterparties (firms or financial institutions) to exchange financial instruments.
- Usually, the two counterparties exchange cash flows instead of the instruments *per se*.
 - ▶ I.e., cash flows of one party's financial instrument is exchanged for those of the other party's financial instrument.
 - ▶ So, the two parties exchange one stream of cash flows against another stream.
 - ▶ These streams are called the **legs** of the swap.

Chapter 7: Swaps

Outline

- 1 Motivation
- 2 Interest Rate Swaps
 - Market
 - Mechanics
 - The Comparative Advantage Argument
 - Using Swap Rates to Bootstrap the LIBOR/Swap Zero Curve
 - Valuation
- 3 Overnight Indexed Swaps
- 4 Foreign Exchange Swaps
- 5 Currency Swaps
- 6 Credit Risk
- 7 Summary

Motivation

- The agreement defines the dates when the cash flows are to be paid and the way in which they are to be calculated.
 - ▶ Usually the calculation of the cash flows involves the future value of an interest rate, an exchange rate, or other market variable.
- Whereas a forward contract is equivalent to the exchange of cash flows on just one future date, swaps typically lead to cash flow exchanges on several future dates.

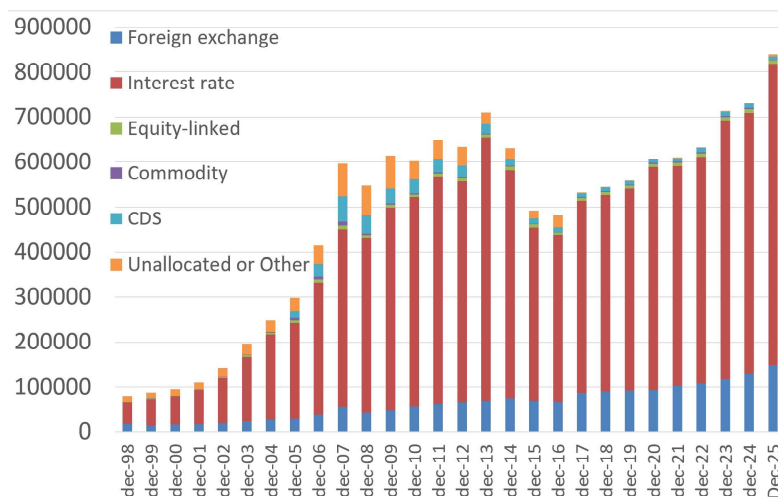
Motivation

- We will examine how swaps are designed, how they are used, and how they are valued.
- We will focus on four popular swaps:
 - ▶ Plain vanilla interest rate swaps;
 - ▶ Overnight indexed swap;
 - ▶ Foreign exchange swap; and
 - ▶ .Currency swaps

Chapter 7: Swaps Outline

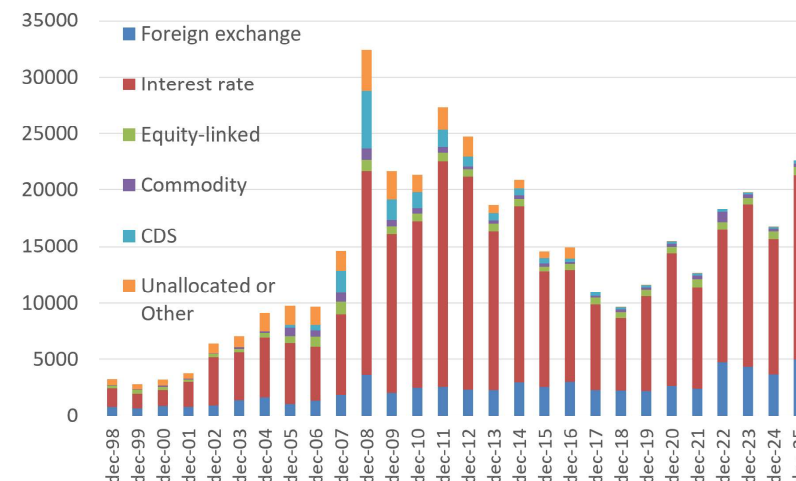
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Global OTC Derivatives Market Notional amounts outstanding



Semiannual (x2), Billions of US\$ — Source: Bank for International Settlements

Global OTC Derivatives Market Gross market values

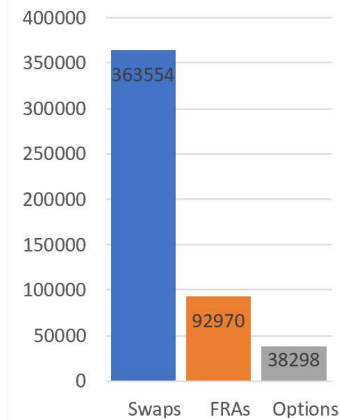


Semiannual (x2), Billions of US\$ — Source: Bank for International Settlements

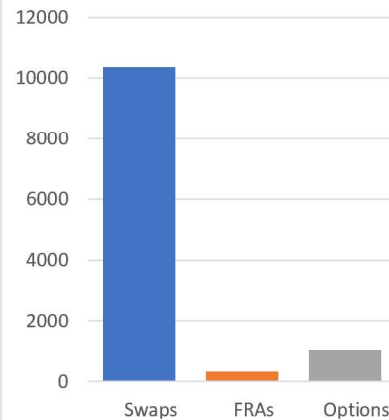
Global OTC Interest rate derivatives by instrument

Notional amounts outstanding and Gross market value (2020, Billions of US\$)

Notional amounts outstanding

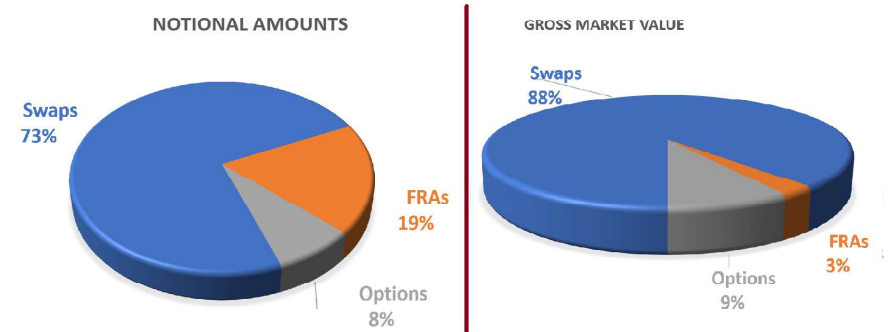


Gross market value



Global OTC Interest rate derivatives by instrument

Share by instruments (2020)

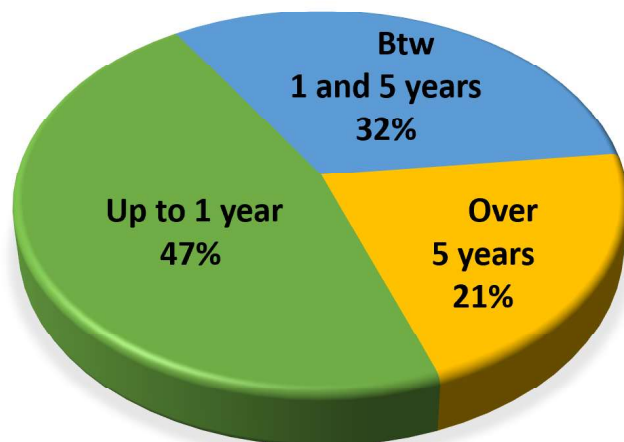


Source: Bank for International Settlements

Global OTC Interest rate derivatives by maturities

Share by maturities (2020)

NOTIONAL AMOUNTS



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Mechanics of Interest Rate Swaps

LIBOR

Definition

An **interest rate swap** is a swap in which the stream of exchanged cash flows comes from a future interest payments based on a specified principal amount.

- An interest rate swap is often utilized if a company can borrow money easily at one type of interest rate but prefers a different type.

Mechanics of Interest Rate Swaps

LIBOR

Example ("Plain Vanilla" Interest Rate Swap)

Consider a 3-year swap initiated on March 5, 2018, between Microsoft and Intel.

We suppose Microsoft agrees to pay Intel an interest rate of 5% per annum on a principal of \$100 million, and in return Intel agrees to pay Microsoft the 6-month LIBOR rate on the same principal.

Microsoft is the **fixed-rate payer**, Intel is the **floating-rate payer**.

We assume the agreement specifies that payments are to be exchanged every 6 months and that the 5% interest rate is quoted with semiannual compounding.

In total, there are six exchanges of payment on the swap.

Mechanics of Interest Rate Swaps

LIBOR

Definition

In a **plain vanilla** interest rate swap a floating interest rate is exchanged for a fixed rate or vice versa.

- A company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for a predetermined number of years.
In return, it receives interest at a floating rate on the same notional principal for the same period of time.
- The floating rate in most interest rate swap agreements is LIBOR.

Mechanics of Interest Rate Swaps

LIBOR

Example

		Floating	Fixed	Net
Date	LIBOR	Cash Flow	Cash Flow	Cash Flow
Mar 5, 2018	4.2%			
Sep 5, 2018	4.8%	+2.10	-2.50	-0.40
Mar 5, 2019	5.3%	+2.40	-2.50	-0.10
Sep 5, 2019	5.5%	+2.65	-2.50	+0.15
Mar 5, 2020	5.6%	+2.75	-2.50	+0.25
Sep 5, 2020	5.9%	+2.80	-2.50	+0.30
Mar 5, 2021		+2.95	-2.50	+0.45

The first exchange of payments would take place on September 5, 2018, 6 months after the initiation of the agreement.

Microsoft would pay Intel \$2.5 million ($=\$100 \text{ million} \times \frac{5\%}{2}$)

Intel would pay Microsoft \$2.1 million ($=\$100 \text{ million} \times \frac{4.2\%}{2}$)

Mechanics of Interest Rate Swaps

Typical Uses of an Interest Rate Swap

- Converting a liability from
 - ▶ floating rate to fixed rate
 - ★ Suppose that Microsoft has arranged to borrow \$100 million at LIBOR plus 10 basis points (i.e., the rate is $\text{LIBOR} + 0.1\%$.)
 - ★ By entering into the swap as a fixed-rate payer, Microsoft receives LIBOR and pays 5%.
 - ★ Thus, for Microsoft, the swap has the effect of transforming borrowings at a floating rate of LIBOR plus 10 basis points into borrowings at a fixed rate of 5.1%.
 - ▶ floating rate to fixed rate
 - ★ By entering into a swap as a floating-rate payer after having borrowed a fixed-rate loan.

Mechanics of Interest Rate Swaps

Role of Financial Intermediary

- Usually two nonfinancial companies such as Intel and Microsoft do not get in touch directly to arrange a swap.
 - ▶ They each deal with a financial intermediary such as a bank or other financial institution.
 - ▶ If one of the companies defaults, the financial institution still has to honor its agreement with the other company.
 - ▶ Plain vanilla fixed-for-floating swaps on US interest rates are usually structured so that the financial institution earns about 3 or 4 basis points (0.03% or 0.04%) on a pair of offsetting transactions.

Mechanics of Interest Rate Swaps

Typical Uses of an Interest Rate Swap

- Converting an investment from
 - ▶ fixed rate to floating rate
 - ★ Suppose that Microsoft owns \$100 million in bonds that will provide interest at 4.7% per annum over the next 3 years.
 - ★ By entering into a swap as a fixed-rate payer, Microsoft receives LIBOR and pays 5%.
 - ★ Thus, for Microsoft, the swap transform an asset earning 4.7% into an asset earning LIBOR minus 30 basis points ($5\% - 4.7\%$).
 - ▶ floating rate to fixed rate
 - ★ By entering into a swap as a floating-rate payer to transform an asset earning LIBOR.

Mechanics of Interest Rate Swaps

Role of Financial Intermediary

- In practice, it is unlikely that two companies will contact a financial institution at the same time and want to take opposite positions in exactly the same swap.
 - ▶ For this reason, many large financial institutions act as market makers for swaps.
 - ▶ This means that they are prepared to enter into a swap without having an offsetting swap with another counterparty.
 - ▶ Market makers must carefully quantify and hedge the risks they are taking.
 - ★ Bonds, forward rate agreements, and interest rate futures are examples of the instruments that can be used for hedging by swap market makers.

Mechanics of Interest Rate Swaps

Role of Financial Intermediary

Definition

The **swap rate** is the average of the bid and offer fixed rates.

- The bid-offer spread is, usually 3 to 4 basis points.

Maturity	Bid (%)	Offer (%)	Swap Rate (%)
2 years	6.03	6.06	6.045
3 years	6.21	6.24	6.225
4 years	6.35	6.39	6.370
5 years	6.47	6.51	6.490
7 years	6.65	6.68	6.665
10 years	6.83	6.87	6.850

Mechanics of Interest Rate Swaps

Day Count

- A day count convention is specified for fixed and floating payment.
 - ▶ The day count convention on the floating leg is generally actual/360, for the U.S. dollar and the euro, or actual/365, for the British pound, Japanese yen and Swiss franc.
 - ▶ For example, LIBOR is likely to be actual/360 in the US because LIBOR is a money market rate.

Example

Consider the first floating payment from Intel to Microsoft in our example.

Based on the LIBOR rate of 4.2%, it is \$2.10 million.

Because there are 184 days between March 5, 2018, and September 5, 2018, it should be $100 \times 4.2\% \times \frac{184}{360} = \$2.1467$ million.

- For clarity of exposition, we will ignore day count issues in the calculations in the rest of this chapter.

Mechanics of Interest Rate Swaps

Role of Financial Intermediary

- The rate of the floating leg of the swap is given by a floating rate index.
- This floating rate index is usually the LIBOR.
- LIBOR is posted for five currencies: the U.S. dollar, euro, Swiss franc, Japanese yen and British pound.
 - ▶ Maturities range from overnight to 12 months.
 - ▶ The rate is set daily by the International Commodities Exchange (ICE) and is based on a survey of between 11 and 18 major banks.
 - ★ Among them: Barclays, HSBC, Royal Bank of Scotland, Deutsche Bank, UBS, JP Morgan, Citigroup, Bank of America, Société Générale, ...
 - ★ A similar process is used for Euribor but the survey includes almost 60 european banks.

Mechanics of Interest Rate Swaps

Day Count

Definition

A **confirmation** is the legal agreement underlying a swap and is signed by representatives of the two parties.

- Confirmations specify the terms of a transaction.
- The *International Swaps and Derivatives Association* (ISDA; www.isda.org) in New York has developed Master Agreements that can be used to cover all agreements (e.g., what happens in the event of default) between two counterparties.
- Governments now require central clearing to be used for most standardized derivatives.

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The Comparative Advantage Argument

	<i>Fixed</i>	<i>Floating</i>
AAACorp	4.0%	6-month LIBOR – 0.1%
BBBCorp	5.2%	6-month LIBOR + 0.6%

- AAACorp can borrow at a lower floating-rate than LIBOR because LIBOR is the rate of interest at which **AA**-rated banks borrow.
- A key feature of the rates offered to AAACorp and BBBCorp is that the difference between the two fixed rates is greater than the difference between the two floating rates.
 - ▶ BBBCorp pays 1.2% more than AAACorp in fixed-rate markets and only 0.7% more than AAACorp in floating-rate markets.
 - ▶ BBBCorp appears to have a comparative advantage in floating-rate markets, whereas AAACorp appears to have a comparative advantage in fixed-rate markets.
 - ★ This does not imply that BBBCorp pays less than AAACorp in this market. It means that the extra amount that BBBCorp pays over the amount paid by AAACorp is less in this market.

The Comparative Advantage Argument

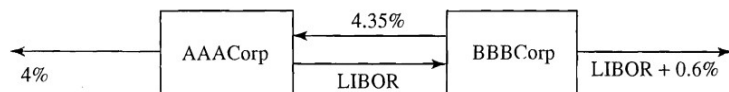
- Suppose that two companies, AAACorp and BBBCorp, both wish to borrow \$10 million for 5 years.
- AAACorp has a AAA credit rating; BBBCorp has a BBB credit rating.
 - ▶ Because it has a worse credit rating than AAACorp, BBBCorp pays a higher rate of interest than AAACorp in both fixed and floating markets.
- Assume they have been offered the following rates:

	<i>Fixed</i>	<i>Floating</i>
AAACorp	4.0%	6-month LIBOR – 0.1%
BBBCorp	5.2%	6-month LIBOR + 0.6%

The Comparative Advantage Argument

- We assume that BBBCorp wants to borrow at a fixed rate of interest, whereas AAACorp wants to borrow at a floating rate of interest linked to 6-month LIBOR.
 - ▶ AAACorp borrows fixed-rate funds at 4% per annum.
 - ▶ BBBCorp borrows floating-rate funds at LIBOR plus 0.6% per annum.
 - ▶ They then enter into a swap agreement to ensure that AAACorp ends up with floating-rate funds and BBBCorp ends up with fixed-rate funds:
 - ★ AAACorp agrees to pay BBBCorp interest at 6-month LIBOR.
 - ★ BBBCorp agrees to pay AAACorp interest at a fixed rate of 4.35% per annum.

The Comparative Advantage Argument



- The net effect is that AAACorp pays LIBOR minus 0.35% per annum.
 - ▶ This is 0.25% per annum less than it would pay if it went directly to floating-rate markets.
- The net effect is that BBBCorp pays 4.95% per annum.
 - ▶ This is 0.25% per annum less than it would pay if it went directly to fixed-rate markets.

The Comparative Advantage Argument

- Why is the difference between the two fixed rates greater than the difference between the two floating rates?
 - ▶ The 4.0% and 5.2% rates available to AAACorp and BBBCorp in fixed rate markets are 5-year rates.
 - ▶ Whereas the LIBOR-0.1% and LIBOR+0.6% rates available in the floating rate market are six-month rates.
 - ★ The lender usually has the opportunity to review the floating rates every 6 months.
 - ▶ The spreads between the rates offered to AAACorp and BBBCorp are a reflection of the extent to which BBBCorp is more likely than AAACorp to default.
 - ★ During the next 6 months, there is very little chance that either AAACorp or BBBCorp will default.
 - ★ As we look further ahead, the probability of a default by a company with a relatively low credit rating (such as BBBCorp) is liable to increase faster than the probability of a default by a company with a relatively high credit rating (such as AAACorp).

The Comparative Advantage Argument

- In this example, the swap has been structured so that the net gain to both sides is the same, 0.25%.
- This need not be the case. However, the total apparent gain from this type of interest rate swap arrangement is always

$$a - b$$

where a is the difference between the interest rates facing the two companies in fixed-rate markets, and b is the difference between the interest rates facing the two companies in floating-rate markets.

- In this case, $a = 1.2\%$ and $b = 0.7\%$, so that the total gain is 0.5%.

Exercise (1)

Companies A and B have been offered the following rates per annum on a \$20 million five- year loan:

	Fixed Rate	Floating Rate
Company A	5.0%	LIBOR+0.1%
Company B	6.4%	LIBOR+0.6%

Company A requires a floating-rate loan; company B requires a fixed-rate loan.

Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

Solution (1)

A has an apparent comparative advantage in fixed-rate markets but wants to borrow floating-rate.

B has an apparent comparative advantage in floating-rate markets but wants to borrow fixed-rate. This provides the basis for the swap.

Solution (1)

Company A borrows at the market .

Company B borrows at the market .

Company A pays at company B (via the Bank).

Company B pays at the Bank and the Bank pays at company A.

Solution (1)

A's net profit writes as: (conditions in the market without SWAP) - (conditions provided by Swap), that is

B's net profit writes as: (conditions in the market without SWAP) - (conditions provided by Swap), that is

To be equally attractive to both companies, x has to satisfy

Solution (1)

The figure becomes:

This means that it should lead to A borrowing at and to B borrowing at .

Solution (1)

We can also find this result by directly notice that there is a (resp.) per annum differential between the fixed (resp. floating) rates offered to the two companies.

The total gain to all parties from the swap is therefore per annum.

Because the bank gets 0.1% per annum of this gain, the swap should make each of A and B per annum better off.

This means that it should lead to A borrowing at and to B borrowing at .

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Using Swap Rates to Bootstrap the LIBOR/Swap Zero Curve

- Swap rates define par yield bonds that can be used to bootstrap the LIBOR (also called LIBOR/swap) zero curve.

Example

Suppose that the 6-month, 12-month, and 18-month LIBOR/swap zero rates have been determined as 4%, 4.5%, and 4.8% with continuous compounding and that the 2-year swap rate (for a swap where payments are made semiannually) is 5%.

This 5% swap rate means that a bond with a principal of \$100 and a semiannual coupon of 5% per annum sells for par.

It follows that, if R is the 2-year zero rate, then

$$2.5e^{-0.04 \times 0.5} + 2.5e^{-0.045 \times 1.0} + 2.5e^{-0.048 \times 1.5} + 102.5e^{-2R} = 100.$$

Solving this, we obtain $R = 4.953\%$.

Valuation of an Interest Rate Swap

- An interest rate swap is worth close to zero when it is first initiated.
 - ▶ After it has been in existence for some time, its value may be positive or negative.
- There are two valuation approaches.
 - ▶ the first regards the swap as the difference between two bonds;
 - ▶ the second regards it as a portfolio of FRAs.

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

- Principal payments are not exchanged in an interest rate swap.
 - However, we can assume that principal payments are both received and paid at the end of the swap without changing its value.
- By doing this, we find that, from the point of view of the floating-rate payer, a swap can be regarded as a long position in a fixed-rate bond and a short position in a floating-rate bond, so that

where:

- V_{swap} is the value of the swap;
- B_{fix} is the value of the fixed-rate bond (corresponding to payments that are received); and
- B_{fl} is the value of the floating-rate bond (corresponding to payments that are made).

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

- We already know how to compute B_{fix} .

Question

How to compute B_{fl} ?

- To answer this question we will first answer simpler questions.

Question (1st step)

Suppose the forward EURIBOR rates (with annual compounding) are worth 8% for every year.

What is the value of a **fixed-rate** bond with annual coupon payments of 8% per annum on a notional principal of 100€ that expires in 3 years?

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

- Similarly, from the point of view of the fixed-rate payer, a swap is a long position in a floating-rate bond and a short position in a fixed-rate bond, so that the value of the swap is

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Solution (1st step)

The fixed rate of 8% will lead to an annual coupon of €.

We then have

$$B_{fix} =$$

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Question (2nd step)

Suppose the forward EURIBOR rates (with annual compounding) worth $r\%$ for every years.

What is the value of a **fixed-rate** bond with annual coupon payments of $r\%$ per annum on a notional principal of $L\text{€}$ that expires in T years?

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Solution (2nd step)

The fixed rate of $r\%$ will lead to an annual coupon of €.

We then have

$$B_{fix} =$$

$$=$$

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Solution (2nd step)

with

$$\sum_{t=1}^T \frac{1}{(1+r)^t} =$$

$$=$$

$$=$$

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Solution (2nd step)

so

$$r \times \sum_{t=1}^T \frac{1}{(1+r)^t} =$$

Hence, we obtain

$$B_{fix} =$$

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Question (3rd step)

Suppose the 1-year, 2-year, and 3-year forward EURIBOR rates for 1 year are $r_1\%$, $r_2\%$, and $r_3\%$ (with annual compounding), respectively.

What is the value of a **floating-rate** bond with annual coupon payments per annum of $r_1\%$, $r_2\%$, and $r_3\%$ respectively in 1 year, 2 years and 3 years, on a notional principal of $L\text{€}$ that expires in 3 years?

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Solution (3rd step)

We have

$$B_{fl} =$$

$$=$$

$$=$$

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Remark

Had the rate r_t , with $t \in \{1, 2, 3\}$, denoted the t -period zero rates (i.e., the rates on a zero coupon bond having a maturity in t periods) per period (e.g., t -year zero rates per annum) with discrete compounding, the corresponding pricing would rather write as

$$B_{fl} =$$

(See Chapter 5, slides 43 for the explanation of the difference between a **forward** LIBOR/EURIBOR rate and a zero rate.)

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Remark

When r_t , with $t \in \{1, 2, \dots, T\}$, denote the t -period zero rates (or the LIBOR or EURIBOR rates for T lower than 12 months) the fixed-rate bond price, and the floating-rate bond price, write as

$$B_{fix} =$$

and

$$B_{fl} =$$

with $\bar{c}$ (resp. c_t) denoting the fixed coupon (resp. floating coupon for a maturity of t).

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Question (4th step)

Suppose the t -year, for $t \in \{1, 2, \dots, T\}$, forward EURIBOR rate for 1 year is worth $r_t\%$ (with annual compounding).

What is the value of a **floating-rate** bond with annual coupon payments per annum of $r_t\%$, in year t , on a notional principal of $L\text{€}$ that expires in T years?

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Solution (4th step)

So

$$B_{fl} =$$

=

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Solution (4th step)

We have

$$B_{fl} =$$

=

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Solution (4th step)

So

$$B_{fl} =$$

=

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Solution (4th step)

And so on... We can repeat the process p times ($p < T$) to obtain

$$B_{fl} =$$

which for $p = T - 1$, writes as

$$B_{fl} =$$

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

- The valuation of a floating rate bond does, at first glance, look more complicated than that of its fixed rate counterpart.
- In reality, the valuation is much easier because all future coupons (except the current one) of the floating rate bond will be determined by the same prevailing market rate in the future than the rate used to discount future payoff.
 - ▶ Any move in the interest rate impacts both the coupon and the discount rate in a way that cancels out any change in terms of present value.
- So a floating-rate bond is worth the notional principal immediately after an interest payment.
 - ▶ This is because at this time the bond is a "fair deal" where the borrower pays LIBOR (or EURIBOR) for each subsequent period.

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

- In our previous examples, the only way a fixed-rate bond may not worth its notional principal is because the fixed-rate coupon differs from the discount rates.
- Similarly, the only way a floated-rate bond may not worth its notional principal is because the floating-rate coupons differ from the discount rates.
 - ▶ But this can only happen in the current period because our present value computation discounts any future cash flow at the same rate as one that gives the coupon.

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

- We already know how to compute B_{fix} .
- To value the floating-rate bond, we note that the bond is worth the notional principal immediately after an interest payment.
 - ▶ This is because at this time the bond is a "fair deal" where the borrower pays LIBOR (or EURIBOR) for each subsequent period.
- Suppose that the notional principal is L , the next exchange of payments is c at time $\tilde{t}$.
 - ▶ Immediately before the payment $B_{fl} = L + \tilde{c}$.
 - ▶ Immediately after the payment $B_{fl} = L$.
 - ▶ The floating-rate bond can therefore be regarded as an instrument providing a single cash flow of $L + \tilde{c}$ at time $\tilde{t}$.
 - ▶ Discounting this, the value of the floating-rate bond today is

$$(L + \tilde{c}) e^{-\tilde{r} \times \tilde{t}}$$

where $\tilde{r}$ is the LIBOR/swap zero rate for a maturity of $\tilde{t}$.

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Remark

When r_t , with $t \in \{k_1, k_2, \dots, k_T\}$, denote the t -period zero rates the fixed-rate bond price, and the floating-rate bond price, write as

$$B_{fix} =$$

and

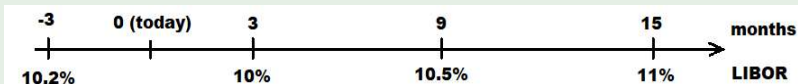
$$B_{fl} =$$

with $\bar{c}$ (resp. c_t) denoting the fixed coupon (resp. floating coupon for a maturity of t).

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Example



Observe that the compounding is semiannual for the fixed rate and continuous for 3-month, 9-month, and 15-month LIBOR rates. So we have

$$B_{fix} = \left(\sum_{t \in \{\frac{3}{12}, \frac{9}{12}, \frac{15}{12}\}} \bar{c} e^{-t \times r_t} \right) + L e^{-\frac{15}{12} \times r_{15/12}}$$

with $\bar{c} = 100 \times \frac{8\%}{2} = \$4.0$ million; $r_{3/12} = 0.1$, $r_{9/12} = 0.105$, and $r_{15/12} = 0.11$; and $L = \$100$ million.

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Example

Suppose that a financial institution has agreed to pay 6-month LIBOR and receive 8% per annum (with semiannual compounding) on a notional principal of \$100 million.

The swap has a remaining life of 1.25 years.

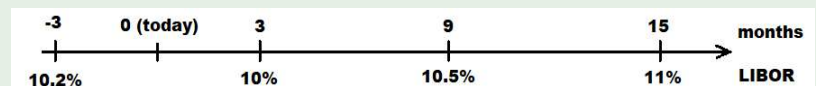
The LIBOR rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively.

The 6-month LIBOR rate at the last payment date was 10.2% (with semiannual compounding).

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Example



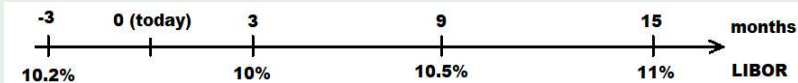
So

$$B_{fix} = 4e^{-0.25 \times 0.1} + 4e^{-0.75 \times 0.105} + 104e^{-1.25 \times 0.11} = 98.238$$

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Example



And

$$B_{fl} = (L + \tilde{c}) e^{-\tilde{r}\tilde{t}}$$

with $L = 100$, $\tilde{c} = 100 \times \frac{10.2\%}{2}$, $\tilde{r} = 10\%$, and $\tilde{t} = 0.25$.

So

$$B_{fl} = 100 \times \left(1 + \frac{0.102}{2}\right) \times e^{-0.25 \times 0.1} = 102.505$$

Valuation of an Interest Rate Swap

Valuation in Terms of Bond Prices

Example

Hence,

$$\begin{aligned} V_{swap} &= B_{fix} - B_{fl} \\ &= 98.238 - 102.505 = -4.267 \end{aligned}$$

or $-\$4,267$ million.

If the financial institution had been in the opposite position of paying fixed and receiving floating, the value of the swap would be $+\$4,267$ million.

Valuation of an Interest Rate Swap

Valuation in Terms of FRAs

- A swap can be characterized as a portfolio of FRAs.
 - The exchanges of payments can be regarded as FRAs.
- For a financial institution paying floating and receiving fixed, we have

$$V_{swap} = \sum_{t=k_1}^{k_T} \frac{\tilde{c} - L \times FRA_{t-1}}{(1 + r_t)^t}$$

Valuation of an Interest Rate Swap

Valuation in Terms of FRAs

Example

Consider the previous example. We have

$$V_{swap} = \sum_{t \in \{\frac{3}{12}, \frac{9}{12}, \frac{15}{12}\}} \left(\tilde{c} - L \times FRA_{t-\frac{6}{12}} \right) e^{-r_t \times t}$$

with $\tilde{c} = 100 \times \frac{8\%}{2} = \$4.0$ million; $r_{3/12} = 0.1$, $r_{9/12} = 0.105$, and $r_{15/12} = 0.11$; $L = \$100$ million; (...)

Valuation of an Interest Rate Swap

Valuation in Terms of FRAs

Example

(...) and $FRA_{-3/12} = \frac{10.2\%}{2}$, and the semiannual compounded forward rate $FRA_{3/12}$ satisfies

$$\left(1 + \frac{FRA_{3/12}}{2}\right) = e^{FRA_{3/12}^{count} \times \frac{6}{12}}$$

where $FRA_{3/12}^{count}$ denotes the continuously compounded forward rate corresponding to the period between 3 and 9 months with

$$FRA_{3/12}^{count} = \frac{0.105 \times 0.75 - 0.10 \times 0.25}{0.5} = 10.75\%. \text{ So}$$

$$FRA_{3/12} = (e^{0.1075 \times 0.5} - 1) \times 2 = 11.044\%$$

and similarly for $FRA_{9/12}$...

Valuation of an Interest Rate Swap

Valuation in Terms of FRAs

Example

Consider the previous example. We then obtain the following table

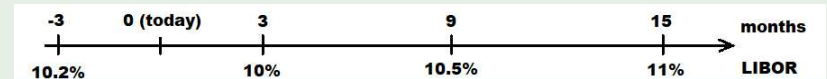


Table 7.6 Valuing swap in terms of FRAs (\$ millions). Floating cash flows are calculated by assuming that forward rates will be realized.

Time	Fixed cash flow	Floating cash flow	Net cash flow	Discount factor	Present value of net cash flow
0.25	4.0	-5.100	-1.100	0.9753	-1.073
0.75	4.0	-5.522	-1.522	0.9243	-1.407
1.25	4.0	-6.051	-2.051	0.8715	-1.787
Total:					-4.267

Column 2: The fixed rate of 8% will lead to a cash inflow of $100 \times \frac{8\%}{2} = \$4.0$ million.

Valuation of an Interest Rate Swap

Valuation in Terms of FRAs

Example



Table 7.6 Valuing swap in terms of FRAs (\$ millions). Floating cash flows are calculated by assuming that forward rates will be realized.

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1.25	4.0	-6.051	-2.051	0.8715	-1.787
Total:					-4.267

Column 3, Row 1: The floating rate of 10.2% (which was set 3 months ago) will lead to a cash outflow of $100 \times \frac{10.2\%}{2} = \$5.1$ million.

Valuation of an Interest Rate Swap

Valuation in Terms of FRAs

Example

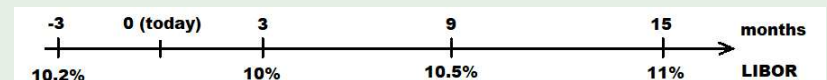


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1.25	4.0	-6.051	-2.051	0.8715	-1.787
Total:					-4.267

Column 3, Row 2: The forward rate corresponding to the period between 3 and 9 months is $\frac{0.105 \times 0.75 - 0.10 \times 0.25}{0.5} = 10.75\%$ with continuous compounding.

Valuation of an Interest Rate Swap

Valuation in Terms of FRAs

Example

Table 7.6 Valuing swap in terms of FRAs (\$ millions). Floating cash flows are calculated by assuming that forward rates will be realized.

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Total:					-4.267

The corresponding semiannual compounding forward rate R is 11.044% (i.e., $e^{0.1075 \times 0.5} = (1 + \frac{R}{2}) \iff R = (e^{0.1075 \times 0.5} - 1) \times 2$).

Valuation of an Interest Rate Swap

Valuation in Terms of FRAs

Example

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1.25	4.0	-6.051	-2.051	0.8715	-1.787
Total:					-4.267

The cash outflow is therefore $100 \times \frac{11.044\%}{2} = \$5.522$ million. The same logic applies to Column 3, Row 3 ...

Valuation of an Interest Rate Swap

Valuation in Terms of FRAs

Example

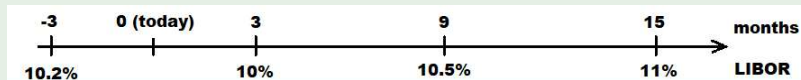


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1.25	4.0	-6.051	-2.051	0.8715	-1.787
Total:					-4.267

The discount factors for the three payment dates are, respectively

$$e^{-0.1 \times 0.25} = 0.9753 \quad e^{-0.105 \times 0.75} = 0.9243 \quad e^{-0.11 \times 1.25} = 0.8715$$

Valuation of an Interest Rate Swap

Exercise on each method

Exercise (3)

A \$100 million interest rate swap has a remaining life of 10 months.

Under the terms of the swap, six-month LIBOR is exchanged for 7% per annum (compounded semiannually).

The average of the bid-offer rate being exchanged for six-month LIBOR in swaps of all maturities is currently 5% per annum with continuous compounding.

The six-month LIBOR rate was 4.6% per annum two months ago.

What is the current value of the swap to the party paying floating?

What is its value to the party paying fixed?

Solve the exercise using the method based on bonds prices.

Valuation of an Interest Rate Swap

Exercise on each method

Solution (3)

First method (viewing the swap as a portfolio of bonds).

In four months will be
received and will be paid.
(We ignore day count issues.)

In 10 months will be received, and the
will be paid.

The value of the fixed-rate bond underlying the swap is

$$B_{fix} =$$

Valuation of an Interest Rate Swap

Exercise on each method

Solution (3)

First method (viewing the swap as a portfolio of bonds).

The value of the floating-rate bond underlying the swap is

$$B_{float} =$$

The value of the swap to the party paying floating is

The value of the swap to the party paying fixed is

Valuation of an Interest Rate Swap

Exercise on each method

Exercise (3, bis)

Solve again Exercise 3 but now using the method based on forward contracts.

Solution (3)

Second method (viewing the swap as a portfolio of forward contracts).

These results can also be derived by decomposing the swap into forward contracts.

Consider the party paying floating.

The first forward contract involves paying

It has a value of

To value the second forward contract, we note that the forward interest rate is per
annum with semiannual compounding.

The value of the forward contract is

The total value of the forward contracts is therefore

Chapter 7: Swaps

Outline

- 1 Motivation
- 2 Interest Rate Swaps
 - Market
 - Mechanics
 - The Comparative Advantage Argument
 - Using Swap Rates to Bootstrap the LIBOR/Swap Zero Curve
 - Valuation
- 3 **Overnight Indexed Swaps**
- 4 Foreign Exchange Swaps
- 5 Currency Swaps
- 6 Credit Risk
- 7 Summary

Overnight Indexed Swaps

Definition

An **overnight indexed swap (OIS)** is a swap where a fixed rate for a period (e.g., 1 month, 3 months, 1 year, or 2 years) is exchanged for the geometric average of the overnight rates over every day of the payment period.

The fixed rate in an OIS is referred to as the **overnight indexed swap rate**.

- An OIS therefore allows overnight borrowing or lending to be swapped for borrowing or lending at a fixed rate.
 - ▶ The floating leg is given by an overnight rate index, such as the overnight federal funds rate.
 - ▶ The fixed leg is agreed among parties.

Overnight Indexed Swaps

- The **overnight market** is the component of the money market involving the shortest term loan: only one night (or one day).
- The **overnight rate** is the interest rate prevailing on the overnight market.
 - ▶ It is the lowest rate at which banks lend money.
 - ▶ In many countries, it is the interest rate the central bank sets to target monetary policy.
- If during a certain period a bank borrows (or lends) funds at the overnight rate (rolling the loan forward each night), then its effective interest rate is the geometric average of the overnight interest rates.

Overnight Indexed Swaps

- A bank (Bank A) can engage in the following transactions:
 - ▶ 1. Borrow \$100 million in the overnight market for 3 months, rolling the loan forward each night;
 - ▶ 2. Lend the \$100 million for 3 months at LIBOR to another bank (Bank B); and
 - ▶ 3. Enter into an OIS to convert the overnight rates into the 3-month OIS rate.
- This will lead to Bank A receiving the 3-month LIBOR rate (from Bank B) and paying the 3-month overnight indexed swap rate (OIS).

Overnight Indexed Swaps

- We might expect the 3-month OIS rate to equal the 3-month LIBOR rate.
 - ▶ However, it is generally lower because Bank A requires some compensation for the risk it is taking that Bank B will default on the LIBOR loan.
 - ▶ LIBOR is “riskier” than OIS.
 - ★ LIBOR: the lending bank loans cash to the borrowing bank.
 - ★ OIS: both counterparties only swap the floating rate of interest for the fixed rate of interest.

Definition

The **LIBOR–OIS spread** is the difference between LIBOR and the OIS rates.

Overnight Indexed Swaps

- It is an important measure of risk and liquidity in the money market, considered by many to be a strong indicator for the relative stress in the money markets.
- A higher spread (high Libor) is typically interpreted as indication of a decreased willingness to lend by major banks, while a lower spread indicates higher liquidity in the market.
 - ▶ As such, the spread can be viewed as indication of banks' perception of the creditworthiness of other financial institutions and the general availability of funds for lending purposes.

Overnight Indexed Swaps

- The LIBOR–OIS spread is considered to be a measure of health of the banking system.
 - ▶ In normal market conditions, it is about 10 basis points.
 - ★ However, it rose sharply during the 2007-2009 credit crisis because banks became less willing to lend to each other.
 - ★ In October 2008, the spread spiked to an all time high of 364 basis points.



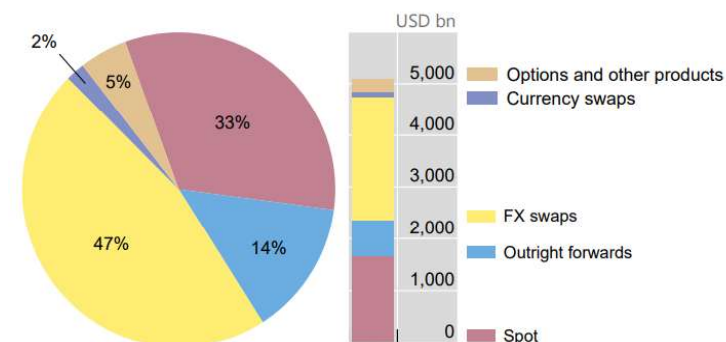
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Spot wrt Forex derivatives

Foreign exchange market turnover by instrument

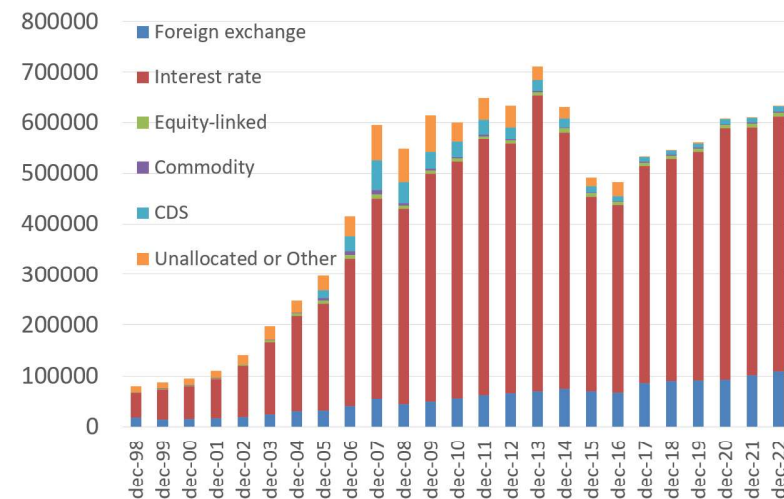
Net-net basis,¹ daily averages in April 2016



¹ Adjusted for local and cross-border inter-dealer double-counting.

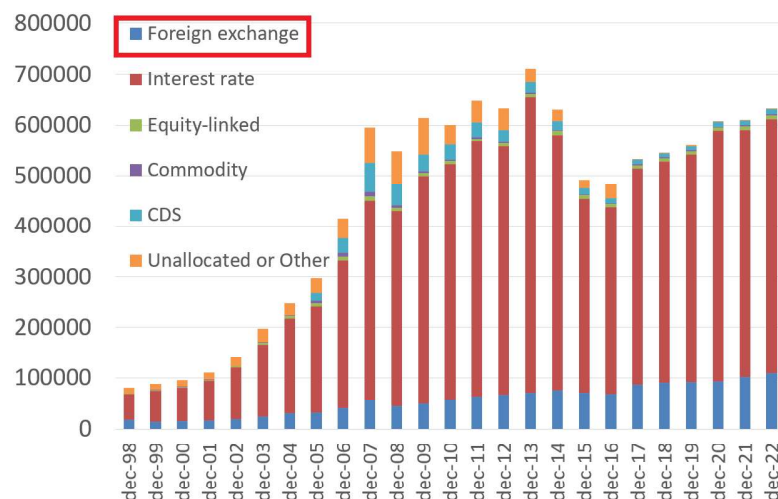
Source: BIS Triennial Central Bank Survey.

Global OTC Derivatives Market Notional amounts outstanding (Billions of US\$)



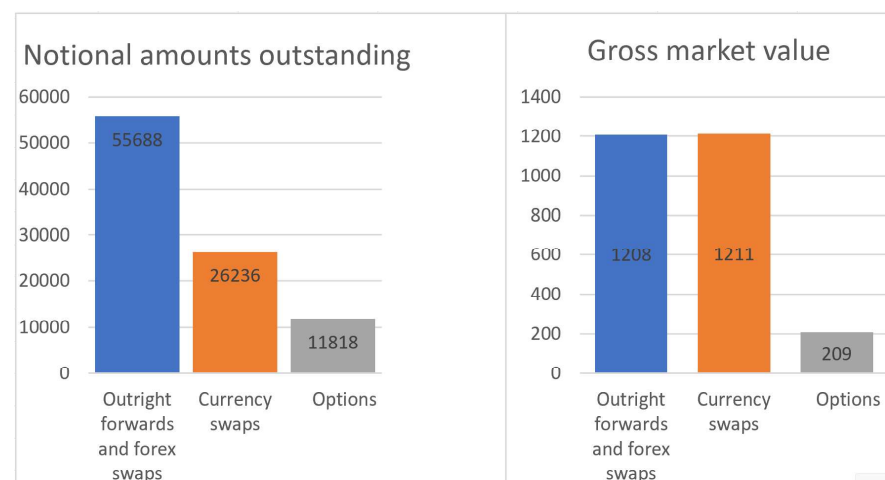
Source: Bank for International Settlements

Global OTC Derivatives Market Notional amounts outstanding (Billions of US\$)



Source: Bank for International Settlements

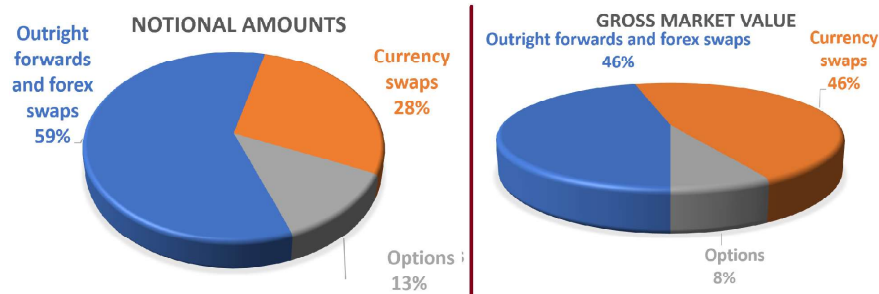
Global OTC Forex derivatives by instrument Notional amounts outstanding and Gross market value (2020, Billions of US\$)



Source: Bank for International Settlements

Global OTC Forex derivatives by instrument

Share by instruments (2020)



Source: Bank for International Settlements

Foreign Exchange Swaps

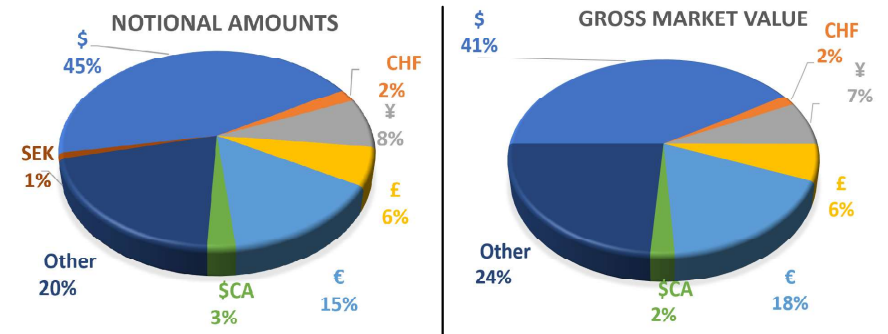
Definition

A **foreign exchange swap**, (aka **forex swap**, or **FX swap**) agreement is a contract in which one party borrows one currency from, and simultaneously lends another to, the second party.

- The amount of repayment is fixed at the FX forward rate as of the start of the contract.
- Thus, FX swaps can be viewed as FX risk-free collateralised borrowing/lending.

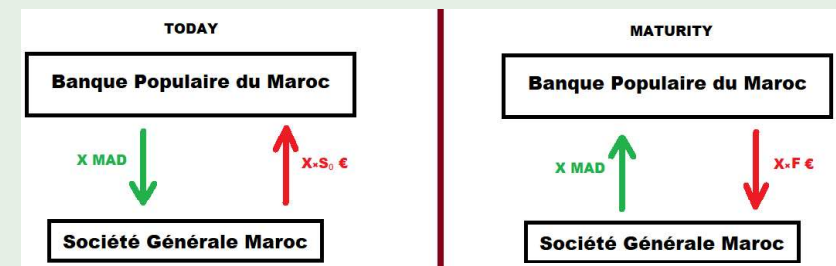
Global OTC Forex derivatives by instrument

Share by currencies (2020)



Foreign Exchange Swaps

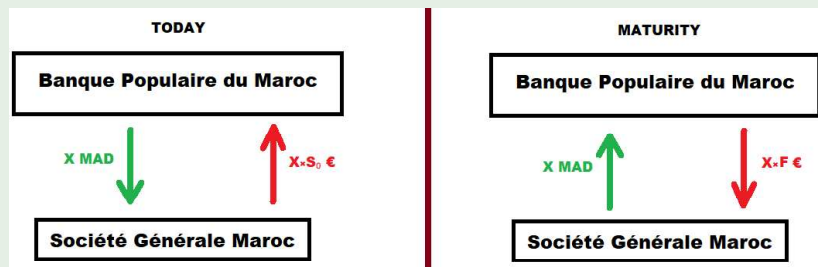
Example



At the start of the contract BPM:

- lends X MAD to SGM; and
 - borrows $X \times S_0$ € from SGM,
- where S_0 is the FX spot rate (1 MAD = S_0 €, e.g., $S_0 = 0.09$).

Example



When the contract expires, BPM:

- receives $X \text{ MAD}$ from SGM; and
 - returns $X \times F \text{ €}$ to SGM,
- where F is the FX forward rate as of the start.

Example

In November 2018, for $T = \frac{1}{52}$, we have $r_1 = r_{BAM_{1w}} = 2,304\%$; $r_2 = EURIBOR_{1w} = -0,379\%$; $S_0 = 0.0906$, so that

$$F_0 = S_0 e^{(r_{BAM_{1w}} - EURIBOR_{1w})T} = 0.0906 e^{(0.02304 + 0.0379) \frac{1}{52}} \simeq 0.0907$$

- As we saw in Chapter 3, we have

$$F_0 = S_0 e^{(r_1 - r_2)T}$$

where:

- ▶ S_0 denotes the current spot price in currency 1 of one unit of the currency 2;
- ▶ F_0 is the forward or futures price in currency 1 of one unit of the currency 2;
- ▶ r_1 is the currency 1 risk-free interest rate;
- ▶ r_2 is the currency 2 risk-free interest rate; and
- ▶ T is the time-to-maturity (aka tenor).

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Currency Swaps

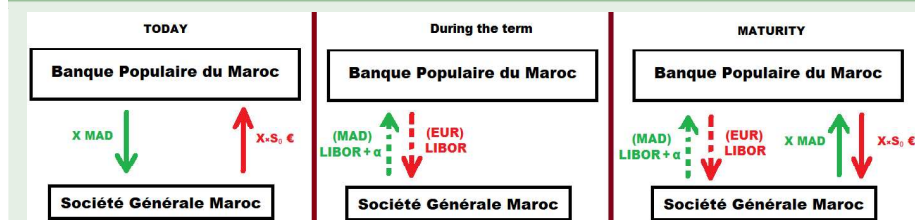
Definition

A **currency swap** (aka **cross-currency basis swap** or **basis swap**) is a swap that consists in exchanging principal and interest payments in one currency for principal and interest payments in another.

- A currency swap agreement requires the principal to be specified in each of the two currencies.
- Though the structure of cross-currency basis swaps differs from FX swaps, the former basically serve the same economic purpose as the latter, except for the exchange of floating rates during the contract term.
- Contrary to an interest rate swap, in a currency swap the principal is usually exchanged (at the beginning and the end of the swap's life).

Currency Swaps

Example

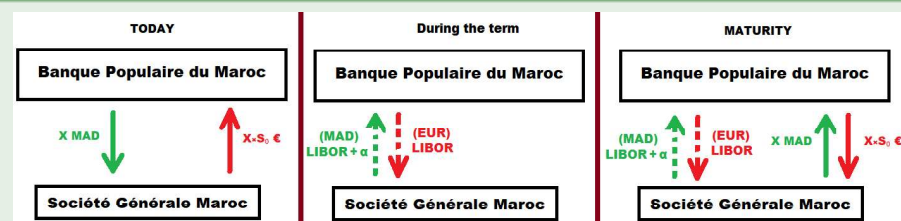


At the start of the contract BPM:

- lends X MAD to SGM; and
 - borrows $X \times S_0$ € from SGM,
- where S_0 is the FX spot rate ($1 \text{ MAD} = S_0 \text{ €}$, e.g., $S_0 = 0.09$).

Currency Swaps

Example

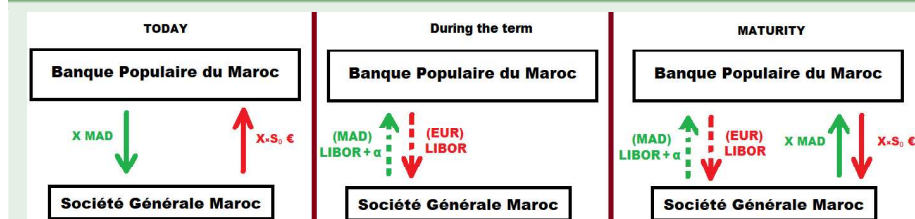


During the contract term BPM:

- pays LIBOR in euros to SGM;
- receives the average interest rate at which a selection of banks in London are prepared to lend in Moroccan Dirham plus $\alpha\%$ from SGM; and
- where α is the price of the basis swap, agreed upon by the counterparties at the start of the contract.

Currency Swaps

Example



When the contract expires, in addition to the interim payments (as defined on the previous slide), BPM:

- receives X MAD from SGM; and
 - returns $X \times S_0$ € to SGM,
- where S_0 is the same FX spot rate as of the start of the contract..

Currency Swaps

Example (Fixed-for-fixed currency swap)

An agreement to pay 5% on a sterling principal of £10,000,000 and receive 6% on a US\$ principal of \$18,000,000 every year for 5 years, starting in February 1, 2018.

Date	Dollar Cash Flows (millions)	Sterling cash flow
Feb 1, 2018	-18.0	+10.0
Feb 1, 2019	+1.08	-0.50
Feb 1, 2020	+1.08	-0.50
Feb 1, 2021	+1.08	-0.50
Feb 1, 2022	+1.08	-0.50
Feb 1, 2023	+19.08	-10.50

Currency Swaps

Typical Uses of a Currency Swap

- Convert a liability in one currency to a liability in another currency.
 - ▶ E.g., the previous swap has the effect of swapping the interest and principal payments from dollars to sterling.
- Convert an investment in one currency to an investment in another currency.
 - ▶ Suppose that IBM can invest £10 million in the UK to yield 5% per annum for the next 5 years, but feels that the US dollar will strengthen against sterling and prefers a US-dollar-denominated investment.
 - ▶ The previous swap has the effect of transforming the UK investment into a \$18 million investment in the US yielding 6%.

Currency Swaps

Comparative Advantage May Be Real Because of Taxes

- One possible source of comparative advantage is tax.

Example

General Electric's position might be such that USD borrowings lead to lower taxes on its worldwide income than AUD (Australian dollars) borrowings. Qantas Airways' position might be the reverse.

- General Electric wants to borrow AUD.
- Qantas wants to borrow USD.
- Cost after adjusting for the differential impact of taxes is

	USD	AUD
General Electric	5.0%	7.6%
Qantas	7.0%	8.0%

Currency Swaps

Comparative Advantage May Be Real Because of Taxes

Example

	USD	AUD
General Electric	5.0%	7.6%
Qantas	7.0%	8.0%

The spreads between the rates paid by General Electric and Qantas Airways in the two markets are not the same.

Qantas Airways pays 2% more than General Electric in the US dollar market and only 0.4% more than General Electric in the AUD market.

General Electric has a comparative advantage in the USD market, whereas Qantas Airways has a comparative advantage in the AUD market.

Both firms can use a currency swap to transform General Electric's loan into an AUD loan and Qantas Airways' loan into a USD loan.

Currency Swaps

Valuation

- Like interest rate swaps, currency swaps can be valued either as:
 - the difference between 2 bonds; or
 - a portfolio of forward contracts.

Currency Swaps

Valuation in Terms of Bond Prices

Example

Suppose all Japanese LIBOR/swap rates are 4%, and all USD LIBOR/swap rates are 9% (both with continuous compounding).

Some time ago a financial institution has entered into a currency swap in which it receives 5% per annum in yen and pays 8% per annum in dollars once a year.

Principals are \$10 million and 1,200 million yen.

Swap will last for 3 more years.

Current exchange rate is 110 yen per dollar.

Currency Swaps

Valuation in Terms of Bond Prices

- If we define V_{swap} as the value in US dollars of an outstanding swap where a domestic currency is received and a foreign currency is paid, then

$$V_{swap} =$$

where:

- B_D is the value of the bond defined by the domestic cash flows on the swap;
- B_F is the value of the bond (measured in the foreign currency) defined by the foreign cash flows on the swap; and
- S_0 is the spot exchange rate (expressed as number of domestic currency per unit of foreign currency).

- Similarly, the value of a swap where the foreign currency is received and domestic currency is paid is

$$V_{swap} =$$

Currency Swaps

Valuation in Terms of Bond Prices

Example

Time	Cash Flows (\$)	PV (\$)	Cash flows (yen)	PV (yen)
1	0.8	0.7311	60	57.65
2	0.8	0.6682	60	55.39
3	0.8	0.6107	60	53.22
3	10.0	7.6338	1,200	1,064.30
Total		9.6439		1,230.55

Column 3: The present values of the US\$ cash flows are $0.8e^{-0.09} = 0.7311$, $0.8e^{-0.09 \times 2} = 0.6682$, ...

Column 4: The present values of the US\$ cash flows are $60e^{-0.04} = 57.65$, $60e^{-0.04 \times 2} = 55.39$, ...

Currency Swaps

Valuation in Terms of Bond Prices

Example

Time	Cash Flows (\$)	PV (\$)	Cash flows (yen)	PV (yen)
1	0.8	0.7311	60	57.65
2	0.8	0.6682	60	55.39
3	0.8	0.6107	60	53.22
3	10.0	7.6338	1,200	1,064.30
Total		9.6439		1,230.55

$$V_{\text{swap}} = S_0 B_F - B_D = \frac{1230.55}{110} - 9.6439 = 1.5430$$

Currency Swaps

Valuation as Portfolio of Forward Contracts

Example

Consider the previous example.

The financial institution then pays $0.08 \times 10 = \$0.8$ million dollars and receives $1,200 \times 0.05 = 60$ million yen each year.

In addition, the dollar principal of \$10 million is paid and the yen principal of 1,200 is received at the end of year 3.

The current spot rate is $\frac{1}{110} = 0.009091$ dollar per yen.

The 1-year forward rate (see, slide 30 Chapter 3) is

$$S_0 e^{(r - r_f) \times 1} = 0.009091 e^{(0.09 - 0.04)} \simeq 0.009557$$

The 2-year forward rate is $0.009091 e^{(0.09 - 0.04) \times 2} \simeq 0.010047$, and the 3-year forward rate is $0.009091 e^{(0.09 - 0.04) \times 3} \simeq 0.010562$.

Currency Swaps

Valuation as Portfolio of Forward Contracts

- Each exchange of payments in a fixed-for-fixed currency swap is a forward foreign exchange contract.
- The forward contracts underlying the swap can be valued by assuming that the forward rates are realized.

Currency Swaps

Valuation as Portfolio of Forward Contracts

Example

If the 1-year forward rate is realized, the yen cash flow in year 1 is worth $60 \times 0.009557 = 0.5734$ million dollars and the net cash flow at the end of year 1 is $0.5734 - 0.8 = -0.2266$ million dollars.

This has a present value of

$$-0.2266 e^{-0.09 \times 1} = -0.2071$$

million dollars.

This is the value of forward contract corresponding to the exchange of cash flows at the end of year 1.

Currency Swaps

Valuation as Portfolio of Forward Contracts

Example

The value of the other forward contracts are calculated similarly. E.g., the yen cash flow in year 2 is worth $60 \times 0.010047 = 0.60282$ million dollars and the net cash flow at the end of year 2 is $0.60282 - 0.8 = -0.19718$ million dollars.

We then obtain the following table

Table 7.10 Valuation of currency swap as a portfolio of forward contracts. (All amounts in millions.)

Time	Dollar cash flow	Yen cash flow	Forward exchange rate	Dollar value of yen cash flow	Net cash flow (\$)	Present value
1	-0.8	60	0.009557	0.5734	-0.2266	-0.2071
2	-0.8	60	0.010047	0.6028	-0.1972	-0.1647
3	-0.8	60	0.010562	0.6337	-0.1663	-0.1269
3	-10.0	1200	0.010562	12.6746	+2.6746	2.0417
Total:						1.5430

Currency Swaps

Valuation as Portfolio of Forward Contracts

Example

Table 7.10 Valuation of currency swap as a portfolio of forward contracts. (All amounts in millions.)

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3	-10.0	1200	0.010562	12.6746	+2.6746	2.0417
Total:						1.5430

The total value of the forward contracts is then \$1.5430 million. This agrees with the previous value calculated for the swap by decomposing it into bonds.

Currency Swaps

Swaps & Forwards

- A swap can be regarded as a convenient way of packaging forward contracts.
- Although the swap contract is usually worth close to zero at the beginning, each of the underlying forward contracts are not worth zero.

Chapter 7: Swaps

Outline

- 1 Motivation
- 2 Interest Rate Swaps
 - Market
 - Mechanics
 - The Comparative Advantage Argument
 - Using Swap Rates to Bootstrap the LIBOR/Swap Zero Curve
 - Valuation
- 3 Overnight Indexed Swaps
- 4 Foreign Exchange Swaps
- 5 Currency Swaps
- 6 Credit Risk
- 7 Summary

- A swap is worth zero to a company initially.
- At a future time its value is liable to be either positive or negative.
- Consider a financial institution that has entered into offsetting contracts with two companies.
 - ▶ If neither party defaults, the financial institution remains fully hedged.
 - ▶ A decline in the value of one contract will always be offset by an increase in the value of the other contract.
 - ▶ However, there is a chance that one party will get into financial difficulties and default.
 - ▶ The financial institution then still has to honor the contract it has with the other party.

Chapter 7: Swaps Outline

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- 7 Summary

- A financial institution clearly has credit-risk exposure from a swap when the value of the swap to the financial institution is positive.
 - ▶ When the value of the swap to the financial institution is negative it is likely that the counterparty that goes bankrupt choose to sell the contract to a third party or rearrange its affairs in some way so that its positive value in the contract is not lost.
- Potential losses from defaults on a swap are much less than the potential losses from defaults on a loan with the same principal.
 - ▶ This is because the value of the swap is usually only a small fraction of the value of the loan.
- Potential losses from defaults on a currency swap are greater than on an interest rate swap.
 - ▶ The reason is that, because principal amounts in two different currencies are exchanged at the end of the life of a currency swap, a currency swap is liable to have a greater value at the time of a default than an interest rate swap.

Summary

- A **swap** is a derivative contract where two parties exchange financial instruments.
 - ▶ Most swaps are derivatives in which two counterparties exchange cash flows of one party's financial instrument for those of the other party's financial instrument.
- Contrary to a future, a forward or an option, the notional amount is usually not exchanged between counterparties.

Summary

- An **interest rate swap** can be used to transform a floating-rate loan into a fixed-rate loan, or vice versa.
 - ▶ It can also be used to transform a floating-rate investment to a fixed-rate investment, or vice versa.
- The most popular types of swaps are **plain vanilla interest rate swaps**.
 - ▶ They allow two parties to exchange fixed and floating cash flows on an interest-bearing investment or loan.
 - ▶ One party agrees to pay the other party interest at a fixed rate on a notional principal for a number of years.
 - ★ In return, it receives interest at a floating rate on the same notional principal for the same period of time.
 - ▶ Principal amounts are not usually exchanged in interest rate swaps.

Summary

- A **foreign exchange swap** is a contract between two parties that simultaneously agrees to buy (or sell) a specific amount of a currency at an agreed on rate, and to sell (or buy) the same amount of currency at a later date at an agreed on rate.
 - ▶ There are two legs in a FX swap transaction.
 - ★ In the first leg of the swap, a specific amount of a currency is bought (or sold) against another currency at the prevailing spot rate.
 - ★ In the second leg of the transaction, an equal amount of currency is sold (or bought) against the other currency at the forward rate.

Summary

- An **overnight index swap** is an interest rate swap involving the overnight rate being exchanged for a fixed interest rate.
 - ▶ An overnight index swap uses an overnight rate index, such as the overnight federal funds rate, as the underlying rate for its floating leg, while the fixed leg would be set at an assumed rate.
- Overnight index swaps are popular among financial institutions because the overnight index is considered to be a good indicator of the interbank credit markets and less risky than other traditional interest rate spreads.

Summary

- A **currency swap** can be used to transform a loan in one currency into a loan in another currency.
 - ▶ It can also be used to transform an investment denominated in one currency into an investment denominated in another currency.
- In a currency swap, one party agrees to pay interest on a principal amount in one currency.
 - ▶ In return, it receives interest on a principal amount in another currency.
 - ▶ Principal amounts are usually exchanged at both the beginning and the end of the life of the swap.
 - ★ For a party paying interest in the foreign currency, the foreign principal is received, and the domestic principal is paid at the beginning of the swap's life.
 - ★ At the end of the swap's life, the foreign principal is paid and the domestic principal is received.

- **Foreign exchange swaps** and **currency swaps** are very similar to one another as they aid in hedging foreign exchange risk and offer corporations a mechanism in which foreign exchange can be obtained with minimal exposure to exchange rate risk.
- Nevertheless, these two derivatives are different to one another in that:
 - ▶ FX swap involves only two transactions (sell or purchase at the spot rate, and repurchase or resell at forward rate);
 - ▶ Currency swap exchanges a series of cash flows (interest payments and principles).

- There are two ways of valuing swaps.
 - ▶ In the first, the swap is decomposed into a long position in one bond and a short position in another bond.
 - ▶ In the second it is regarded as a portfolio of forward contracts.
- When a financial institution enters into a pair of offsetting swaps with different counterparties, it is exposed to credit risk.
 - ▶ If one of the counterparties defaults when the financial institution has positive value in its swap with that counterparty, the financial institution loses money because it still has to honor its swap agreement with the other counterparty.