Introduction to Derivative Instruments

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Slides on book: John C. Hull, “Options, Futures, and Other Derivatives”, Pearson ed.

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Chapter 10

Chapter 10: Trading Strategies Involving Options
Outline

1 Introduction
2 Principal Protected Note
3 Positions in an Option and the Underlying Asset
4 Bull, Bear, Box, Butterfly, and Calendar Spreads
5 Combination
6 Summary

Introduction
Motivation

- We shall look at what can be achieved when an option is traded in conjunction with other assets.
- In particular, we examine the properties of portfolios consisting of positions in
  - (a) an option and a zero-coupon bond;
  - (b) an option and the asset underlying the option; and
  - (c) two or more options on the same underlying asset.

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Principal Protected Note

- Options are often used to create what are termed **principal-protected notes** for the retail market.
  - These are products that appeal to conservative investors.
  - The return earned by the investor depends on the performance of a stock, a stock index, or other risky asset, but the initial principal amount invested is not at risk.

Example

Suppose that the 3-year interest rate is 6% with continuous compounding. This means that

$$\$1,000e^{-0.06\times 3} = \$835.27$$

will grow to \$1,000 in 3 years.

The difference between \$1,000 and \$835.27 is \$164.73.

Principal Protected Note

Example

Suppose that a stock portfolio is worth \$1,000 and a 3-year at-the-money European call option on this stock portfolio can be purchased for less than \$164.73.

A bank can offer clients a \$1,000 investment opportunity consisting of:

1. A 3-year zero-coupon bond with a principal of \$1,000; and
2. A 3-year at-the-money (i.e., strike=\$1,000) European call option on the stock portfolio.

In any case, the bond gives back the original \$1,000 principal invested.

If the value of the porfolio increases above \$1,000, the call is exercised to purchase the stock portfolio at the strike price of \$1,000.

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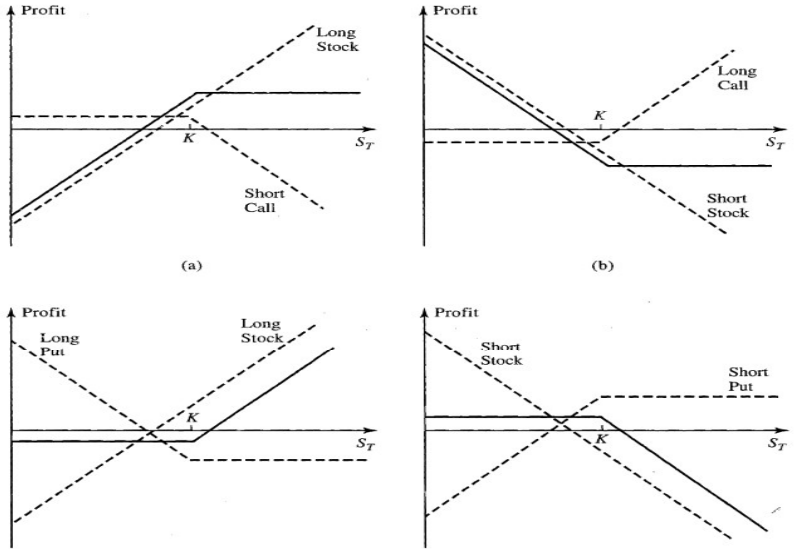
Positions in an Option and the Underlying Asset



Positions in an Option and the Underlying Asset

- All the previous relationships come from the Call-Put parity:

$$p + S_0 = c + \left(Ke^{-rt} + D\right)$$

- This equation says that a long position in a European put (p) combined with a long position in the underlying asset (S_0) is equivalent to a long position in a European call (c) plus a certain amount of cash $\left(Ke^{-rt} + D\right)$.
- This is depicted in the bottom-left previous figure.
- Similarly, all previous diagrams can be obtained by arranging the terms of the Call-Put parity.
  - E.g., top-right previous figure comes from

$$c - S_0 = p - \left(Ke^{-rt} + D\right)$$

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Bull, Bear, Box, Butterfly, and Calendar Spreads
Bull Spread Using Calls

- A **spread trading strategy** involves taking a position in two or more options of the same type (i.e., two or more calls or two or more puts).
- One of the most popular types of spreads is a **bull spread**.
  - This limits the investor's upside as well as downside risk.
  - An investor who enters into a bull spread is hoping that the stock price will increase.
    - The strike price of the option purchased is always less than the strike price of the option sold.
    - Both options have the same expiration.

Bull, Bear, Box, Butterfly, and Calendar Spreads
Bull Spread Using Calls

- This can be created by buying a European call option on a stock with a certain strike price (K_1) and selling a European call option on the same stock with a higher strike price ($K_2 > K_1$).
  - The strategy is illustrated in the Figure

Figure 11.2 Profit from bull spread created using call options.

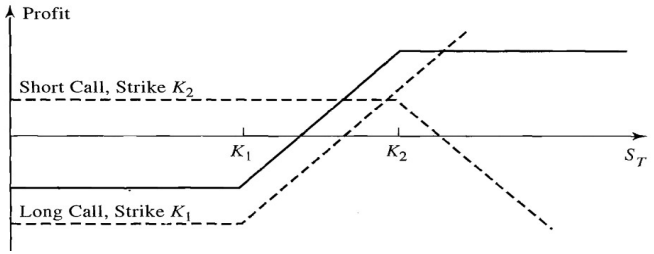
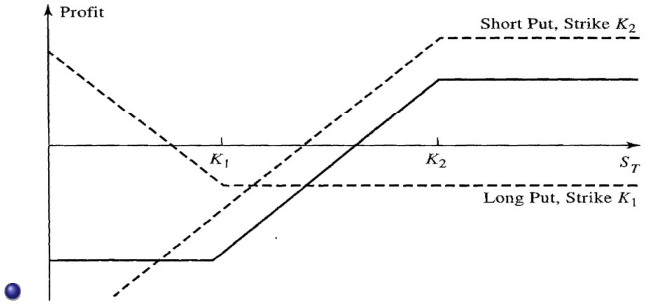


Table 11.1 Payoff from a bull spread created using calls.

Stock price range	*Payoff from long call option*	*Payoff from short call option*	*Total payoff*
$S_T \leqslant K_1$	0	0	0
$K_1 < S_T < K_2$	$S_T - K_1$	0	$S_T - K_1$
$S_T \geqslant K_2$	$S_T - K_1$	$-(S_T - K_2)$	$K_2 - K_1$

- Bull spreads can also be created by buying a European put with a low strike price (K_1) and selling a European put with a high strike price ($K_2 > K_1$).

Figure 11.3 Profit from bull spread created using put options.



- By contrast to Bull spread, an investor who enters into a **Bear spread** is hoping that the stock price will decline.
  - Like bull spreads, bear spreads limit both the upside profit potential and the downside risk.
  - Bear spreads can be created by buying a European put with one strike price and selling a European put with another strike price.
  - The strike price of the option purchased is greater than the strike price of the option sold.

Figure 11.4 Profit from bear spread created using put options.

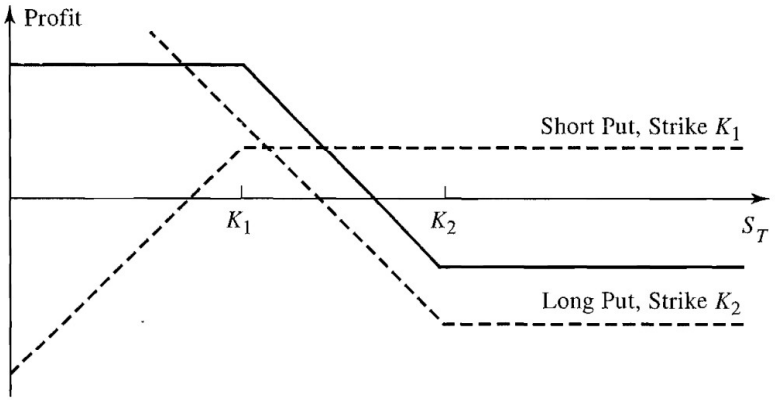
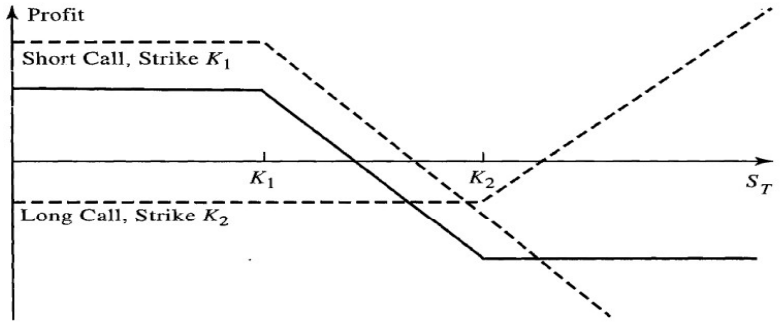
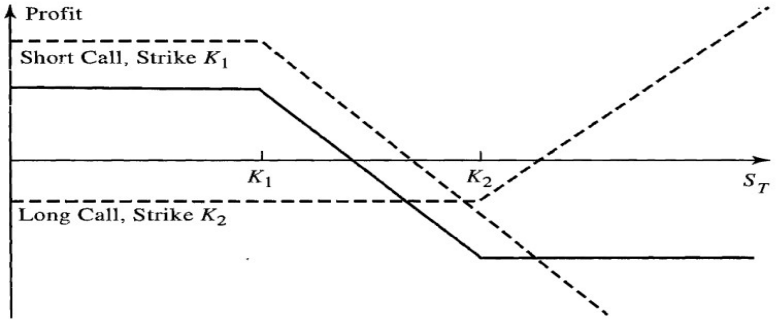


Figure 11.5 Profit from bear spread created using call options.



- Bear spreads can be created using calls instead of puts.
- The investor buys a call with a high strike price and sells a call with a low strike price.

Figure 11.5 Profit from bear spread created using call options.



Exercise (3)

Suppose that put options on a stock with strike prices \$30 and \$35 cost \$4 and \$7, respectively.

How can the options be used to create (a) a bull spread and (b) a bear spread?
Construct a table that shows the profit and payoff for both spreads.

Solution (3)

Solution (3)

- A **Box spread** is a combination of a bull call spread with strike prices K_1 and K_2 and a bear put spread with the same two strike prices.
- If all options are European a box spread is worth the present value of the difference between the strike prices $(K_2 - K_1)e^{-rT}$.
  - If they are American this is not necessarily so a box-spread arbitrage only works with European options.

Table 11.3 Payoff from a box spread.

Stock price range	*Payoff from bull call spread*	*Payoff from bear put spread*	*Total payoff*
$S_T \leqslant K_1$	0	$K_2 - K_1$	$K_2 - K_1$
$K_1 < S_T < K_2$	$S_T - K_1$	$K_2 - S_T$	$K_2 - K_1$
$S_T \geqslant K_2$	$K_2 - K_1$	0	$K_2 - K_1$

- A **Butterfly spread** involves positions in options with three different strike prices.
  - It can be created by buying a European call option with a relatively low strike price K_1 buying a European call option with a relatively high strike price K_3, and selling two European call options with a strike price K_2 that is halfway between K_1 and K_3.
  - Generally, K_2 is close to the current stock price.
  - A butterfly spread leads to a profit if the stock price stays close to K_2, but gives rise to a small loss if there is a significant stock price move in either direction.
    - ⋆ It is therefore an appropriate strategy for an investor who feels that large stock price moves are unlikely.

Figure 11.6 Profit from butterfly spread using call options.

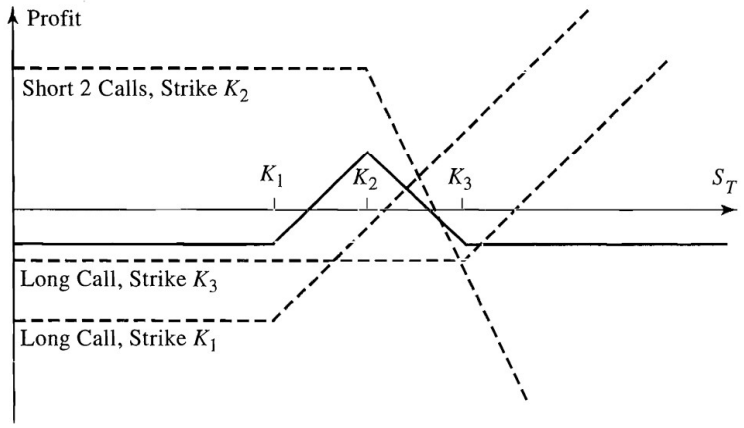


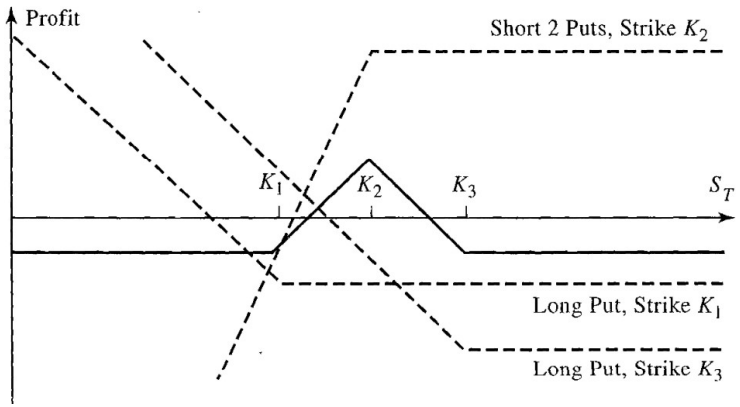
Table 11.4 Payoff from a butterfly spread.

Stock price range	*Payoff from first long call*	*Payoff from second long call*	*Payoff from short calls*	*Total payoff**
$S_T \leqslant K_1$	0	0	0	0
$K_1 < S_T \leqslant K_2$	$S_T - K_1$	0	0	$S_T - K_1$
$K_2 < S_T < K_3$	$S_T - K_1$	0	$-2(S_T - K_2)$	$K_3 - S_T$
$S_T \geqslant K_3$	$S_T - K_1$	$S_T - K_3$	$-2(S_T - K_2)$	0

* These payoffs are calculated using the relationship $K_2 = 0.5(K_1 + K_3)$.

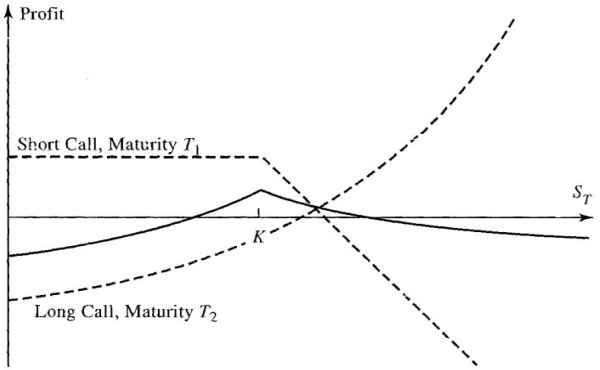
- A butterfly spread can be sold or shorted by following the reverse strategy.
  - Options are sold with strike prices of K_1 and K_3, and two options with the middle strike price K_2 are purchased.
  - This strategy produces a modest profit if there is a significant movement in the stock price.

Figure 11.7 Profit from butterfly spread using put options.



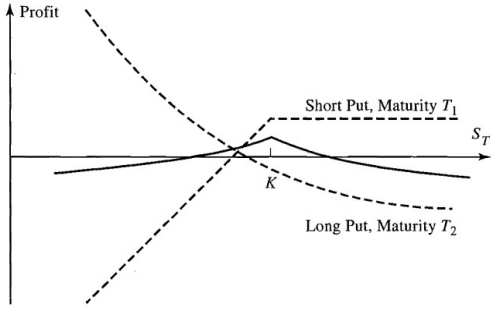
- In a **calendar spreads** the options have the same strike price and different expiration dates.
  - A calendar spread can be created by selling a European call option with a certain strike price and buying a longer-maturity Eurpean call option with the same strike price.
  - The longer the maturity of an option, the more expensive it usually is.
- Profit diagrams for calendar spreads are usually produced so that they show the profit when the short-maturity option expires on the assumption that the long-maturity option is closed out at that time.
  - Since the long-maturity option is closed out before maturity it has a time value (the call-curve is convex).
- In a **neutral calendar spread**, a strike price close to the current stock price is chosen.
  - A **bullish calendar spread** involves a higher strike price, whereas a **bearish calendar spread** involves a lower strike price.

Figure 11.8 Profit from calendar spread created using two call options, calculated at the time when the short-maturity call option expires.



- Calendar spreads can be created with put options as well as call options.
  - The investor buys a long-maturity put option and sells a short-maturity put option.

Figure 11.9 Profit from calendar spread created using two put options, calculated at the time when the short-maturity put option expires.



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- A **combination** is an option trading strategy that involves taking a position in both calls and puts on the same stock.
- We will consider straddles, strips, straps, and strangles.
- A **straddle** involves buying a European call and put with the same strike price and expiration date.
  - A straddle is appropriate when an investor is expecting a large move in a stock price but does not know in which direction the move will be.

Figure 11.10 Profit from a straddle.

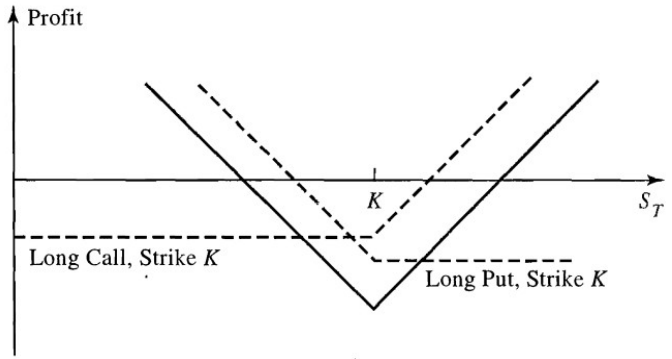
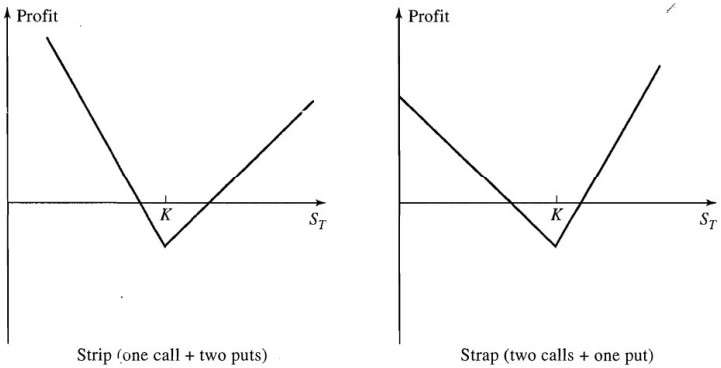


Table 11.5 Payoff from a straddle.

Range of stock price	*Payoff from call*	*Payoff from put*	*Total payoff*
$S_T \leq K$	0	$K - S_T$	$K - S_T$
$S_T > K$	$S_T - K$	0	$S_T - K$

- A **strip** (resp. **strap**) consists of a long position in one (resp. two) European call and two (resp. one) European puts with the same strike price and expiration date.
  - In a strip (resp. strap) the investor is betting that there will be a big stock price move and considers a decrease (resp. increase) in the stock price to be more likely than an increase (resp. decrease).

Figure 11.11 Profit from a strip and a strap.

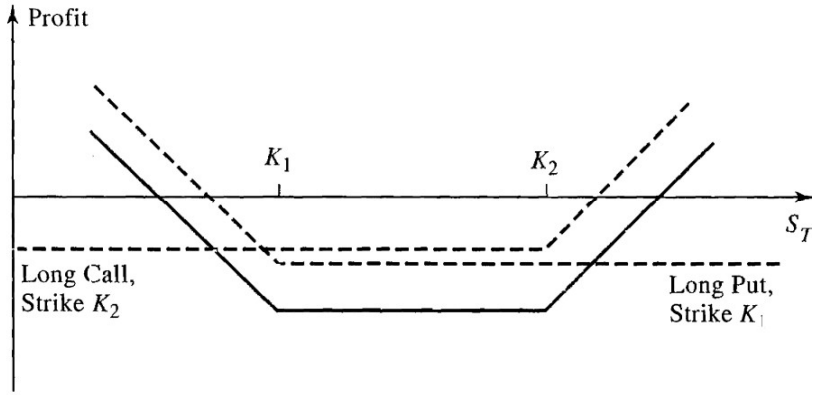


Combination
Strangle

- In a **strangle**, sometimes called a **bottom vertical combination**, an investor buys a European put and a European call with the same expiration date and different strike prices.
  - The call strike price, K_2, is higher than the put strike price, K_1.
  - A strangle is a similar strategy to a straddle.
    - The investor is betting that there will be a large price move, but is uncertain whether it will be an increase or a decrease.
    - The stock price has to move farther in a strangle than in a straddle for the investor to make a profit.
    - However, the downside risk if the stock price ends up at a central value is less with a strangle.

Combination
Strangle

Figure 11.12 Profit from a strangle.



Combination
Strangle

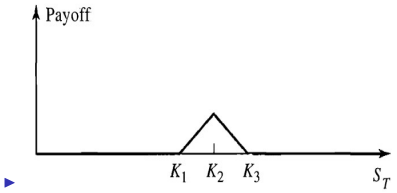
Table 11.6 Payoff from a strangle.

Range of stock price	*Payoff from call*	*Payoff from put*	*Total payoff*
$S_T \leqslant K_1$	0	$K_1 - S_T$	$K_1 - S_T$
$K_1 < S_T < K_2$	0	0	0
$S_T \geqslant K_2$	$S_T - K_2$	0	$S_T - K_2$

Combination
Other Payoff Patterns

- When the strike prices are close together a butterfly spread provides a payoff consisting of a small “spike”:

Figure 11.13 “Spike payoff” from a butterfly spread that can be used as a building block to create other payoffs.



- If options with all strike prices were available any payoff pattern could (at least approximately) be created by combining the spikes obtained from different butterfly spreads.

Exercise (2)

A call option with a strike price of \$50 costs \$2.
A put option with a strike price of \$45 costs \$3.

Explain how a strangle can be created from these two options. What is the pattern of profits from the strangle?

Solution (2)

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Summary

- Principal-protected notes can be created from a zero-coupon bond and a European call option.
  - They are attractive to some investors because the issuer of the product guarantees that the purchaser will be receive his or her principal back regardless of the performance of the asset underlying the option.
- A number of common trading strategies involve a single option and the underlying stock.

Summary

- *Spreads* involve either taking a position in two or more calls or taking a position in two or more puts.
  - A *bull spread* can be created by buying a call (put) with a low strike price and selling a call (put) with a high strike price.
  - A *bear spread* can be created by buying a put (call) with a high strike price and selling a put (call) with a low strike price.
  - A *butterfly spread* involves buying calls (puts) with a low and high strike price and selling two calls (puts) with some intermediate strike price.
  - A *calendar spread* involves selling a call (put) with a short time to expiration and buying a call (put) with a longer time to expiration.

Summary

- *Combinations* involve taking a position in both calls and puts on the same stock.
  - A *straddle* combination involves taking a long position in a call and a long position in a put with the same strike price and expiration date.
  - A *strip* consists of a long position in one call and two puts with the same strike price and expiration date.
  - A *strap* consists of a long position in two calls and one put with the same strike price and expiration date.
  - A *strangle* consists of a long position in a call and a put with different strike prices and the same expiration date.
- There are many other ways in which options can be used to produce interesting payoffs.
  - It is not surprising that option trading has steadily increased in popularity and continues to fascinate investors.