

Industrial Organization - Final Exam

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Duration: 90 mn. No document, no calculator allowed.

Exercise 1. Bertrand Competition Between Artificial Intelligence Platforms (10 pts)

Two AI platforms, A and B, compete by setting subscription prices for access to their large language models (LLMs). The two models are differentiated: they may differ in reasoning quality, inference speed, tool integrations, safety parameters, or interface design.

Demand for each platform depends on its own price and the competitor's price:

$$D_A = a - bp_A + cp_B; \text{ and } D_B = a - bp_B + cp_A$$

The parameters satisfy:

- $a > 0$: baseline market size or intrinsic demand for AI platforms;
- $b > 0$: own-price sensitivity (how demand falls when a platform raises its price); and
- $c > 0$: cross-price sensitivity (how demand rises when the competitor raises price), with $c < b$ to ensure differentiation.

The platforms play a Bertrand pricing game: each simultaneously chooses its price taking the rival's price as given. Let $m > 0$ denote the marginal cost per subscriber (compute, inference load, infrastructure).

a) (1 pt) Write the profit maximization problems for Platform A and Platform B, and derive the best-response functions $p_A^*(p_B)$ and $p_B^*(p_A)$.

b) (2 pts) Solve for the Bertrand–Nash equilibrium prices p_A^N and p_B^N .

c) (2 pts) Give a graphical representation of this price equilibrium in the (p_A, p_B) space.

d) (1 pt) Perform comparative statics: how do equilibrium prices change with the baseline demand parameter a , the own-price sensitivity b , the cross-price sensitivity parameter c , and the marginal cost m ?

e) (2 pts) Calculate the equilibrium profits for both platforms. How do profits change with a and m ?

f) (1 pt) Suppose the platforms collude and set a joint (symmetric) monopoly price. Derive the collusive price, denoted as p^c , and the collusive profit, denoted as Π^c .

g) (1 pt) Numerical application: compare the Bertrand–Nash to the colluding solutions (prices and aggregated profits) when $a = 100$, $b = 2$, $c = 1$, and $m = 10$.

Exercise 2. Collusion in a quantity oligopoly (10 pts)

Consider a repeated game where in each period $t \in 1, 2, \dots, T$, n firms (with the same discount factor $\delta \in [0, 1]$) independently choose the quantity of an identical good they produce. We denote

- $N = \{1, 2, \dots, n\}$: the set of n firms;
- $C(q) = cq$: identical production cost function for each firm, with $c \in [0, 1]$;
- $\pi_i(p, q) = (p - c)q$: firm i 's profit when producing q units and selling it at price p ;
- $q_i(t)$: the quantity produced by firm i at period t ; and
- $Q(t) = \sum_{i=1}^N q_i(t)$: aggregate output at period t .

Once the quantities produced have been observed, all firms sell the good at market price $p(t)$ which is established at $1 - Q(t)$ if $Q(t) \leq 1$ and zero otherwise.

a) (1 pt) In the stage game, determine the aggregate quantity that maximizes the aggregate profit that the firms can be led to share within a cartel.

b) (2 pts) Determine the unique pure strategies Nash equilibrium payoff of the stage game. What is the corresponding market price?

c) (1 pt) Is collusion sustainable in (subgame perfect) equilibrium when the game is finitely repeated? Why?

d) (1 pt) In the infinitely repeated game define a grim-trigger strategy devoted to sustain full collusion.

e) (2 pts) For which discount factors is the corresponding strategy profile a Nash equilibrium of the infinitely repeated game? A subgame perfect Nash equilibrium?

f) (1 pt) What is the influence of the number of firms on the difficulty to maintain the cartel?

g) (2 pts) Is collusion at a quantity yielding a per-period aggregate profit $\bar{\pi}$ that lies between the Cournot and monopoly profits sustainable in equilibrium? Under what conditions? How does the threshold on the discount factor vary with $\bar{\pi}$?