Arbitrage and Pricing – Exam

Université Paris Dauphine-PSL - Master 1 I.E.F. (272)

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Answers can be formulated in English or French.

Exercise 1 European Down-and-out and Down-and-in Calls (12 pts)

Consider a stock whose price starts at $S_0 = 20$€ and evolves annually according to a two-steps binomial tree where each upward (resp. downward) move doubles (resp. divides by two) the previous value for the next 3 years ($T = 3$). The risk-free interest rate is $r = 25\%$ and is annually compounded. This stock is the underlying asset of all the following derivatives instruments.

At year $t = 0$, a financial institution issues an exotic European call named "Down-and-out" (aka "knock-out barrier"). This call behaves in every way like a vanilla (i.e., regular) European call, except if the spot price of the underlying stock ever moves below a certain "barrier" price, in which case the call is knocked out and becomes null and void. The call then expires worthless and does not reactivate, even if the spot price rises above the barrier price again. The maturity, strike, and barrier are set at $T = 3$, $K = 30$€ and $B = 18$€ respectively.

a) (1 pt) *Draw the binomial tree that depicts the evolution of the stock price through time* $t \in \{0, 1, 2, 3\}$.

b) (1 pt) *Is the binomial tree that depicts the evolution of the derivative price recombining ? Why ?*

c) (1 pt) *Compute the risk neutral probability.*

d) (3 pts) *Draw the binomial tree that depicts the evolution of the derivative no-arbitrage price, denoted as* Do_t*, through time* t*, with* $t \in \{0, 1, 2, 3\}$.

Assume now the financial institution issues an European "Down-and-in" (aka "knock-in barrier") call option that behaves in every way like a vanilla European call, except that the spot price of the underlying stock has to move below the barrier for this exotic option to be activated, otherwise the call stays null and void. The maturity, the strike and the barrier are set as in the previous option.

e) (3 pts) *Draw the binomial tree that depicts the evolution of the derivative no-arbitrage price, denoted as* Di_t*, through time* $t \in \{0, 1, 2, 3\}$.

Assume now the financial institution issues a vanilla European call with the same maturity and strike than in the previous option.

f) (2 pts) *Draw the binomial tree that depicts the evolution of the derivative no-arbitrage price, denoted as* c_t*, through time* $t \in \{0, 1, 2, 3\}$.

g) (1 pts) *What is the relationship between the price of these three options ?*

Exercise 2 Greeks put-call parity (4 pts)

Consider a Black–Scholes–Merton framework. Use the put–call parity relationship to derive, for a non-dividend-paying stock, the relationship between :

(a) *The delta of a European call and the delta of a European put.*

(b) *The gamma of a European call and the gamma of a European put.*

(c) *The vega of a European call and the vega of a European put.*

(d) *The theta of a European call and the theta of a European put.*

Exercise 3 Black-Scholes Portfolio Insurance Delta (4 pts)

Consider a Black–Scholes–Merton framework. Suppose that 70€ billion of equity assets with volatility $\sigma = 0.25$ are the subject of portfolio insurance schemes. Assume that the schemes are designed to provide insurance against the value of the assets declining by more than 5% within one year. The risk-free rate is $r = 3\%$. We want to calculate the value of the stock that the administrators of the portfolio insurance schemes will attempt to sell if the market falls by 23% in a single day.

a) (2 pts) *Calculate the value (in billion euros) of the stock that the administrators of the portfolio insurance schemes will attempt to sell before the decline. (Hint : Regard the position of all portfolio insurers taken together as a single put option that the administrators replicates, and compute the corresponding delta.)*

b) (2 pts) *Same question after the market falls by* 23% *in a single day.*