Industrial Organization - Additional Exercises
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1 Product differentiation

Consider the two stages game in which two players, firm 1 and firm 2, compete in quality and price as follows:

- Stage 1: Both firms simultaneously choose a quality $\theta \in [\underline{\theta}, \overline{\theta}]$;
- Stage 2: Both firms simultaneously choose a price.

Firm i, $i \in \{1, 2\}$, produces a good of quality θ_i, and charges a price p_i. The unit cost of production is c. Let us order the firms such that if $\theta_1 \neq \theta_2$ then $\theta_1 < \theta_2$.
For a given pair of qualities (θ_1, θ_2) where $\theta_1 < \theta_2$, the reaction and residual demand functions are given by:

$$p_1(p_2, \theta_1, \theta_2) = \frac{p_2 + c}{2}$$

$$p_2(p_1, \theta_1, \theta_2) = \frac{p_1 + c + \theta_2 - \theta_1}{2}$$

$$D_1(p_1, p_2, \theta_1, \theta_2) = \min\{1; \frac{p_2 - p_1}{\theta_2 - \theta_1}\}$$

$$D_1(p_1, p_2, \theta_1, \theta_2) + D_2(p_1, p_2, \theta_1, \theta_2) = 1$$

A_1) *Give the pure strategy Nash equilibrium of the price competition (for a given pair of qualities (θ_1, θ_2) where $\theta_1 < \theta_2$).*

A_2) *To which situation would correspond the price equilibrium resulting from identical qualities?*

A_3) *Give a graphical representation of this price equilibrium in the (p_1, p_2) space.*

A_4) *Graphically illustrate how the price equilibrium would move with an increase in quality differentiation.*

A_5) *What are the corresponding residual demands and profits?*

A_6) *(1 pt) Assume the quality is costless. Give a pure strategy Nash equilibrium of the quality choice.*

A_7) *Give the two-stage game equilibrium and corresponding profits. Does this equilibrium exhibit minimal or maximal differentiation?*

A_8) *Is this equilibrium unique? Explain.*

A_9) Is the whole equilibrium subgame perfect? Explain.

A_{10}) Why do firms use product differentiation?

2 Tacit collusion

Consider two firms that compete in price $T + 1$ times. At each date t, $t = 0, 1, \ldots, T$, the firms choose their prices (p_{1_t}, p_{2_t}) simultaneously. Let $\pi^i(p_{i_t}, p_{j_t})$ be firm i's profit at date t when it charges p_{i_t} and its rival charges p_{j_t}. Consider a firm i's discount factor $\delta_i \in (0, 1)$, so that the discounted value of firm i's profits write as:

$$\sum_{t=0}^{T} \delta_i^t \pi^i(p_{i_t}, p_{j_t}).$$

At date t, if $p_{i_t} > p_{j_t}$ then firm i makes zero profit; if $p_{i_t} = p_{j_t}$ then $\pi^i(p_{i_t}, p_{j_t}) = \alpha_i \pi(p_{i_t}, p_{j_t})$, where $\pi(p_{i_t}, p_{j_t})$ denotes the aggregate profits at date t, that is shared between the firms according to α_1 and α_2, two positive real numbers satisfying $\alpha_1 + \alpha_2 = 1$. The unit cost of production is c.

B_1) *Is any collusion sustainable as a (time-invariant) equilibrium in finite horizon? Why?*

B_2) *In infinite horizon, give the smaller discount factors δ_1 and δ_2 such that any pair (δ_1, δ_2) satisfying $\delta_1 \geq \delta_1$ and $\delta_2 \geq \delta_2$ allows to fully collude at (time-invariant) equilibrium. Start by exhibiting an adapted trigger strategies profile.*

B_3) *Explain why δ_i is a decreasing function of α_i.*

3 Markets for loans: Cournot vs Stackelberg

We consider an oligopolistic banking sector with two banks, bank 1 and bank 2, that operate in the market for loans.

- The inverse demand function for loans of bank $i \in \{1, 2\}$ is:

$$r_i(L_i, L_j) = m - 2bL_i - bL_j,$$

  with $b > 0$, where $L_i \in R^+$ is bank i's loan volume.
- Cost function: $C_i(L_i) = c_L L_i$.
- Balance-sheet constraint: $M_i(L_i) = -(1 - \rho)L_i$, with $\rho \in (0, 1)$.
- Banks borrow their negative net position at the interbank rate r_{CB}.
- Profit:

$$\pi_i(L_i, L_j) = r_i(L_i, L_j)L_i - C_i(L_i) + r_{CB}M_i(L_i).$$

3.1 Part A. Cournot game

We first consider the Cournot game where the banks simultaneously compete in loan quantities.

a) Determine the volume of bank i's loan L_i that maximizes its profit given its competitor loan volume $\bar{L}_j$.

b) Determine the pure strategies Nash equilibrium (L_1^C, L_2^C).

c) How does it vary with c_L, r_{CB}, m and ρ? Explain.

d) What are the resulting lending rates (r_1^C, r_2^C)?

e) Numerical application for: $m = 1.5$, $b = 1/20$, $r_{CB} = 1.05$, $\rho = 15\%$, $c_L = 0.1075$.

3.2 Part B. Stackelberg game

Now assume Bank 1 acts as leader choosing L_1 first, and Bank 2 as follower chooses L_2 after observing L_1.

f) Determine the subgame perfect Nash equilibrium (L_1^S, L_2^S).

g) Provide a graphical representation of both equilibrium points, (L_1^C, L_2^C) and (L_1^S, L_2^S), in the (L_1, L_2) plane.

h) What are the resulting lending rates (r_1^S, r_2^S)?

i) Numerical application for: $m = 1.5$, $b = 1/20$, $r_{CB} = 1.05$, $\rho = 15\%$, $c_L = 0.1075$.

j) By comparing answers e) and i), what kind of competition would a borrower prefer?