Derivative Instruments (Produits dérivés) - Solution to the Exam

Université Paris Dauphine-PSL - Master 1 I.E.F. (272)

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December 2025. Duration: 2h. No document allowed. Calculator allowed.
Answers can be formulated in English or French.

Solution 1 (1 pt)

(a) The fair delivery price per unit is

$$F_0 = (S_0 - D)(1+r)^T$$

with $S_0 = 150$; $T = 5/12$; $r = 2\%$; and

$$D = \frac{2.5\% \times 150}{4}(1.02)^{-3/12} \simeq 0.9329$$

So,

$$F_0 = (150 - 0.9329)(1.02)^{5/12} \simeq \$150.3021$$

(b) The fair delivery price for the total contract is then $F_0 \times n = 150.3 \times 10,000 = \$1,503,021$.

Solution 2 (1 pt)

The trader sells for 35 cents per pound something that is worth 35.83 cents per pound. The loss is $\$(0.3583 - 0.35) \times 30,000 \times 2 = \$498$.

Solution 3 (2 pts)

The present value of the storage costs for nine months are

$$\frac{0.15}{4} + \frac{0.15}{4}e^{-0.024\times\frac{3}{12}} + \frac{0.15}{4}e^{-0.024\times\frac{6}{12}} \simeq 0.112$$

The futures price of coffee for delivery in nine months is $1.2075 + 0.112 = 1.341$

Solution 4 (1 pt)

The initial margin is 10% of the contract: $80 \times 1,000 \times 10\% = 8,000$. There will be a margin call when 25% has been lost from the margin account, that is $\$2,000$. For a short futures position, losses occur when the futures price increases. This will occur when the new price p satisfies

$$(p - 80) \times 1,000 = 2,000$$

so the Crude Oil price must therefore rise to \$82.

Solution 5 (1 pt)

Let ρ denotes the correlation between the futures price and the commodity price, and σ_S (resp. σ_F) denotes the standard deviation of monthly changes in the price of commodity A (resp. in a futures price for a contract on commodity B).

The optimal hedge ratio is $h^* = \rho\frac{\sigma_S}{\sigma_F} = 0.45\frac{0.13}{0.16} \simeq 0.3656$.

Solution 6 (1 pt) Answer: B (Most futures are cash-settled or closed before delivery.)

Solution 7 (1 pt) When the term structure of interest rates is upward sloping, we have $b < a < c$.

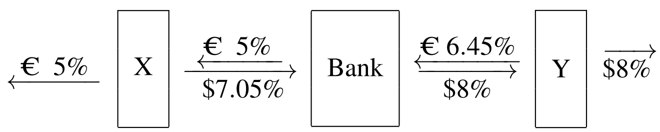
Solution 8 (1 pt)

Answer : B) Because the future cash flows of the fixed and floating legs change in present value due to interest rate movements

Solution 9 (3 pts)

a) X has a comparative advantage in euro markets but wants to borrow dollars. Y has a comparative advantage in dollar markets but wants to borrow euro. This provides the basis for the swap. There is a 1.5% per annum differential between the euro rates and a 0.4% per annum differential between the dollar rates. The total gain to all parties from the swap is therefore $2 - 0.4 = 1.6\%$ per annum. The bank requires 0.5% per annum, leaving 0.55% per annum for each of X and Y.

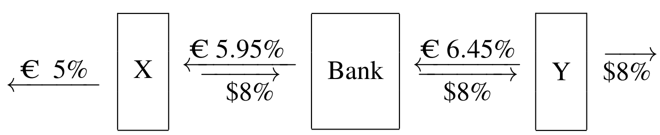
The swap should lead to X borrowing dollars at $7.6 - 0.55 = 7.05\%$ per annum and to Y borrowing euro at $7 - 0.55 = 6.45\%$ per annum. The appropriate arrangement is therefore as shown in the following figure. (All foreign exchange risk is borne by the bank.)



All foreign exchange risk is assumed by the bank. Indeed, in terms of cashflows, the net payoff is :

- for compagny X to pay \$7.05%;
- for compagny Y to pay €6.45%; and
- for the Bank is to receive €1.45% and pay \$0.95%.

b) When the currency risk is taken over by company X, compagny Y and the Bank have each a net payoff that involves only one currency. The appropriate arrangement is therefore as shown in the following figure.



Indeed, in terms of cashflows, the net payoff is:

- for compagny X to receive €0.95% and pay \$8% ;
- for compagny Y to pay €6.45%; and
- for the Bank is to receive €0.5%.

Solution 10 (6 pts)

a) **(0.5 pts)** The bond yield of Bond 2 is 1% because flat yield curve at 1%.

b) **(1 pt)** The duration of Bond 1 is 1 year because zero coupon. The duration of Bond 2 is

$$D_2 = \frac{\left(c_1 e^{-0.01} + c_2 \cdot 2 \cdot e^{-0.01 \cdot 2}\right)}{B_2} = \frac{\left(5e^{-0.01} + 105 \cdot 2 \cdot e^{-0.01 \cdot 2}\right)}{107.88} \simeq 1.95 \text{ years.}$$

c) **(1 pt)** If the yield curve shifts up, both bond prices decrease. Since the duration of the second bond is higher than the duration of the first bond, price of bond 1 decreases by less than price of bond 2. So, the value of Portfolio P: **(i) increase**.

d) **(1 pt)** If the 1-year interest rate goes up and the 2-year interest rate goes down, the present value of P: **(ii) decrease**. Indeed, P has positive cash flows at $t = 1$, whose present value decreases when r_1 increases, and negative cash flows at $t = 2$, whose present value increases when r_1 decreases.

e) **(1 pt)** The zero coupon bond can be replicated with Bond 1 and Bond 2 according to units x and y that satisfy:

$$\begin{cases} 100x + 5y = 0 \\ 105y = 100 \end{cases}$$

which yield $x = \frac{-5}{105} \simeq -0.0476$ and $y = \frac{100}{105} \simeq 0.9523$.

f) **(1 pt)** The no-arbitrage price of a 2-year risk-free zero coupon bond is $100e^{-0.01 \cdot 2} \simeq$ €98.02 < €99 so there is an arbitrage opportunity. According to the previous answer, an arbitrage strategy that uses one unit of the zero coupon bond consists in taking:

- a short position in one unit of the 2-year zero coupon bond;
- a short position in 0.0476 units of Bond 1; and
- a long position in 0.9524 units of Bond 2

g) **(0.5 pts)** Yes, the previous arbitrage opportunity is still valid if the previous 2-year zero coupon bond is with default risk because a risky bond should be even less expensive than a risk-free one.

Solution 11 (2 pts)

a) Creating a calendar spread using these options above consists in selling the near-term call, and buying the long-term call. The cost of this strategy is 5–2 = 3€.

b) The diagram representing the profit as a function of the share price at the December 31st expiration is

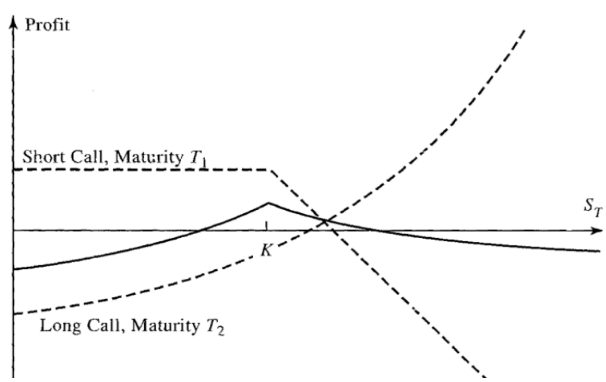


*Figure*1