

# Derivative Instruments (Produits dérivés) - Exam

Université Paris Dauphine-PSL - Master 1 I.E.F. (272)

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Answers can be formulated in English or French.

**Exercise 1 (1 pt)** One of your clients wishes to enter into a forward contract to purchase 10,000 shares of Acme Inc. in five months. The stock is currently trading for \$150 on the New York Stock Exchange. Its dividend is equal to 2.5% per year, it is paid quarterly and the next dividend will be paid in three months. The risk-free rate is equal to 2% per year. All interest rates are compounded annually.

- (a) *What is the fair delivery price per unit ?*
- (b) *What is the fair delivery price for the total contract.*

**Exercise 2 (1 pt)** A trader enters in two short cotton futures contracts when the futures price is 35 cents per pound. The contract is for the delivery of 30,000 pounds.

*How much does the trader gain or lose if the cotton price at the end of the contract is 35.83 cents per pound ?*

**Exercise 3 (2 pts)** The spot price of coffee is 120.75 cents per pound. The storage costs are \$0.15 per pound per year payable quarterly in advance. Assume that interest rates are 2.4% per annum for all maturities.

*What is the futures price of coffee for delivery in nine months ?*

**Exercise 4 (1 pt)** Suppose that you enter into a short futures contract to sell the January WTI Crude Oil futures at \$80 per barrel. The size of the contract is 1,000 barrels. The initial margin is 10% of the contract, and the maintenance margin is 75% of the initial margin.

*What change in the futures price will lead to a margin call ?*

**Exercise 5 (1 pt)** Suppose that the standard deviation of quarterly changes in the price of commodity A is \$0.13. The standard deviation of quarterly changes in a futures price for a contract on commodity B (which is similar to commodity A) is \$0.16. The correlation between the futures price and the commodity price is 0.45.

*What hedge ratio should be used when hedging a three-month exposure to the price of commodity A ?*

**Exercise 6 (1 pt)** *Which of the following answer is correct ?* Futures contracts : A. End with physical delivery ; B. Are cash-settled or closed before delivery ; C. Require storage ; D. Cannot be closed early ; or E. None of the previous answers.

**Exercise 7 (1 pt)** The term structure of interest rates is upward sloping. Put the following in order of magnitude (from smallest to largest) :

- (a) The 2-year zero rate
- (b) The yield on a 2-year coupon-bearing bond
- (c) The forward rate corresponding to the period between 1.75 and 2 years in the future

**Exercise 8 (1 pt)** *Which of the following best describes why the value of an interest rate swap changes over time ?* A) Because the notional principal changes with market conditions ; B) Because the future cash flows of the fixed and floating legs change in present value due to interest rate movements ; C) Because the swap contract is renegotiated daily ; D) Because swaps always generate a fixed profit for the payer same each year.

**Exercise 9 (3 pts)** Company X wishes to borrow U.S. dollars at a fixed rate of interest. Company Y wishes to borrow euros at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. The companies have been quoted the following interest rates, which have been adjusted for the impact of taxes :

	Euros (€)	Dollars (\$)
Company X	5.0%	7.6%
Company Y	7.0%	8.0%

We want to design a swap that will net a bank, acting as intermediary, 50 basis points per annum.

a) *Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank. Draw the corresponding diagram.*

b) *What will the swap contract look like if the currency risk is taken over by company X? Draw the corresponding diagram.*

**Exercise 10 (6 pts)** The term structure of risk-free interest rates is flat at 1% per year. Consider the following risk-free bonds with annual coupon payments :

	Coupon rate	Face value (par value)	Maturity	Price
Bond 1	0%	100 €	1 year	99.01 €
Bond 2	5%	100 €	2 years	107.88 €

We assume the bonds are divisible, so they can be purchased in units of any size (not necessarily whole units).

a) *What is the bond yield of Bond 2 ?*

b) *Determine the duration of Bond 1 and the duration of Bond 2 ?*

In questions c) and d) you will consider Portfolio P which is composed of a long position in Bond 1 and a short position in Bond 2, with the position sizes chosen to make the portfolio's initial value equal to zero.

c) *If the yield curve shifts up, does the value of Portfolio P : (i) increase ; (ii) decrease ; (iii) does not change ; (iv) may either increase or decrease ? Why ?*

d) *If the 1-year interest rate goes up and the 2-year interest rate goes down, does the value of Portfolio P : (i) increase ; (ii) decrease ; (iii) does not change ; (iv) may either increase or decrease ? Why ?*

e) *Assume that a 2-year risk-free zero coupon bond with face value €100 is trading at €99. How can the zero coupon bond be replicated with Bond 1 and Bond 2 ? [Compute the respective units of Bond 1 and Bond 2 that replicate one unit of the zero coupon bond].*

f) *Is there an arbitrage opportunity ? If yes, describe an arbitrage strategy that uses one unit of the zero coupon bond.*

g) *Assume the previous 2-year zero coupon bond is with default risk (with same face value €100 still trading at €99). Is the previous arbitrage opportunity still valid ? Why ?*

**Exercise 11 (2 pts)** You are analyzing options on XYZ stock, currently trading at 100€. The implied volatility is stable, and you expect the stock price to remain near 100€ for the next month but possibly move afterward. Two options are available. A near-term call option : Expiration December 31, Strike 100€, Premium 2€ ; and a long-term call option : Expiration March 31, Strike 100€, Premium 5€.

a) *Describe how to create a calendar spread using the options above. Indicate which option you buy and which you sell. What is the cost (net debit) of this strategy ?*

b) *Draw the diagram representing the profit as a function of the share price at the December 31st expiration.*