

# Industrial Organization - Exercises Chapter 1

Université Paris Dauphine-PSL

Jérôme MATHIS (LEDa)

## Exercise 1: Bertrand equilibrium with subadditive different costs (Dastidar, Economics Letters, 2011)

Consider a simultaneous move price choice game in a homogeneous product, asymmetric cost duopoly. The cost function for firm  $i$ ,  $i = 1, 2$ , when producing quantity  $q \in [0; 1]$  is  $C_i(q) = \bar{c}_i + c_i q$  if  $q > 0$  and zero otherwise, where  $\bar{c}_i \in (0; \frac{(1-c_i)^2}{4})$  (resp.  $c_i \in (0; 1)$ ) is the fixed (resp. variable) cost of production, so that the cost function is strictly subadditive (i.e.,  $C_i(x + y) < C_i(x) + C_i(y)$  for any two quantities  $x$  and  $y$ ).

Firm  $i$  chooses which price  $p_i$  to quote in the interval  $[0, 1]$ . There is a demand function for the lowest posted price  $p$ ,  $D(p) = 1 - p$ .

In price competition firms have to meet the demand that they face at the posted price. The firm which quotes the lowest price gets all the demand. Any firm which quotes a price higher than its rival gets no demand. If there is a tie at any price, the two firms share the demand equally.

- 1) How does write  $\pi_i(p_i, p_j)$  the profit going to firm  $i$  when it quotes price  $p_i$  and its competitor charges price  $p_j$ ?
- 2) What price  $p_i^m$  would firm  $i$  quote if it was a monopoly? Which assumption does guarantee that firm  $i$ 's monopoly profit is strictly positive?
- 3) What price  $\tilde{p}_i$  is firm  $i$ 's "monopoly breakeven price" (i.e., the price at which firm  $i$ 's monopoly profit is zero and just below (resp. above) which it is negative (resp. positive))?
- 4) Can we have altogether  $\tilde{p}_1 > p_2^m$  and  $\tilde{p}_2 > p_1^m$ ? Why?
- 5) Suppose  $\tilde{p}_2 \geq p_1^m$ . Give a pure strategy Bertrand equilibrium  $(p_1^*, p_2^*)$ . Prove that there is no unilateral profitable deviation.
- 6) Conversely, suppose  $\tilde{p}_1 \geq p_2^m$ . Give a pure strategy Bertrand equilibrium  $(p_1^*, p_2^*)$ . (The proof is not required.)
- 7) Suppose  $\tilde{p}_1 \neq \tilde{p}_2$ ,  $\tilde{p}_1 < p_2^m$ , and  $\tilde{p}_2 < p_1^m$ . We want to show that there is no pure strategy Bertrand equilibrium  $(p_1^*, p_2^*)$ . Find a unilateral profitable deviation in each of the following case. (Without loss of generality, when  $p_1^* \neq p_2^*$  we will assume that  $p_1^* < p_2^*$ .)
  - 7.a)  $p_1^* = p_2^* < \max\{\tilde{p}_1, \tilde{p}_2\}$ .
  - 7.b)  $p_1^* = p_2^* \geq \max\{\tilde{p}_1, \tilde{p}_2\}$ .
  - 7.c)  $p_1^* < p_2^*$  and  $p_1^* \leq \tilde{p}_2$ .
  - 7.d)  $p_1^* < p_2^*$  and  $p_1^* > \tilde{p}_2$ .
- 8) Suppose again  $\tilde{p}_1 \neq \tilde{p}_2$ ,  $\tilde{p}_1 < p_2^m$ , and  $\tilde{p}_2 < p_1^m$ . Without loss of generality, assume  $\tilde{p}_1 < \tilde{p}_2$ . We want to show that there is a Bertrand equilibrium  $(p_1^*, p_2^*)$  that relies on a firm 2's mixed strategy that consists in randomizing uniformly on the interval  $[\tilde{p}_2; \tilde{p}_2 + a]$ , with  $a > 0$  small enough.
  - 8.a) Does  $p_1^* \geq \tilde{p}_2 + a$ ? Explain.
  - 8.b) Does  $p_1^* < \tilde{p}_2$ ? Explain.
  - 8.c) Does  $p_1^* \in (\tilde{p}_2; \tilde{p}_2 + a)$ ? Explain. (Hint: assume  $p_1^* \in (\tilde{p}_2; \tilde{p}_2 + a)$ , compute firm 1's expected payoff and show that it has a unilateral profitable deviation when the parameter  $a$  is small enough.)
  - 8.d) So what is firm 1's best response?

**Exercise 2: Is a horizontal merger privately and/or socially beneficial? (Liu and Wang, Economics Letters, 2015)**

Consider an industry with  $m + k$  symmetric firms producing homogeneous goods at a constant marginal cost  $\bar{c} \in (0, 1)$ . Firm  $i = 1, \dots, m$  is an insider that may agree to merge and firm  $i = m + 1, \dots, m + k$  is an outsider. The inverse market demand is linear and is given by  $P(Q) = 1 - Q$ , where  $P$  is price and  $Q$  is industry output.

Conditional on the insiders merge or not, these firms compete in different modes. If all firms behave non-cooperatively, i.e., there is no merger, we assume that these firms compete like Cournot oligopolists. If the  $m$  insiders merge, we assume that the merger results in two changes. First, the insiders act like a single firm and choose their aggregate output to maximize their joint profits. Second, the merger obtains a strategic advantage of becoming the leader in the market (Stackelberg competition).

**Part A. Cournot competition**

**A.1)** Give the symmetric equilibrium of the Cournot competition in which the equilibrium output of the  $i$ th firm,  $i = 1, \dots, m+k$ , and the industry output are represented as  $q_i^c = q^c$  and  $Q^c = (m + k)q^c$ , respectively.

**A.2)** What are the corresponding market price and  $i$ th firm profit ( $i = 1, \dots, m + k$ ), represented as  $P^c$  and  $\pi_i^c = \pi^c$ , respectively?

**Part B. Stackelberg competition**

We consider now the game with the following sequence of moves and assumption. In the first stage, the insiders merge and choose their aggregate output  $q^m$  to maximize their joint profits. In the second stage, each outsider simultaneously chooses its output to maximize its own profit. We assume the outsiders output choice is symmetric, so that  $q_i = q^o$ , for all  $i = m + 1, \dots, m + k$ .

**B.1)** Give the subgame perfect Nash equilibrium (SPNE) of this game in which the industry output is represented as  $Q^S = q^m + kq^o$ .

**B.2)** What are the associated industry output  $Q^s$ , profit of the merger  $\pi^m$ , and profit of each outsider  $\pi^o$ ?

**Part C. Profits and consumer welfare effects of a leading merger**

**C.1)** Is becoming a leader and merging profitable to the insiders? Why?

**C.2)** Under what condition is the merger profitable to the outsiders?

**C.3)** While a merger generally changes all firms' outputs, consumers care about the changes in industry output only. Under what condition does the industry output (and the resulting consumer welfare) increase?

**C.4)** Conclude.