

Industrial Organization - Exercises Chapter 1

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Exercise 1: Bertrand equilibrium with subadditive different costs (Dastidar, Economics Letters, 2011)

Consider a simultaneous move price choice game in a homogeneous product, asymmetric cost duopoly. The cost function for firm i , $i = 1, 2$, when producing quantity $q \in [0; 1]$ is $C_i(q) = \bar{c}_i + c_i q$ if $q > 0$ and zero otherwise, where $\bar{c}_i \in (0; \frac{(1-c_i)^2}{4})$ (resp. $c_i \in (0; 1)$) is the fixed (resp. variable) cost of production, so that the cost function is strictly subadditive (i.e., $C_i(x + y) < C_i(x) + C_i(y)$ for any two quantities x and y).

Firm i chooses which price p_i to quote in the interval $[0, 1]$. There is a demand function for the lowest posted price p , $D(p) = 1 - p$.

In price competition firms have to meet the demand that they face at the posted price. The firm which quotes the lowest price gets all the demand. Any firm which quotes a price higher than its rival gets no demand. If there is a tie at any price, the two firms share the demand equally.

- 1) How does write $\pi_i(p_i, p_j)$ the profit going to firm i when it quotes price p_i and its competitor charges price p_j ?
- 2) What price p_i^m would firm i quote if it was a monopoly? Which assumption does guarantee that firm i 's monopoly profit is strictly positive?
- 3) What price \tilde{p}_i is firm i 's “monopoly breakeven price” (i.e., the price at which firm i 's monopoly profit is zero and just below (resp. above) which it is negative (resp. positive))?
- 4) Can we have altogether $\tilde{p}_1 > p_2^m$ and $\tilde{p}_2 > p_1^m$? Why?
- 5) Suppose $\tilde{p}_2 \geq p_1^m$. Give a pure strategy Bertrand equilibrium (p_1^*, p_2^*) . Prove that there is no unilateral profitable deviation.
- 6) Conversely, suppose $\tilde{p}_1 \geq p_2^m$. Give a pure strategy Bertrand equilibrium (p_1^*, p_2^*) . (The proof is not required.)
- 7) Suppose $\tilde{p}_1 \neq \tilde{p}_2$, $\tilde{p}_1 < p_2^m$, and $\tilde{p}_2 < p_1^m$. We want to show that there is no pure strategy Bertrand equilibrium (p_1^*, p_2^*) . Find a unilateral profitable deviation in each of the following case. (Without loss of generality, when $p_1^* \neq p_2^*$ we will assume that $p_1^* < p_2^*$.)
 - 7.a) $p_1^* = p_2^* < \max\{\tilde{p}_1, \tilde{p}_2\}$.
 - 7.b) $p_1^* = p_2^* \geq \max\{\tilde{p}_1, \tilde{p}_2\}$.
 - 7.c) $p_1^* < p_2^*$ and $p_1^* \leq \tilde{p}_2$.
 - 7.d) $p_1^* < p_2^*$ and $p_1^* > \tilde{p}_2$.
- 8) Suppose again $\tilde{p}_1 \neq \tilde{p}_2$, $\tilde{p}_1 < p_2^m$, and $\tilde{p}_2 < p_1^m$. Without loss of generality, assume $\tilde{p}_1 < \tilde{p}_2$. We want to show that there is a Bertrand equilibrium (p_1^*, p_2^*) that relies on a firm 2's mixed strategy that consists in randomizing uniformly on the interval $[\tilde{p}_2; \tilde{p}_2 + a]$, with $a > 0$ small enough.
 - 8.a) Does $p_1^* \geq \tilde{p}_2 + a$? Explain.
 - 8.b) Does $p_1^* < \tilde{p}_2$? Explain.
 - 8.c) Does $p_1^* \in (\tilde{p}_2; \tilde{p}_2 + a)$? Explain. (Hint: assume $p_1^* \in (\tilde{p}_2; \tilde{p}_2 + a)$, compute firm 1's expected payoff and show that it has a unilateral profitable deviation when the parameter a is small enough.)
 - 8.d) So what is firm 1's best response?

Exercise 2: Is a horizontal merger privately and/or socially beneficial ? (Liu and Wang, Economics Letters, 2015)

Consider an industry with $m + k$ symmetric firms producing homogeneous goods at a constant marginal cost $\bar{c} \in (0, 1)$. Firm $i = 1, \dots, m$ is an insider that may agree to merge and firm $i = m + 1, \dots, m + k$ is an outsider. The inverse market demand is linear and is given by $P(Q) = 1 - Q$, where P is price and Q is industry output.

Conditional on the insiders merge or not, these firms compete in different modes. If all firms behave non-cooperatively, i.e., there is no merger, we assume that these firms compete like Cournot oligopolists. If the m insiders merge, we assume that the merger results in two changes. First, the insiders act like a single firm and choose their aggregate output to maximize their joint profits. Second, the merger obtains a strategic advantage of becoming the leader in the market (Stackelberg competition).

Part A. Cournot competition

A.1) Give the symmetric equilibrium of the Cournot competition in which the equilibrium output of the i th firm, $i = 1, \dots, m+k$, and the industry output are represented as $q_i^c = q^c$ and $Q^c = (m+k)q^c$, respectively.

A.2) What are the corresponding market price and i th firm profit ($i = 1, \dots, m+k$), represented as P^c and $\pi_i^c = \pi^c$, respectively ?

Part B. Stackelberg competition

We consider now the game with the following sequence of moves and assumption. In the first stage, the insiders merge and choose their aggregate output q^m to maximize their joint profits. In the second stage, each outsider simultaneously chooses its output to maximize its own profit. We assume the outsiders output choice is symmetric, so that $q_i = q^o$, for all $i = m+1, \dots, m+k$.

B.1) Give the subgame perfect Nash equilibrium (SPNE) of this game in which the industry output is represented as $Q^S = q^m + kq^o$.

B.2) What are the associated industry output Q^s , profit of the merger π^m , and profit of each outsider π^o ?

Part C. Profits and consumer welfare effects of a leading merger

C.1) Is becoming a leader and merging profitable to the insiders ? Why ?

C.2) Under what condition is the merger profitable to the outsiders ?

C.3) While a merger generally changes all firms' outputs, consumers care about the changes in industry output only. Under what condition does the industry output (and the resulting consumer welfare) increase ?

C.4) Conclude.