Industrial Organization - Solution to the Final Exam

Paris Dauphine University - Master Industries de Réseau et Economie Numérique (IREN), December 2024

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Duration: 105 mn. No document, no calculator allowed.

Exercise 1. Stackelberg Competition in the Telecommunications Industry

a) (2 pts) Firm B maximizes its profit, given by:

$$\begin{aligned}\pi_B(q_A, q_B) &= P(Q) \times q_B - C_B(q_B) \\ &= (100 - (q_A + q_B))\, q_B - (10q_B + 5) \\ &= -q_B^2 + (90 - q_A)\, q_B - 5\end{aligned}$$

F.O.C. :

$$\frac{\partial\left(\pi_B(q_A, q_B)\right)}{\partial q_B} = 0 \iff q_B^*(q_A) = \frac{90 - q_A}{2}$$

(S.O.C. is satisfied : $\frac{\partial^2(\pi_B(q_A,q_B))}{\partial q_B^2} = -2 < 0$, so F.O.C. is sufficient.)

b) (2 pts) Firm A is the leader, so it maximizes its profit given that Firm B will choose its output according to the reaction function we derived. The profit for Firm A is:

$$\begin{aligned}\pi_A(q_A, q_B^*(q_A)) &= P(q_A + q_B^*(q_A)) \times q_A - C_A(q_A) \\ &= \left(100 - (q_A + \frac{90 - q_A}{2}\right) q_A - (20q_A + 10) \\ &= -\frac{q_A^2}{2} + 35q_A - 10\end{aligned}$$

F.O.C. :

$$\frac{\partial\left(\pi_A(q_A, q_B^*(q_A))\right)}{\partial q_A} = 0 \iff q_A^S = 35$$

(S.O.C. is satisfied : $\frac{\partial^2\left(\pi_A(q_A,q_B^*(q_A))\right)}{\partial q_A^2} = -1 < 0$, so F.O.C. is sufficient.)

c) (2 pts) Now that we know $q_A^S = 35$, substitute it into Firm B's reaction function to find $q_B^S = q_B^*(35) = \frac{90-35}{2} = 27.5$.

The total quantity supplied in the market is:

$$Q^S = q_A^S + q_B^S = 35 + 27.5 = 62.5$$

d) (2 pts) Substitute $Q^S = 62.5$ into the demand function to find the market price:

$$P^S = P(Q^S) = 100 - Q^S = 100 - 62.5 = 37.5$$

Firm A's profit is thus worth

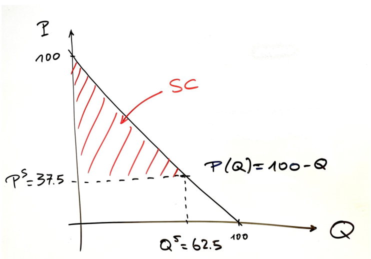
$$\pi_A(q_A^S, q_B^S) = P(Q^S) \times q_A^S - C_A(q_A^S) = 37.5 \times 35 - (20 \times 35 + 10) = 17.5 \times 35 - 10 = 602.5$$

Firm B's profit is worth

$$\pi_B(q_A^S, q_B^S) = P(Q^S) \times q_B^S - C_B(q_B^S) = 37.5 \times 27.5 - (10 \times 27.5 + 5) = 27.5 \times 27.5 - 5 = 751.25$$

e) (2 pts) Consumer surplus is the area between the demand curve and the price line, up to the total quantity $Q^S = 62.5$. The formula for consumer surplus is:

$$\begin{aligned} CS &= Q^S \times \frac{100 - P(Q^S)}{2} \\ &= 62.5 \frac{100 - 37.5}{2} = \frac{62.5 \times 62.5}{2} = 1953.125 \end{aligned}$$



Total social welfare is the sum of consumer surplus and the total profits of the firms.

$$SW = CS + \pi_A + \pi_B = 1953.125 + 602.5 + 751.25 = 3306.875$$

Exercise 2. Product differentiation (10 pts).

1) (1 pt) Fix a pair of qualities (θ_1, θ_2) where $\theta_1 < \theta_2$. The pure strategy Nash equilibrium in price $({p_1}^*, {p_2}^*)$ is given by:

$$p_1(p_2({p_1}^*, \theta_1, \theta_2), \theta_1, \theta_2) = {p_1}^* \tag{1}$$

$$p_2({p_1}^*, \theta_1, \theta_2) := {p_2}^* \tag{2}$$

(1) yields to

$$\frac{{p_1}^* + c + \theta_2 - \theta_1}{2} + c = 2{p_1}^*$$

That is

$${p_1}^* = c + \frac{\theta_2 - \theta_1}{3}$$

Using (1) into (2) we obtain

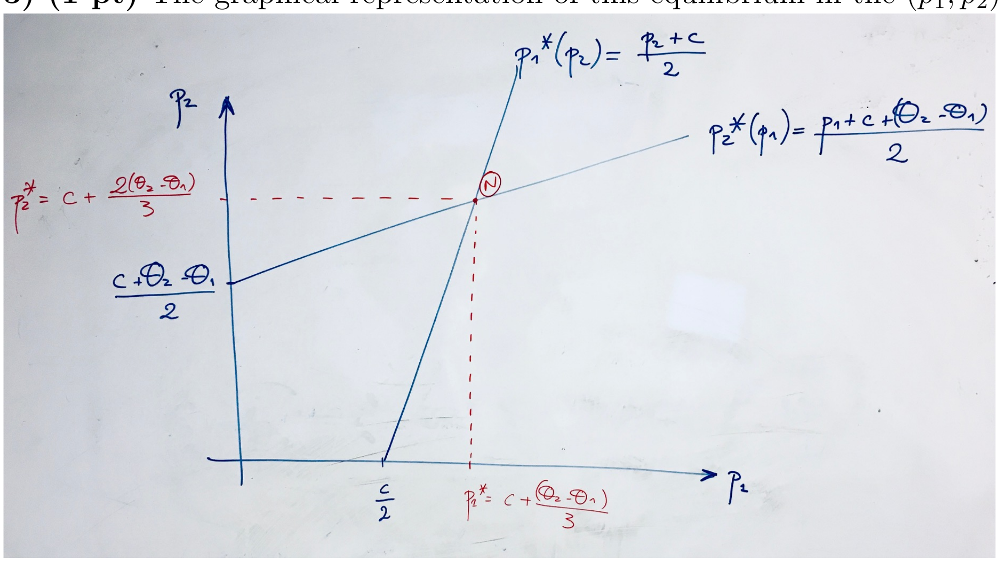
$${p_2}^* = c + \frac{2}{3}(\theta_2 - \theta_1)$$

The pure strategy Nash equilibrium in price is

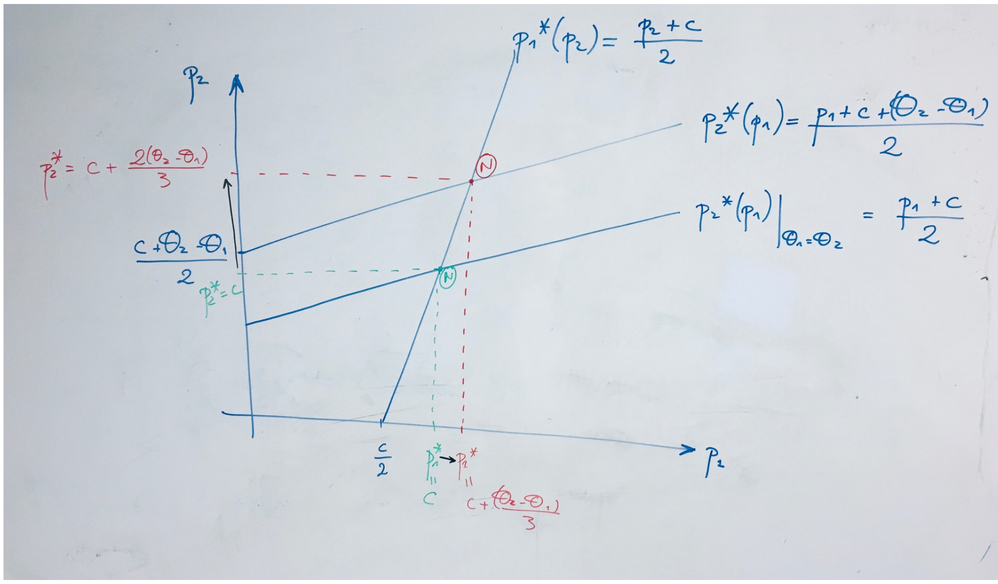
$$({p_1}^*, {p_2}^*) = \left(c + \frac{\theta_2 - \theta_1}{3}, c + \frac{2}{3}(\theta_2 - \theta_1)\right).$$

2) (1 pt) When the qualities are almost identical (i.e. when $\theta_2 = \lim_{\epsilon \to 0,\ \epsilon > 0}(\theta_1 + \epsilon)$, there is almost no differentiation ($\theta_2 - \theta_1 \simeq 0$), both firms price almost at marginal cost (i.e., ${p_1}^* = {p_2}^* \simeq c$) and one obtain Bertrand paradox: two firms are sufficient to replicate pure and perfect competition.

3) (1 pt) The graphical representation of this equilibrium in the (p_1, p_2) space is given by:



4) (1 pt) With an increase in quality differentiation, the price equilibrium would move in the following way:



5) (1 pt) From our first answer, the fraction $\frac{{p_2}^*-{p_1}^*}{\theta_2-\theta_1}$ worth $\frac{c+\frac{\theta_2-\theta_1}{3}-(c+\frac{2}{3}(\theta_2-\theta_1))}{\theta_2-\theta_1} = \frac{1}{3}$. So, the corresponding residual demands write as

$D_1({p_1}^*, {p_2}^*, \theta_1, \theta_2) = \frac{1}{3}$ and $D_2({p_1}^*, {p_2}^*, \theta_1, \theta_2) = \frac{2}{3}$.

Using the residual demand functions we can compute firm i's profits as $\pi_i^* = (p_i^* - c) D_i({p_1}^*, {p_2}^*, \theta_1, \theta_2)$. This yields to

$$ {\pi_1}^* = \frac{1}{9}(\theta_2 - \theta_1) \text{ and } {\pi_2}^* = \frac{4}{9}(\theta_2 - \theta_1). $$

6) (1 pt) Reasoning by backward induction, we use $({p_1}^*, {p_2}^*)$ as the solution for the second period. From, $\frac{\partial \pi_i^*}{\partial(\theta_2-\theta_1)} > 0$, $i = 1, 2$. If the quality is costless then each firm's profits increase with the product differentiation. So whatever the firm 1's (resp. 2's) quality choice θ_1 (resp. θ_2) the firm 2's (resp. 1's) best response is to choose the quality $\bar{\theta}$ (resp. $\underline{\theta}$). Choosing $\bar{\theta}$ (resp. $\underline{\theta}$) is a firm 2's (resp. 1's) dominant strategy. A pure strategy Nash equilibrium of the quality choice is given by $(\theta_1, \theta_2) = (\underline{\theta}, \bar{\theta})$.

7) (1 pt) The two-stage game equilibrium is given by $(\theta_1, \theta_2, {p_1}^*, {p_2}^*) = (\theta, \theta, c + \frac{\bar{\theta}-\underline{\theta}}{3}, c + \frac{2}{3}(\bar{\theta} - \underline{\theta}))$. The corresponding profits are ${\pi_1}^* = \frac{1}{9}(\bar{\theta} - \underline{\theta})$ and ${\pi_2}^* = \frac{2}{3}(\bar{\theta} - \underline{\theta})$. This pure strategy Nash equilibrium of the quality choice exhibits maximal differentiation.

8) (1 pt) This equilibrium is unique because it relies on strictly dominant strategy at the first stage and a unique best-response at the second stage.

9) (1 pt) This equilibrium has been obtained by backward induction. Each firm plays a best-response to the other in each possible subgame. It is subgame perfect.

10) (1 pt) Firms use product differentiation to soften price competition (escape-competition effect). As we have seen in the second answer, without product differentiation ($\theta_2 - \theta_1 = 0$) the firms would obtain the Bertrand competitive zero profits.