

Derivative Instruments

Paris Dauphine University - Master IEF (272)

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LEDa

Exercises + Solutions Chapter 6

Exercise (1)

Let x denote a rate that is quarterly compounded in actual/360. Let z denote the equivalent rate that is continuously compounded in actual/365.

- a) Compute z by first transforming x in a rate y that is continuously compounded rate in actual/360.
- b) Compute z by first transforming x in a rate y' that is quarterly compounded rate in actual/365.
- c) Compare the values obtained in a) and b) when $x = 25\%$. Compare it again when $x = 2.5\%$. Conclude.

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$$z = y \left(\frac{90}{360} \right) \times \frac{365}{90} = \frac{y}{4} \times \frac{365}{90} = \frac{365}{90} \times \ln \left(1 + \frac{x}{4}\right).$$

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$$f(x) := z_a - z_b = \frac{365}{90} \times \ln \left(1 + \frac{x}{4} \right) - 4 \ln \left(1 + \frac{x}{4} \frac{365}{360} \right)$$

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This comes from the approximation of the continuously compounding rate that is obtained from the Taylor's serie. The lower the rate the more accurate is the approximation.

Exercise (2)

It is January 9. The price of a Treasury bond with a 12% coupon that matures on October 12, in four years, is quoted as 102-07.

What is the cash price?

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Between October 12 (last year) and January 9 (current year), there are 89 days.

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The cash price is therefore $102.21875 + \frac{89}{182} \times 6 = \105.15

Exercise (3)

A Eurodollar futures price changes from 96.76 to 96.82.

What is the gain or loss to an investor who is long two contracts?

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The Eurodollar futures price has increased by 6 basis points.

The investor makes a gain per contract of $25 \times 6 = \$150$ or \$300 in total.

Exercise (4, Done)

The 350-day LIBOR rate is 3% with continuous compounding and the forward rate calculate from a Eurodollar futures contract that matures in 350 days is 3.2% with continuous compounding.

Estimate the 440-day zero rate.

Solution (4)

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$$R_2 =$$

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with $F_1 = 3.2\%$, $T_1 = 350$, $T_2 = 440$, and $R_1 = 3\%$. So

$$R_2 = \frac{3.2\% \times 90 + 3\% \times 350}{440} = 3.0409\%.$$

Exercise (5)

It is January 30. You are managing a bond portfolio worth \$6 million. The duration of the portfolio in six months will be 8.2 years. The September Treasury bond futures price is currently 108-15, and the cheapest-to-deliver bond will have a duration of 7.6 years in September.

How should you hedge against changes in interest rates over the next six months?

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Rounding to the nearest whole number, 60 contracts should be shorted. The position should be closed out at the end of July.

Exercise (6)

Suppose that the Treasury bond futures price is 101-12. Which of the following four bonds is cheapest to deliver?

Bond	Price	Conversion Factor
1	125-05	1.2131
2	142-15	1.3792
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$$\text{Quoted Price} - \text{Futures Price} \times \text{Conversion Factor}$$

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Calculating this factor for each of the 4 bonds we get:

$$\text{Bond 1: } 125.15625 - 101.375 \times 1.2131 = 2.178$$

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Bond 4 is therefore the cheapest to deliver.

Exercise (7)

Suppose that the 300-day LIBOR zero rate is 4% and Eurodollar quotes for contracts maturing in 300, 398 and 489 days are 95.83, 95.62, and 95.48.

Calculate 398-day and 489- day LIBOR zero rates.

Assume no difference between forward and futures rates for the purposes of your calculations.

(Hint: The forward rates calculated from the Eurodollar futures are expressed with an actual/360 day count and quarterly compounding. The use of our formula then require these rates to be expressed with continuous compounding and an actual/365 day count)

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Solution (7)

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$$R_2 = \frac{F_1 (T_2 - T_1) + R_1 T_1}{T_2}.$$

with $F_1 =$

Solution (7)

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$$R_2 = \frac{F_1 (T_2 - T_1) + R_1 T_1}{T_2}.$$

with $F_1 = 4.2060\%$, $T_1 = 300$, $T_2 =$

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$$R_2 = \frac{F_1 (T_2 - T_1) + R_1 T_1}{T_2}.$$

with $F_1 = 4.2060\%$, $T_1 = 300$, $T_2 = 398$, and $R_1 =$

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$$\frac{4.2060 \times 98 + 4 \times 300}{398} =$$

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$$\frac{4.2060 \times 98 + 4 \times 300}{398} = 4.0507$$

The 489 day rate is obtained with $F_1 =$

Solution (7)

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$$\frac{4.4167 \times 91 + 4.0507 \times 398}{489} =$$

Solution (7)

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$$\frac{4.4167 \times 91 + 4.0507 \times 398}{489} = 4.1188$$

We are assuming that the first futures rate applies to 98 days rather than the usual 91 days. The third futures quote is not needed.

Exercise (8, Done)

On August 1 a portfolio manager has a bond portfolio worth \$10 million. The duration of the portfolio in October will be 7.1 years. The December Treasury bond futures price is currently 91-12 and the cheapest-to-deliver bond will have a duration of 8.8 years at maturity.

- a) How should the portfolio manager immunize the portfolio against changes in interest rates over the next two months?
- b) How can the portfolio manager change the duration of the portfolio to 3.0 years?

Solution (8)

a)

Exercise (8, Done)

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Solution (8)

a) The treasurer should short Treasury bond futures contract. If bond prices go down, this futures position will provide offsetting gains. The number of contracts that should be shorted is

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$$\frac{10,000,000}{91,375} \frac{7.1}{8.8} = 88.30$$

Exercise (8)

On August 1 a portfolio manager has a bond portfolio worth \$10 million. The duration of the portfolio in October will be 7.1 years. The December Treasury bond futures price is currently 91-12 and the cheapest-to-deliver bond will have a duration of 8.8 years at maturity.

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Solution (8)

Rounding to the nearest whole number 88 contracts should be shorted.
b)

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b) In a) the problem is designed to reduce the duration to zero. To reduce the duration from 7.1 to 3.0 instead of from 7.1 to 0, the treasurer should short

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$$\frac{4.1}{7.1} \times 88.30 = 50.99$$

Exercise (9)

The three-month Eurodollar futures price for a contract maturing in six years is quoted as 95.20. The standard deviation of the change in the short-term interest rate in one year is 1.1%.

Estimate the forward LIBOR interest rate for the period between 6.00 and 6.25 years in the future.

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$$\text{Forward Rate} = \text{Futures Rate} - \frac{\sigma^2}{2} T_1 T_2$$

with $\sigma =$

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From

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with $\sigma = 0.011$, $T_1 = 6$, and $T_2 =$

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with $\sigma = 0.011$, $T_1 = 6$, and $T_2 = 6.25$, we have

$$\text{Forward Rate} = \text{Futures Rate} -$$

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$$\begin{aligned} \text{Forward Rate} &= \text{Futures Rate} - \frac{0.011^2}{2} \times 6 \times 6.25 \\ &= \text{Futures Rate} - \end{aligned}$$

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with $\sigma = 0.011$, $T_1 = 6$, and $T_2 = 6.25$, we have

$$\begin{aligned} \text{Forward Rate} &= \text{Futures Rate} - \frac{0.011^2}{2} \times 6 \times 6.25 \\ &= \text{Futures Rate} - 0.002269 \end{aligned}$$

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The convexity adjustment $\frac{\sigma^2}{2} T_1 T_2$ is then about 23 basis points. The futures Rate is $\frac{100-95.20}{100} = 4.8\%$ with quarterly compounding and an actual/360 day count. This becomes

$$4.8 \times \frac{365}{360} = 4.867\%$$

with an actual/360 day count (see Exercise 1 b)).
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$$4 \ln \left(1 + \frac{0.04867}{4} \right) =$$

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It is

$$4 \ln \left(1 + \frac{0.04867}{4} \right) = 4.84\%$$

with continuous compounding. The forward rate is therefore

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It is

$$4 \ln \left(1 + \frac{0.04867}{4} \right) = 4.84\%$$

with continuous compounding. The forward rate is therefore

$$4.84 - 0.23 =$$

Solution (9)

The convexity adjustment $\frac{\sigma^2}{2} T_1 T_2$ is then about 23 basis points. The futures Rate is $\frac{100-95.20}{100} = 4.8\%$ with quarterly compounding and an actual/360 day count. This becomes

$$4.8 \times \frac{365}{360} = 4.867\%$$

with an actual/360 day count (see Exercise 1 b)).

It is

$$4 \ln \left(1 + \frac{0.04867}{4} \right) = 4.84\%$$

with continuous compounding. The forward rate is therefore

$$4.84 - 0.23 = 4.61\%$$

with continuous compounding.