Introduction to Derivative Instruments

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Slides on book: John C. Hull, “Options, Futures, and Other Derivatives”, Pearson ed.

LEDa

Chapter 1

Chapter 1: Introduction
Outline

What is a Derivative?

- A derivative is an instrument whose value depends on (or derives from) the value of another asset.
  - Examples of underlying assets: stocks, currencies, interest rates, commodities, debt instruments, electricity, insurance payouts, the weather, . . .
  - Examples of derivatives: futures, forwards, swaps, options, exotics . . .
- Derivatives exchanges have existed for a long time.
  - A board of trade was established at Chicago in 1848 to bring farmers and merchants together.
  - Initially its main task was to standardize the quantities and qualities of the grains that were traded.
  - Within a few years, the first futures-type contract was developed.
  - Speculators soon became interested in the contract and found trading the contract to be an attractive alternative to trading the grain itself.

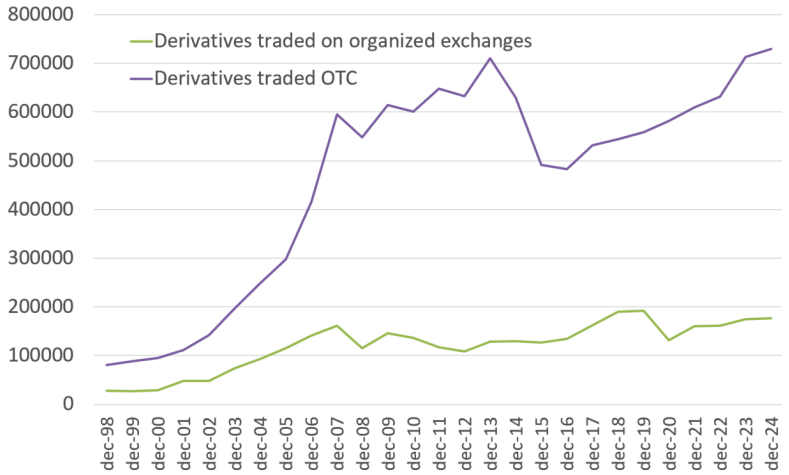
Why Derivatives Are Important?

- Derivatives play a key role in transferring risks in the economy.
- Many financial transactions have embedded derivatives.
- The real options approach to assessing capital investment decisions has become widely accepted.

How Derivatives Are Traded?

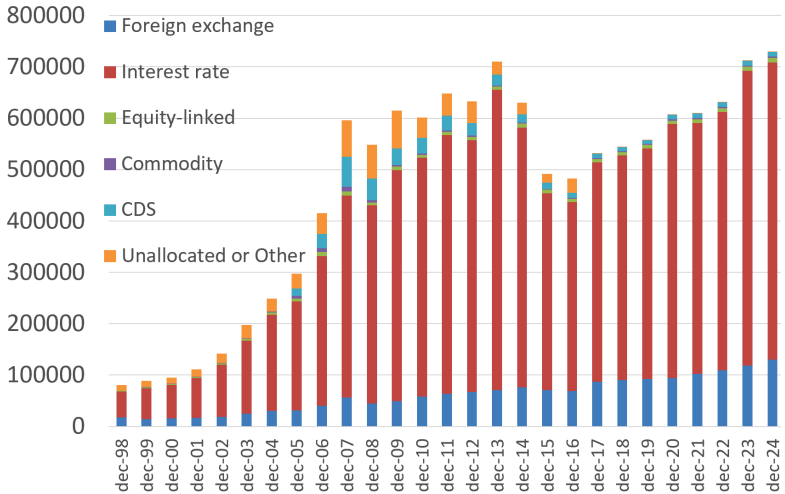
- On exchange-traded markets such as the Chicago Board Options Exchange (CBOE).
- In the over-the-counter (OTC) market where traders working for banks, fund managers and corporate treasurers contact each other directly.
- Both the over-the-counter and the exchange-traded market for derivatives are huge.
- The over-the-counter market is much larger than the exchange-traded market.

Size of OTC and Exchange-Traded Markets
Notional amounts outstanding



Semiannual, Billions of US$ — Source: Bank for International Settlements

Global OTC Derivatives Market
Notional amounts outstanding



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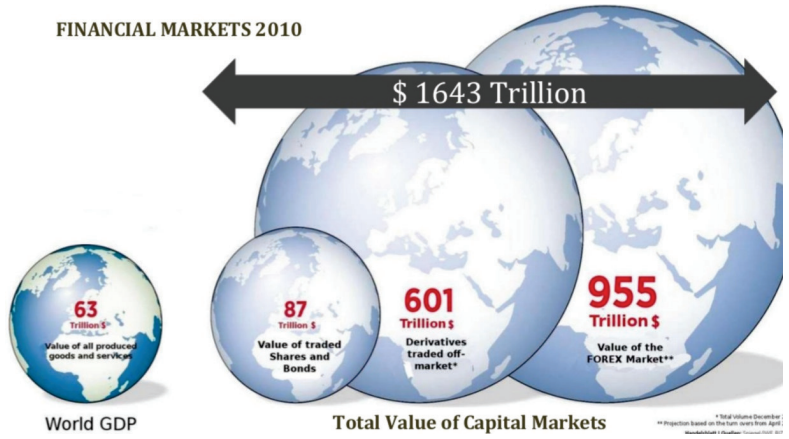
Notional amounts outstanding



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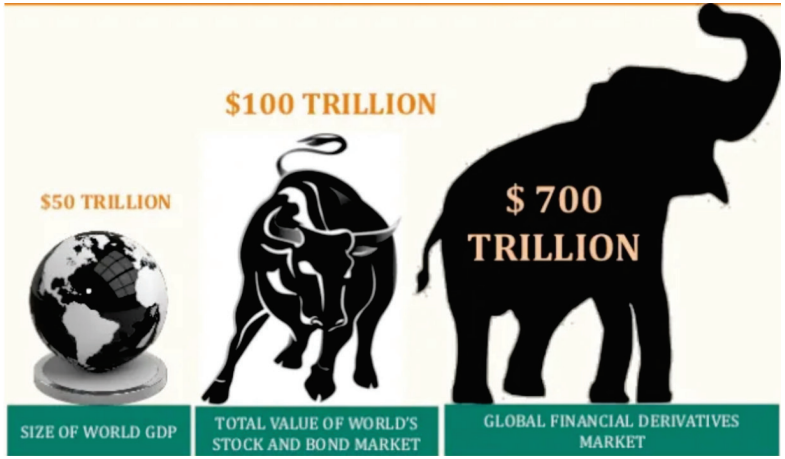
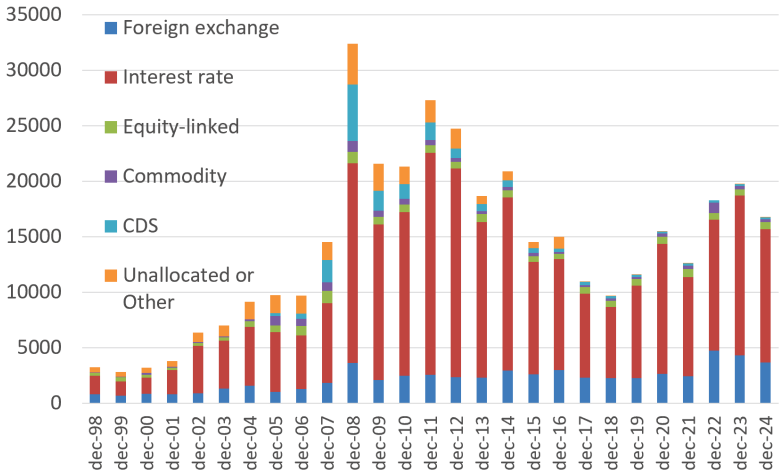
Notional amounts outstanding



Global OTC Derivatives Market
Notional vs Gross market values

- The notional amount of outstanding OTC derivatives contracts (Forward, Swap, Options) corresponds to the face value of the underlying asset.
  - But parties to a derivative contract are almost never required to pay out the full value of the asset
  - Hence the notional amount outstanding is seen as a poor reflection of the actual risk.
- The gross market value, in contrast, is the total amount paid by companies for outstanding contracts.

Global OTC Derivatives Market
Gross market values



Semiannual, Billions of US$ — Source: Bank for International Settlements

How Derivatives are Used

- To hedge risks
- To speculate (take a view on the future direction of the market)
- To lock in an arbitrage profit
- To change the nature of a liability
- To change the nature of an investment without incurring the costs of selling one portfolio and buying another

Chapter 1: Introduction
Outline

Forward Contracts

- A **forward contract** is an agreement to buy or sell an asset at a certain future time for a certain price.
  - It can be contrasted with a **spot contract**, which is an agreement to buy or sell an asset *today*.
- A forward contract is traded in the over-the-counter market—usually between two financial institutions or between a financial institution and one of its clients.
- The party that has agreed *to buy* (resp. *sell*) has what is termed a **long** *(resp.* **short***)* **position**.
- The forward price for a contract is the delivery price that would be applicable to the contract if were negotiated today (i.e., it is the delivery price that would make the contract worth exactly zero)
- The forward price may be different for contracts of different maturities (as shown by the next table).

Example of Forward Contract

	Bid	Offer
Spot	1.4407	1.4411
1-month forward	1.4408	1.4413
3-month forward	1.4410	1.4415
6-month forward	1.4416	1.4422

Table 1.1 Exchange rate between the British pound (GBP) and the US dollar (USD) on May 24, 2018

- The quote is for the number of USD per GBP.
- The first row indicates that the bank is prepared to buy GBP (also known as sterling) in the spot market (i.e., for virtually immediate delivery) at the rate of $1.4407 per GBP and sell sterling in the spot market at $1.4411 per GBP.

Example of Forward Contract

	Bid	Offer
Spot	1.4407	1.4411
1-month forward	1.4408	1.4413
3-month forward	1.4410	1.4415
6-month forward	1.4416	1.4422

Table 1.1 Exchange rate between the British pound (GBP) and the US dollar (USD) on May 24, 2018

- The second, third, and fourth rows indicate that the bank is prepared:
  - to buy sterling in 1, 3, and 6 months at $1.4408, $1.4410, and $1.4416 per GBP, respectively; and
  - to sell sterling in 1, 3, and 6 months at $1.4413, $1.4415, and $1.4422 per GBP, respectively.

Using forward contract to hedge foreign currency risk

- Suppose that, on May 24, 2018, the treasurer of a US corporation knows that the corporation will pay £1 million in 6 months (i.e., on November 24, 2018) and wants to hedge against exchange rate moves.
  - Using the quotes in Table 1.1, the treasurer can agree to buy £1 million 6 months forward at an exchange rate of 1.4422.
  - The corporation then has a long forward contract on GBP.
    - ⋆ It has agreed that on November 24, 2018, it will buy £1 million from the bank for $1.4422 million.
  - The bank has a short forward contract on GBP.
    - ⋆ It has agreed that on November 24, 2018, it will sell £1 million for $1.4422 million.
  - Both sides have made a binding commitment.
  - What are the possible outcomes in the previous example?

Payoffs from Forward Contracts

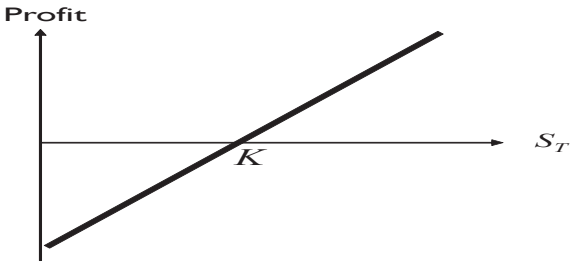


Figure: **Figure 1.a** Profit from a Long Forward Position.

- The payoff from a long position in a forward contract on one unit of an asset is

$$S_T - K$$

where K is the delivery price and S_T is the spot price of the asset at maturity of the contract.

Payoffs from Forward Contracts

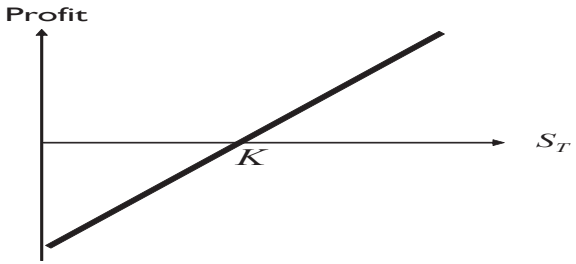


Figure: **Figure 1.a** Profit from a Long Forward Position.

- In the example just considered, $K = 1.4422$ and the corporation has a long contract.
  - When $S_T = 1.5000$, the payoff is $\$0.0578$ per $\$1$;
  - when $S_T = 1.3500$, it is $-\$0.0922$ per $\$1$.

Payoffs from Forward Contracts

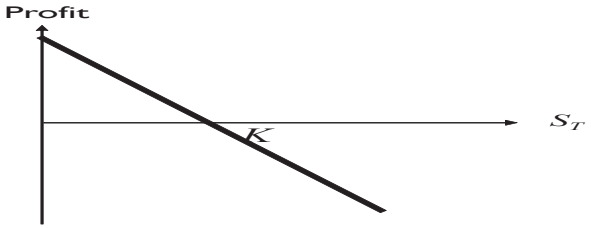


Figure: **Figure 1.b** Profit from a Short Forward Position. K: Delivery price; S_T: Price of asset at contract maturity.

- The payoff from a long position in a forward contract on one unit of an asset is

$$K - S_T$$

1. Gold: An Arbitrage Opportunity?

Question

Suppose that:

- The spot price of gold is US$1,400
- The 1-year forward price of gold is US$1,500
- The 1-year US$ interest rate is 5% per annum
- You can borrow or lend money for 1 year at 5%.

Is there an arbitrage opportunity?

Solution

2. Gold: Another Arbitrage Opportunity?

Question

Suppose that:

- The spot price of gold is US$1,400
- The 1-year forward price of gold is US$1,400
- The 1-year US$ interest rate is 5% per annum
- You are an investor owning gold as part of a portfolio.

Is there an arbitrage opportunity?

Solution

The Forward Price of Gold

- If the spot price of gold is S and the forward price for a contract deliverable in T years is F, then the no-arbitrage condition writes as

$$F = S(1+r)^T$$

where r is the 1-year (domestic currency) risk-free rate of interest.

- In our examples, $S = 1400$, $T = 1$, and $r = 0.05$ so that $F = 1400(1+0.05) = 1470$.
- If $F > S(1+r)^T$ (resp. $F < S(1+r)^T$) there is an arbitrage opportunity to be short (resp. long).

Chapter 1: Introduction
Outline

Futures Contracts

- Like forward contracts, a **futures contract** is an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price.
- Unlike forward contracts, futures contracts are normally traded on an exchange.
- The futures price (i.e., the delivery price) is determined in the same way as other prices (i.e., by the laws of supply and demand).
  - If more traders want to go long than to go short, the price goes up;
  - if the reverse is true, then the price goes down.
- A very wide range of commodities and financial assets form the underlying assets in the various contracts.
  - The commodities include pork bellies, live cattle, sugar, wool, lumber, copper, aluminum, gold, and tin.
  - The financial assets include stock indices, currencies, and Treasury bonds.

Major Exchanges Trading Futures

- CME Group (formerly Chicago Mercantile Exchange and Chicago Board of Trade)
  - The largest exchanges on which futures contracts are traded
- NYSE Euronext
- BM&F (Sao Paulo, Brazil)
- TIFFE (Tokyo)
- ...

Examples of Futures Contracts

- Agreement to:
  - Buy 100 oz. of gold at US$1400/oz. in December
  - Sell £62,500 at 1.4500 US$/£ in March
  - Sell 1,000 bbl. of oil at US$90/bbl. in April

Chapter 1: Introduction
Outline

Options

- An **option** gives the holder the *right* to buy or sell the underlying asset by a certain date for a certain price.
  - It can be contrasted with a **forward contract**, which gives the holder the *obligation* to buy or sell at a certain price.
- Options are traded both on exchanges and in the over-the-counter market.
- There are two types of option.
  - A **call** (resp. **put**) option gives the holder the right to *buy* (resp. *sell*) the underlying asset by a certain date for a certain price.
- The price in the contract is known as the **exercise price** or **strike price**

Options

- The date in the contract is known as the **expiration date** or **maturity**.
  - **American options** can be exercised at any time up to the expiration date.
  - **European options** can be exercised only on the expiration date itself.
  - Note that the terms *American* and *European* do not refer to the location of the option or the exchange.

Participants

- At this stage we note that there are four types of participants in options markets:
  - Buyers of calls;
  - Sellers of calls;
  - Buyers of puts; and
  - Sellers of puts.
- Buyers are referred to as having *long positions*; sellers are referred to as having *short positions*.
- Selling an option is also known as *writing the option*.

Example of call option

Strike Price	Jul 2018 Bid	Jul 2018 Offer	Sept 2018 Bid	Sept 2018 Offer	Dec 2018 Bid	Dec 2018 Offer
460	43.30	44.00	51.90	53.90	63.40	64.80
480	28.60	29.00	39.70	40.40	50.80	52.30
500	17.00	17.40	28.30	29.30	40.60	41.30
520	9.00	9.30	19.10	19.90	31.40	32.00
540	4.20	4.40	12.70	13.00	23.10	24.00
560	1.75	2.10	7.40	8.40	16.80	17.70

Table 1.2 Google Call Option Prices
(June 15, 2018; Stock Price is bid 497.07, offer 497.25)

Example of call option

- Suppose an investor instructs a broker to buy one December call option contract on Google with a strike price of $520.
- The (offer) price is $32.00, as indicated in Table 1.2.
  - This is the price for an option to buy one share.
  - In the United States, an option contract is a contract to buy or sell 100 shares.
  - Therefore, the investor must arrange for $3,200 to be remitted to the exchange through the broker.
- In our example, the investor has obtained at a cost of $3,200 the right to buy 100 Google shares for $520 each.
- If the price of Google does not rise above $520 by December 18, 2018, the option is not exercised and the investor loses $3,200.
- But if Google does well and the option is exercised when the bid price for the stock is $600, the investor is able to buy 100 shares at $520 and immediately sell them for $600 for a profit of $8,000, or $4,800 when the initial cost of the options is taken into account.

Example of put option

Strike Price	Jul 2018 Bid	Jul 2018 Offer	Sept 2018 Bid	Sept 2018 Offer	Dec 2018 Bid	Dec 2018 Offer
460	6.30	6.60	15.70	16.20	26.00	27.30
480	11.30	11.70	22.20	22.70	33.30	35.00
500	19.50	20.00	30.90	32.60	42.20	43.00
520	31.60	33.90	41.80	43.60	52.80	54.50
540	46.30	47.20	54.90	56.10	64.90	66.20
560	64.30	66.70	70.00	71.30	78.60	80.00

Table 1.3 Google Put Option Prices
(June 15, 2018; Stock Price is bid 497.07, offer 497.25).

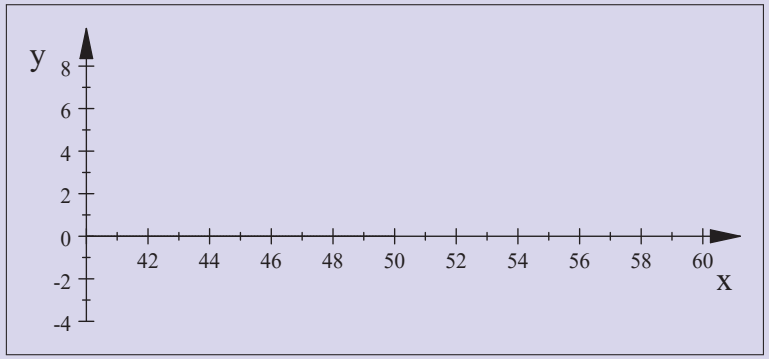
Example of put option

- An alternative trade for the investor would be to sell one September put option contract with a strike price of $480.
  - This would lead to an immediate cash inflow of 100 x 22.20 = $2,220.
- If the Google stock price stays above $480, the option is not exercised and the investor makes a profit of this amount.
- However, if stock price falls and the option is exercised when the stock price is $420, then there is a loss.
  - The investor must buy 100 shares at $480 when they are worth only $420.
  - This leads to a loss of $6,000, or $3,780 when the initial amount received for the option contract is taken into account.

Exercise (5)

Suppose that a March **call option** to buy a share for $50 costs $2.50 and is held until March. Draw a diagram showing how the profit on a **long position** in the option depends on the stock price at the maturity of the option.

Solution (5)

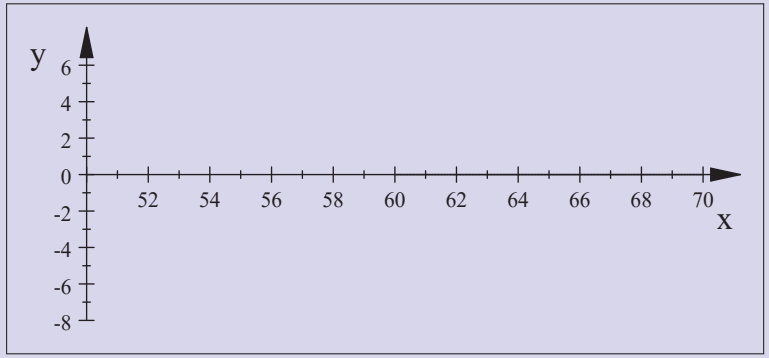


x − axis: Stock price at maturity; y − axis: Profit on long position.

Exercise (6)

Suppose that a June **put option** to sell a share for $60 costs $4 and is held until June. Draw a diagram showing how the profit from a **short position** in the option depends on the stock price at the maturity of the option.

Solution (6)



x − axis: Stock price at maturity; y − axis: Profit on short position.

Properties

- Observes that Tables 1.2 and 1.3 illustrate a number of properties of options.
  - The price of a call (resp. put) option decreases (resp.increases) as the strike price increases
  - Both types of option tend to become more valuable as their time to maturity increases.
    - Markets where the futures price is an increasing (resp. decreasing) function of the time to maturity are known as *normal markets* (resp. *inverted markets*, which generally occurs because a good is in short supply).
- These properties of options will be discussed further.

Chapter 1: Introduction
Outline

Types of Traders

- Three broad categories of traders can be identified:
  - **Hedgers** use derivatives to reduce the risk that they face from potential future movements in a market variable.
  - **Speculators** use them to bet on the future direction of a market variable.
  - **Arbitrageurs** lock in a riskless profit by simultaneously entering into transactions in two or more markets.
- Hedge funds have become big users of derivatives for all three purposes.

Hedging examples: using forward contract and option

- The purpose of hedging is to reduce risk.
  - There is no guarantee that the outcome with hedging will be better than the outcome without hedging.
- (Hedging using forward contract.) A US company will pay £10 million for imports from Britain in 3 months and decides to hedge using a long position in a forward contract.
  - This would have the effect of fixing the price to be paid to the British exporter.
- (Hedging using option.) An investor owns 1,000 Microsoft shares currently worth $28 per share. A two-month put with a strike price of $27.50 costs $1. The investor decides to hedge by buying 10 contracts.
  - The value of Microsoft shares with and without hedging is given in the next figure.

Hedging examples: using forward contract and option

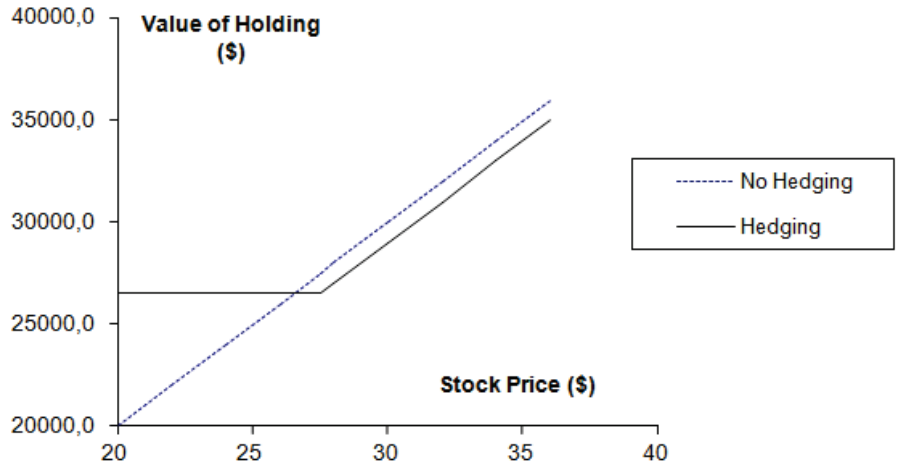


Figure: Value of Microsoft Shares with and without Hedging

Hedging examples: using forward contract and option

- There is a fundamental difference between the use of forward contracts and options for hedging.
  - Forward contracts are designed to neutralize risk by fixing the price that the hedger will pay or receive for the underlying asset.
  - Option contracts, by contrast, provide insurance.
    - ⋆ They offer a way for investors to protect themselves against adverse price movements in the future while still allowing them to benefit from favorable price movements.
  - Unlike forwards, options involve the payment of an up-front fee.

Speculation example: using option

- Suppose that it is October and a specultator is willing to invest $2,000 feels that a stock price will increase over the next 2 months.
- The current stock price is $20 and the price of a 2-month call option with a strike of 22.50 is $1
- What are the alternative strategies?
  - One alternative is to purchase 100 shares.
  - The other alternative involves the purchase of 2,000 call options (i.e., 20 call option contracts).

Speculation example: using option

- Suppose that the speculator's hunch is correct and the price of the stock rises to \$27 by December.
  - The first alternative of buying the stock yields a profit of $100 \times (\$27 - \$20) = \$700$.
  - However, the second alternative is far more profitable.
  - A call option on the stock with a strike price of \$22.50 gives a payoff of \$4.50, because it enables something worth \$27 to be bought for \$22.50.
  - The total payoff from the 2,000 options that are purchased under the second alternative is $2,000 \times \$4.50 = \$9,000$.
  - Subtracting the original cost of the options yields a net profit of $\$9,000 - \$2,000 = \$7,000$
- The options strategy is, therefore, 10 times more profitable than directly buying the stock.
- Options also give rise to a greater potential loss!

Speculation example: using option

- Futures and options are similar instruments for speculators in that they both provide a way in which a type of leverage can be obtained.
- However, there is an important difference between the two.
  - When a speculator uses futures, the potential loss as well as the potential gain is very large. When options are used for long position, no matter how bad things get, the speculator's loss is limited to the amount paid for the options.

Arbitrage Example

Question

A stock price is quoted as £100 in London and \$140 in New York.

The current exchange rate is \$1.4300 per pound.

What is the arbitrage opportunity?

Solution

Chapter 1: Introduction
Outline

Dangers

- Derivatives are very versatile instruments.
- Sometimes traders who have a mandate to hedge risks or follow an arbitrage strategy become (consciously or unconsciously) speculators.
  - The results can be disastrous.
    - One example of this is provided by the activities of Jerome Kerviel at *Société Générale*
- It is important to set up controls to ensure that trades are using derivatives in for their intended purpose.

Chapter 1: Introduction
Outline

Summary

- One of the exciting developments in finance over the last 30 years has been the growth of derivatives markets.
- In many situations, both hedgers and speculators find it more attractive to trade a derivative on an asset than to trade the asset itself.
- Some derivatives are traded on exchanges; others are traded by financial institutions, fund managers, and corporations in the over-the-counter market.
- In this chapter we have taken a first look at forward, futures, and options contracts.
  - A forward or futures contract involves an obligation to buy or sell an asset at a certain time in the future for a certain price.
  - There are two types of options: calls and puts.
    - A call (resp. put) option gives the holder the right to buy (resp. sell) an asset by a certain date for a certain price.
  - Forwards, futures, and options trade on a wide range of different underlying assets.

Summary

- Three main types of traders can be identified: hedgers, speculators, and arbitrageurs.
  - Hedgers are in the position where they face risk associated with the price of an asset.
    - They use derivatives to reduce or eliminate this risk.
  - Speculators wish to bet on future movements in the price of an asset.
    - They use derivatives to get extra leverage.
  - Arbitrageurs are in business to take advantage of a discrepancy between prices in two different markets.
    - If, for example, they see the futures price of an asset getting out of line with the cash price, they will take offsetting positions in the two markets to lock in a profit.