Introduction to Derivative Instruments

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Slides on book: John C. Hull, "Options, Futures, and Other Derivatives", Pearson ed.

LEDa

Chapter 11

Chapter 11: Credit Derivatives

Outline

Introduction

Motivation

- Banks are reluctant to keep loans on their balance sheets.
  - Due to the capital required by regulators, the average return earned on loans is often less attractive than that on other assets.
- *Credit derivatives* are a type of derivatives instrument that allows the transfer of credit risk from a lender to a third party against payment of a fee.
- There are three parties to a credit derivative contract:
  - borrower (*reference entity*);
    - ⋆ E.g., one or more companies or countries
  - lender (*protection buyer*); and
    - ⋆ E.g., bondholders, investors, banks, asset managers, hedge funds, corporate treasuries...
  - third party (*protection seller*).
    - ⋆ E.g., banks, insurance companies, hedge funds...

Introduction

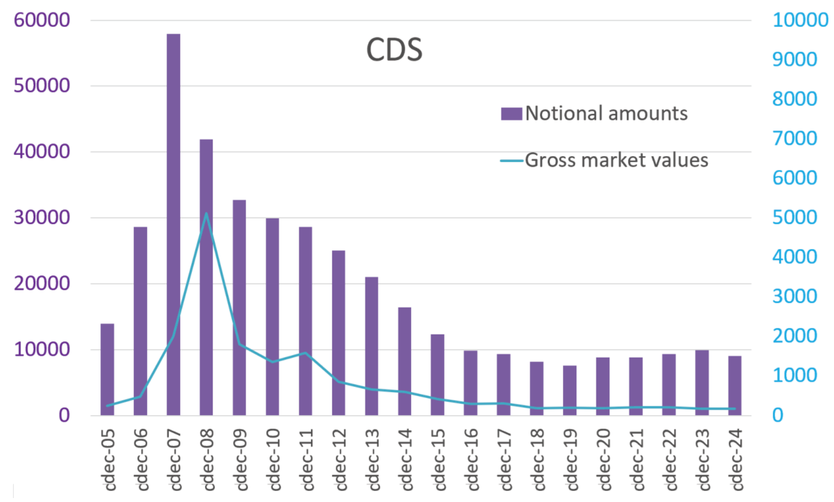
Categories

- Credit derivatives can be categorized as "single-name" or "multi-name".
  - The payoff from a single-name credit derivative depends on the creditworthiness of **one** reference entity.
    - ⋆ The most popular one is a *credit default swap*.
  - The payoff from a multi-name credit derivative depends on the creditworthiness of **several** reference entities.
    - ⋆ The most popular one is a *collateralized debt obligation*.

Introduction
Evolution

- The market in credit derivatives started in 1993 after having been pioneered by J.P. Morgan.
  - J.P. Morgan sold the credit risk (to the European Bank for Reconstruction and Development (EBRD)), from its $4.8 billion credit line to Exxon which faced the threat of $5 billion in punitive damages for the Exxon Valdez oil spill.
- By 1996 there was around $40 billion of outstanding transactions, half of which involved the debt of developing countries.
- In 1998 and 1999, the International Swaps and Derivatives Association (ISDA) developed a standard contract for trading CDS in the OTC market.
- In 2000, the total notional principal for outstanding credit derivatives contracts was about $800 billion.
- By the credit crisis of 2007, this had become $50 trillion.

Introduction
Notional amounts outstanding and Gross market values



Semiannual, Billions of US$ — Source: Bank for International Settlements

Introduction
Subprime crisis

- During the Subprime crisis.
  - Insurance giant AIG was a big seller of protection on the AAA-rated tranches created from mortgages.
    - The protection proved very costly to AIG and the company was bailed out by the U.S. government.
  - Lehman, AIG and other sellers of swaps did not have enough funds to cover them.
    - When Lehman defaulted in September 2008, there was about $400 billion of CDS contracts and $155 billion of Lehman debt outstanding.
- In 2009, the U.S. government came up with the Dodd-Frank Wall Street Report Act to regulate the CDS.
  - The regulation did away with the riskiest swaps and laid restrictions on banks on using the customer deposits for investing in CDS and other derivatives.
  - The Act also requires setting up a central clearinghouse for trading and pricing of swaps.

Introduction
Motivation

- What are the different credit derivatives?
- How do credit derivatives work?
- How are they valued?

Chapter 11: Credit Derivatives
Outline

Credit Default Swap

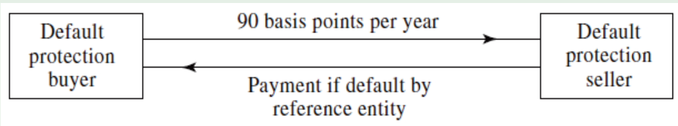
Definition
A **credit default swap** (CDS) is a contract that provides insurance against the risk of a default by particular company.

- The buyer of the CDS makes periodic payments (every 1, 6, or 12 months) to the seller until the end or until a credit event occurs.
- When a credit event occurs:
  - The buyer of the insurance obtains the right to sell bonds (*physical settlement*) issued by the company for their face value to the seller of the insurance.
    - The total face value of the bonds that can be sold is known as the CDS's *notional principal*.
  - Or, as is now usual, the buyer obtains a *cash settlement.*
    - An ISDA-organized auction process is used to determine the mid-market value of the cheapest deliverable bond several days after the credit event.

Credit Default Swap

Example
Suppose that two parties enter into a 5-year CDS on March 20, 2025.

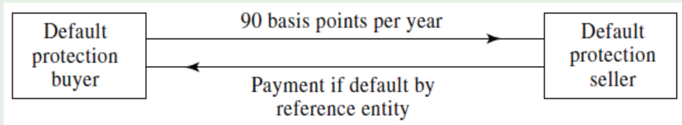
Assume that the notional principal is \$100 million and the buyer agrees to pay 90 basis points per annum for protection against default by the reference entity, with payments being made quarterly in arrears.



(...)

Credit Default Swap

Example



If the reference entity does not default (i.e., there is no credit event):

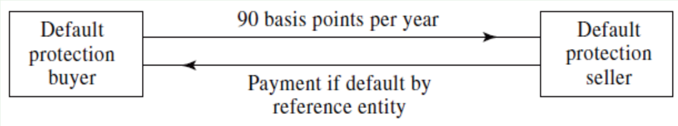
- the buyer receives no payoff; and

- he pays 22.5 ($=\frac{90}{4}$) basis points on \$100 million on June 20, 2025, and every quarter thereafter until March 20, 2030.

The amount paid each quarter is $0.00225 \times 100,000,000 = \$225,000$.

Credit Default Swap

Example



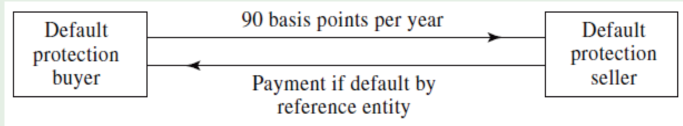
Suppose there is a credit event on May 20, 2028 (2 months into the fourth year):

- In case of physical settlement, the buyer receives $100 million.

- In case of cash settlement, if the auction indicates that the bond is worth $35 per $100 of face value, the buyer receives $65 million (...)

Credit Default Swap

Example



Because these payments are made in arrears, a final accrual payment by the buyer is usually required.

- the buyer would be required to pay to the seller the amount of the annual payment accrued between March 20, 2028, and May 20, 2028 (approximately $150,000), but no further payments would be required.

Credit Default Swap

Definition

The **CDS spread** *(aka* **CDS fee**) is the total amount paid per year, as a percent of the notional principal, the buyer makes to the seller in exchange of receiving a payoff if the asset defaults.

- It is 90 basis points in our previous example.
- When quoting on a new CDS on a company, a market maker might bid 250 basis points and offer 260 basis points.
  - The market maker is then prepared to buy protection by paying 2.5% (resp. sell protection for 2.6%) of the principal per year.

Credit Default Swap
CDS and Bond Yields

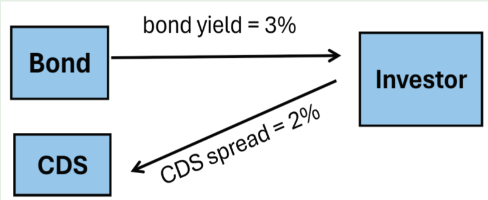
- A CDS can be used to hedge a position in a corporate bond.
  - The effect of the CDS is to convert the corporate bond to a risk-free bond (at least approximately).
- A CDS contract is like an insurance contract in many ways, but there is one key difference.
  - An insurance contract provides protection against losses on an asset that is owned by the protection buyer.
  - In the case of a CDS, the underlying asset does not have to be owned.
- It is common for the volume of CDSs on a company to be greater than its debt.
  - Cash settlement of contracts is then clearly necessary.

Credit Default Swap
CDS and Bond Yields

Example (A)

Suppose that an investor buys a 5-year corporate bond yielding 3% per year for its face value; and

at the same time enters into a 5-year CDS to buy protection at 200 basis points against default.



Credit Default Swap
CDS and Bond Yields

Example (A)

- If the bond issuer does not default, the investor earns $3\% - 2\% = 1\%$ per year.

- If the bond does default, the investor earns $3\% - 2\% = 1\%$ up to the time of the default, then exchange the bond for its face value and invest it at the risk-free rate.

	No default	Default
Bond+CDS	3%-2%=1%	risk-free rate

Credit Default Swap
Default probability

Definition

A **bond's yield spread** is the excess of the promised yield on the bond over the risk-free rate.

- It is the difference between a bond yield and the risk-free rate (e.g., LIBOR/swap rate).
  - It measures the credit risk priced in the bond market.
- The usual assumption is that the excess yield is compensation for the possibility of default.[1]

[1] This assumption is not perfect, as the price of a corporate bond may also be affected by its liquidity. The lower the liquidity, the lower its price.

Credit Default Swap
CDS and Bond Yields

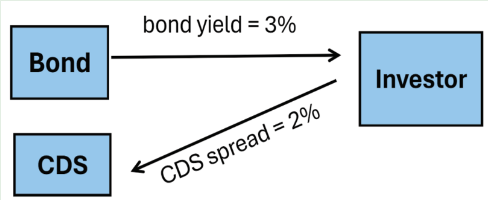
- The CDS spread measures the credit risk priced in the CDS market.
- The difference between the two spreads is the *CDS–bond basis.*
  - It is defined as

$$\text{CDS-bond basis} = \text{CDS spread} - \text{Bond yield spread}$$

- In relatively stable market conditions, CDS–bond basis $\simeq 0$.
  - No difference between the credit risk priced in the bond market (bond yield spread) and in the CDS market (CDS spread).
  - Investors could earn near riskless return by buying a physical bond and CDS on that same bond with equal maturity.

Credit Default Swap
CDS and Bond Yields

Example (A)



So, Bond yield spread should be 2% and risk-free rate 1%.

	No default	Default
Bond	3%	0%
Bond+CDS	3%-2%=1%	1%

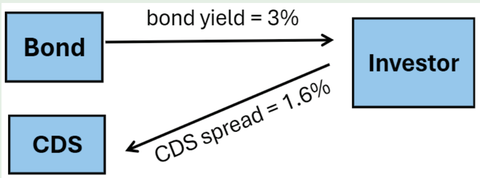
Credit Default Swap
CDS and Bond Yields

- Negative bond-CDS basis (i.e., CDS spread$<$Bond yield spread) leads to approximate arbitrage opportunities.
  - investors can take advantage by simultaneously buying the corporate bond and CDS default protection on the underlying bond.

Credit Default Swap
CDS and Bond Yields

Example (A')

Assume Bond yield spread is 2%, and CDS spread is 1.6% instead of 2%. So, CDS-bond basis$= 1.6\% - 2\% = -0.4\% < 0$



	No default	Default
Bond	3%	0%
Bond+CDS	3%-1.6%=1.4%	1%

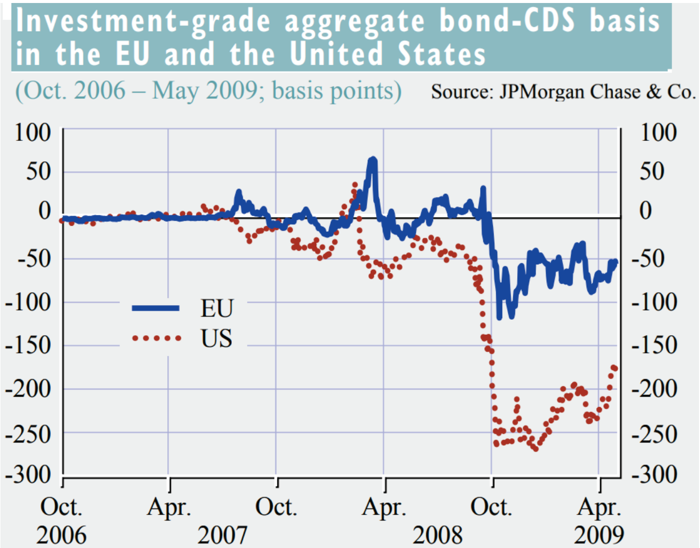
(3%-1.6%=1.4%: > risk-free rate)

Credit Default Swap
CDS and Bond Yields

- Negative bond-CDS basis may be a good indicator of overall deteriorating credit conditions in the corporate bond market.
  - The basis abruptly turned negative following the default of Lehman Brothers in mid-September 2008.
    - The corporate bond market experienced severe stress, whereby issuance conditions deteriorated significantly, funding costs increased markedly, as evidenced by the shift in the average investment-grade[2] bond spread.
  - Liquidity deteriorated also during the Covid-19
    - This is reflected mainly in physical bonds as opposed to CDS indices which are usually far more liquid.

[2] An *investment grade* (IG) is a rating (from *AAA* to BBB^-) that signifies a bond presents a relatively low risk of default.

Credit Default Swap
CDS and Bond Yields



Credit Default Swap
CDS and Bond Yields

Credit Default Swap
CDS and Bond Yields

- Such arbitrage opportunities are not always immediately exploited by market participants.
  - Investors may face credit constraints owing to a worsening of funding conditions.
  - When basis is substantial and volatile, there is a risk that it shifts further into negative territory after an investor has entered into the basis trade.
  - Low liquidity in one or both markets may boost bid-ask spreads to levels that would make arbitrage opportunities less profitable than they appear.
- As liquidity gradually coming back, this dislocation closes down.
  - Central banks' Quantitative Easing programmes contributes reducing the gap as they are buying physical bonds and not CDS instruments.

Credit Default Swap
Default probability

- The CDS spread for a particular reference entity can be calculated from *default probability* estimates.

Definition

A **recovery rate** is the estimated percent of a loan or an obligation that will still be repaid to creditors in the event of a default or bankruptcy.

- The recovery rate of a bond is expressed as a percentage of face value.
  - A recovery rate of 40% on a bond with notional principal of $100 means that in case of default, the bond worth $40.
  - The payoff from a CDS is then $L(1-R)$, where L is the notional principal and R is the recovery rate.

Credit Default Swap
Default probability

Definition

The **loss given default (LGD)** is the estimated loss on an exposure at risk when the reference entity defaults.

- Most of the times[3], it is expressed as an amount of money:

$$LGD(\$) = \text{Exposure at Risk} * (1 - \text{Recovery Rate})$$

  - LGD corresponding to the previous numbers is then \$60.

[3] It is sometimes expressed as a percentage. In this case, LGD typically includes collateral while it ignores it when expressed in dollars amount.

Credit Default Swap
Default probability

- Suppose that the bond yield spread for a T-year bond is $s(T)$ per annum.
  - This means that the average loss rate on the bond between time 0 and time T should be approximately $s(T)$ per annum.
  - Suppose that the average hazard rate[4] during this time is $\bar{\lambda}(T)$.
  - Another expression for the average loss rate is $\bar{\lambda}(T)(1-R)$, where R is the estimated recovery rate.
- This means that it is approximately true that

$$\bar{\lambda}(T)(1-R) = s(T)$$

[4] The *hazard rate* (aka *default intensity*) is the probability of default for a certain time period conditional on no earlier default.

Credit Default Swap
Default probability

Example

Suppose that 1-year bond issued by a corporation yields 150 basis points more than the risk-free rate.

If the recovery rate is estimated at 40%, the average hazard rate for year 1 is

$$\bar{\lambda}(1) = \frac{s(1)}{1-R} = \frac{0.0150}{0.6} = 2.5\%$$

Credit Default Swap
Default probability

Exercise (1)

Suppose further that 2-year and 3-year bonds yield 180 and 195 bp more than the risk-free rate, respectively.

a) What are the average hazard rate $\bar{\lambda}(2)$ for years 1 and 2, and $\bar{\lambda}(3)$ for all three years?

Solution (1)

a)

Exercise (1)

Suppose further that 2-year and 3-year bonds yield 180 and 195 bp more than the risk-free rate, respectively.

a) What are the average hazard rate $\bar{\lambda}(2)$ for years 1 and 2, and $\bar{\lambda}(3)$ for all three years?

b) What is the average hazard rate for the second year?

Solution (1)

b)

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a) What are the average hazard rate $\bar{\lambda}(2)$ for years 1 and 2, and $\bar{\lambda}(3)$ for all three years?

b) What is the average hazard rate for the second year?

c) What is the average hazard rate for the third year?

Solution (1)

c)

- Default probabilities can also be deduced from matching hazard rates to bond prices.

Definition

A **risk-neutral value of a bond** (aka **equivalent risk-free bond's value**) is the value obtained by discounting the expected cash-flows of the bond at the risk-free rate.

- Let $B_{risk-neutral}$ denote the risk-neutral value of a bond with market price B.
  - Let p denote the bond's probability of default.
  - Let rec denote the amount recovered in case of default.
  - We must then have

$$B = p \times rec + (1-p) \times B_{risk-neutral}$$

    - E.g., if $p = 0.5$ and $rec = 0$, $B = \frac{B_{risk-neutral}}{2}$.

- This is equivalent to

$$B_{risk-neutral} - B = p(B_{risk-neutral} - rec)$$

  - LHS = Bond risk premium; RHS = Expected loss.
- More generally, let t_1, t_2, ... t_n, denote the different dates at which a bond may default.
  - Let p_t denote the default probability at time $t \in \{t_1, t_2, ..., t_n\}$.
  - Let $B^t_{risk-neutral}$ denote the (forward) risk-free value of the bond at time $t \in \{t_1, t_2, ..., t_n\}$.
    - I.e., the present value at time t of the sum of cash-flows after time t.
  - Let rec^t denote the present value at time t of the recovery amount at time t.
    - The recovery amount at time t is usually a constant = $L \times R$, where L is the notional principal.

Credit Default Swap
Default probability

- At date 0, we then have

$$B_{risk-neutral} - B = \sum_{t=t_1}^{t_n} p_t (B^t_{risk-neutral} - rec^t) e^{-r \times t}$$

  - LHS = Bond risk premium; RHS = present value of the expected default loss.

Credit Default Swap
Default probability

Exercise (2)

Suppose that the LIBOR/swap rate curve (risk free rate) is flat equal to 3%.

A 1-year corporate bond provides a coupon of 4% per year payable semiannually and it has a yield of 5% (continuous compounding).

Assume that defaults can take place at the beginning of the year and that the recovery rate is 30%.

(a) What is the market price of the bond?

Solution (2)

(a)

Credit Default Swap
Default probability

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(a) What is the market price of the bond?

(b) What is the equivalent risk-free bond's value?

Solution (2)

(b)

Credit Default Swap
Default probability

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(a) What is the market price of the bond?

(b) What is the equivalent risk-free bond's value?

(c) What is the bond risk premium?

Solution (2)

(c)

Credit Default Swap
Default probability

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(a) What is the market price of the bond?

(b) What is the equivalent risk-free bond's value?

(c) What is the bond risk premium?

(d) What is the loss in case of default?

Solution (2)

(d)

Credit Default Swap
Default probability

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(c) What is the bond risk premium?

(d) What is the loss in case of default?

(e) Deduce the risk-neutral default probability p.

Solution (2)

(e)

Credit Default Swap
Default probability

- Let $F(t)$ denote the bond cumulative default rate at time t.
  - The probability that the bond will survive until the end of year n is $1 - F(n)$.
  - Default probability during the n^{th} year writes as $F(n) - F(n-1)$.
    - This probability of default during the n^{th} year as seen today is an *unconditional default probability.*
  - The probability of default during the n^{th} year *conditional on* no earlier default is $\frac{F(n)-F(n-1)}{1-F(n)}$.

Credit Default Swap
Default probability

Example (B)

Consider the following average cumulative default rates

Table 24.1 Average cumulative default rates (%), 1970–2012, from Moody's.

Term (years):	*1*	*2*	*3*	*4*	*5*	*7*	*10*	*15*	*20*
Aaa	0.000	0.013	0.013	0.037	0.106	0.247	0.503	0.935	1.104
Aa	0.022	0.069	0.139	0.256	0.383	0.621	0.922	1.756	3.135
A	0.063	0.203	0.414	0.625	0.870	1.441	2.480	4.255	6.841
Baa	0.177	0.495	0.894	1.369	1.877	2.927	4.740	8.628	12.483
Ba	1.112	3.083	5.424	7.934	10.189	14.117	19.708	29.172	36.321
B	4.051	9.608	15.216	20.134	24.613	32.747	41.947	52.217	58.084
Caa–C	16.448	27.867	36.908	44.128	50.366	58.302	69.483	79.178	81.248

E.g., default probability during the 3^{rd} year for a bond rated Caa or below is $F(3) - F(2) = 36.908\% - 27.867\% = 9.041\%$.

Example (B)

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Term (years):	*1*	*2*	*3*	*4*	*5*	*7*	*10*	*15*	*20*
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Caa–C	16.448	27.867	36.908	44.128	50.366	58.302	69.483	79.178	81.248

E.g., the probability that the bond will survive until the end of year 2 is
$1 - F(2) = 1 - 27.867 = 72.133\%$.
The probability that it will default during the third year conditional on no earlier default is
$\frac{F(3)-F(2)}{1-F(2)} = \frac{9.041\%}{72.133\%} \simeq 12.53\%$.

- Let $V(t)$ denote the cumulative probability of the reference entity surviving to time t (i.e., no default by time t).
  - $V(t) = 1 - F(t)$
  - The probability of default between time t and $t + \Delta(t)$ conditional on no earlier default is

$$\frac{F(t+\Delta(t)) - F(t)}{1 - F(t)} = \frac{V(t) - V(t+\Delta(t))}{V(t)} = \lambda(t)\Delta(t)$$

    where $\lambda(t)$ denote the *hazard rate* at time t.
- We then have

$$V(t+\Delta(t)) - V(t) = -\lambda(t)\Delta(t)V(t)$$

- Taking limits

$$\frac{dV(t)}{dt} = \lim_{\Delta(t)\to 0} \frac{V(t+\Delta(t)) - V(t)}{\Delta(t)} = -\lambda(t)V(t)$$

  from which
$$V(t) = e^{-\int_0^t \lambda(\tau)d\tau} = e^{-\bar{\lambda}(t)}$$
  where $\bar{\lambda}(t)$ denote the average hazard rate between time 0 and time t.
- So,
$$F(t) = 1 - V(t) = 1 - e^{-\bar{\lambda}(t)} \tag{1}$$

Example (C)

Suppose that the hazard rate of the reference entity is 2% per annum for the whole of the 5-year life of a CDS.
From $V(t) = e^{-\bar{\lambda}(t)}$, and $\bar{\lambda}(t) = 2\% \times t$, the probability of survival to time t is $e^{-0.02t}$. We then obtain the following table

Year	*Probability of surviving to year end*	*Probability of default during year*
1	0.9802	0.0198
2	0.9608	0.0194
3	0.9418	0.0190
4	0.9231	0.0186
5	0.9048	0.0183

E.g., the probability of survival to time 3 years is $e^{-0.02\times 3} = 0.9418$.
The probability of default during the 3^{rd} year is
$e^{-0.02\times 2} - e^{-0.02\times 3} = 0.9608 - 0.9418 = 0.0190$.

Credit Default Swap
Valuation

Example (C)

Assuming that payments are made at the rate of s per year, the notional principal is $\$1$, and the risk-free interest rate is 5% per annum (with continuous compounding), we obtain

Time (years)	*Probability of survival*	*Expected payment*	*Discount factor*	*PV of expected payment*
1	0.9802	0.9802s	0.9512	0.9324s
2	0.9608	0.9608s	0.9048	0.8694s
3	0.9418	0.9418s	0.8607	0.8106s
4	0.9231	0.9231s	0.8187	0.7558s
5	0.9048	0.9048s	0.7788	0.7047s
Total				4.0728s

E.g., there is a 0.9418 probability that the third payment of s is made.
The expected payment is therefore $0.9418s$ and its present value is $0.9418s \times e^{-0.05*3} = 08106s$.

Credit Default Swap
Valuation

Example (C)

Assuming that the recovery rate is 40% and defaults always happen halfway through a year, we obtain

Time (years)	*Probability of default*	*Recovery rate*	*Expected payoff (\$)*	*Discount factor*	*PV of expected payoff (\$)*
0.5	0.0198	0.4	0.0119	0.9753	0.0116
1.5	0.0194	0.4	0.0116	0.9277	0.0108
2.5	0.0190	0.4	0.0114	0.8825	0.0101
3.5	0.0186	0.4	0.0112	0.8395	0.0094
4.5	0.0183	0.4	0.0110	0.7985	0.0088
Total					0.0506

E.g., there is a 0.0190 probability of a default halfway through the 3^{rd} year.
The expected payoff at this time is $0.0190 \times 0.6 \times 1 = 0.0114$.
The present value of the expected payoff is $0.0114 \times e^{-0.05*2.5} = 0.0101$.

Credit Default Swap
Valuation

Example (C)

As a final step, assuming that the payments on the CDS are made once a year, at the end of each year, we obtain

Time (years)	*Probability of default*	*Expected accrual payment*	*Discount factor*	*PV of expected accrual payment*
0.5	0.0198	0.0099s	0.9753	0.0097s
1.5	0.0194	0.0097s	0.9277	0.0090s
2.5	0.0190	0.0095s	0.8825	0.0084s
3.5	0.0186	0.0093s	0.8395	0.0078s
4.5	0.0183	0.0091s	0.7985	0.0073s
Total				0.0422s

E.g., there is a 0.0190 probability that there will be a final accrual payment halfway through the third year ($t = 2.5$). The accrual payment is $0.5s$. The expected accrual payment at this time is therefore $0.0190 \times 0.5s = 0.0095s$.
Its present value is $0.0095s \times e^{-0.05*2.5} = 0.0084s$.

Credit Default Swap
Valuation

Example (C)

Hence, aggregating these tables we have

Time (years)	*PV of expected payment*	*Time (years)*	*PV of expected accrual payment*	*PV of expected payoff (\$)*
1	0.9324s	0.5	0.0097s	0.0116
2	0.8694s	1.5	0.0090s	0.0108
3	0.8106s	2.5	0.0084s	0.0101
4	0.7558s	3.5	0.0078s	0.0094
5	0.7047s	4.5	0.0073s	0.0088
Total	4.0728s	*Total*	0.0422s	0.0506

The PV of the expected payments is $4.0728s + 0.0422s = 4.1150s$.
Equating the PV of the expected payoff and payments gives: $4.1150s = 0.0506$.
So, $s = 0.0123$.
The CDS spread for the 5-year deal we have considered should be 0.0123 times the principal or 123 basis points per year.

Credit Default Swap
Marking to Market a CDS

- A CDS, like most other swaps, is marked to market daily.
  - It may have a positive or negative value.
- Suppose, for example the CDS in Example C had been negotiated some time ago for a spread of 150 basis points.
  - The present value of the payoff would be 0.0506 as above.
  - The present value of the payments by the buyer would be $4.1150s = 4.1150 \times 150 = 0.0617$.
  - The value of swap to the seller would therefore be $0.0617 - 0.0506 = 0.0111$ times the principal.
  - Similarly the mark-to-market value of the swap to the buyer of protection would be -0.0111 times the principal.

Credit Default Swap
Legs

Definition

The total (regular fee) payments is called the **fix leg** (aka **premium leg**).

- It is paid by the protection buyer.
- In Example C, the value of the fix leg is $4.1150s$.

Definition

The total payoffs is called the **variable leg** (aka **contingent leg**, or **protection leg**).

- It is the payment at the time of default made by the protection seller.
- In Example C, the value of the variable leg is 0.0506.

Credit Default Swap
Binary CDS

- A *binary* CDS is structured similarly to a regular CDS except that the payoff is a fixed dollar amount.

Example (C')

Suppose the PV of the expected payments is still $4.1150s$, but the payoff is now $\$1$ instead of $\$(1 - R)$. The table that deals with the recovery rate $R = 0.4$ becomes

Time (years)	*Probability of default*	*Expected payoff (\$)*	*Discount factor*	*PV of expected payoff (\$)*
0.5	0.0198	0.0198	0.9753	0.0193
1.5	0.0194	0.0194	0.9277	0.0180
2.5	0.0190	0.0190	0.8825	0.0168
3.5	0.0186	0.0186	0.8395	0.0157
4.5	0.0183	0.0183	0.7985	0.0146
Total				0.0844

By equating the expected payoff and payments, the CDS spread S satisfies $4.1150s = 0.0844$, so $s = 0.0205$, or 205 basis points.

Credit Default Swap
CDS Forwards

Definition

A **forward CDS** is the obligation to buy or sell a particular CDS on a particular reference entity at a particular future time T.

- If the reference entity defaults before time T, the forward contract ceases to exist.
- E.g., a bank could enter into a forward contract to sell 5-year protection on a company for 280 basis points starting in 1 year.
  - If the company defaulted before the 1-year point, the forward contract would cease to exist.

Credit Default Swap
CDS Options

Definition
A **CDS option** is an option to buy or sell a particular CDS on a particular reference entity at a particular future time T.

- Like CDS forwards, CDS options are usually structured so that they cease to exist if the reference entity defaults before option maturity.
- E.g., a trader could negotiate the right to buy 5-year protection on a company starting in 1 year for 280 basis points.
  - This is a call option.
  - If the 5-year CDS spread for the company in 1 year turns out to be more than 280 basis points, the option will be exercised;
    - otherwise it will not be exercised.
  - The cost of the option would be paid up front.

Credit Default Swap
CDS Options

- Similarly an investor might negotiate the right to sell 5-year protection on a company for 280 basis points starting in 1 year.
  - This is a put option.
  - If the 5-year CDS spread for the company in 1 year turns out to be less than 280 basis points, the option will be exercised;
    - otherwise it will not be exercised.

Credit Default Swap
CDS Basket

Definition
A **basket CDS** is a CDS with several reference entities.

Definition
A **first-to-default CDS** provides a payoff when the first default occurs.

Definition
A **second-to-default CDS** provides a payoff only when the second default occurs.

Credit Default Swap
CDS Basket

Definition
A **kth-to-default CDS** provides a payoff only when the kth default occurs.

- Payoffs are calculated in the same way as for a regular CDS.
  - After the relevant default has occurred, there is a settlement.
    - The swap then terminates and there are no further payments by either party.

Chapter 11: Credit Derivatives
Outline

Total Return Swaps

Definition

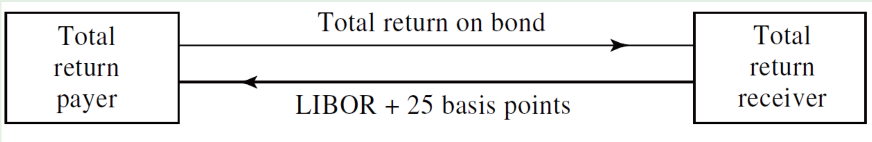
A **total return swap (TRS)** (aka **total rate of return swap (TRORS)**, or **cash-settled equity swap**) is an agreement to exchange the total return on an underlying asset (bond, portfolio of assets...) for LIBOR plus a spread.

- These swaps transfers both the *credit risk* and *market risk* of an underlying asset.
  - They allow the party receiving the total return to gain exposure and benefit from a reference asset without actually having to own it.
    - ⋆ They are popular with hedge funds because they get the benefit of a large exposure with a minimal cash outlay.
- When the underlying asset is a bond, LIBOR is usually set on one coupon date and paid on the next as in a plain vanilla interest rate swap.
- Transfers includes coupons, interest, and the gain or loss on the asset over the life of the swap.

Total Return Swaps

Example (D)

Total return swap: a 5-year agreement with a notional principal of $100 million to exchange the total return on a corporate bond for LIBOR+25 bp.



On coupon payment dates the payer pays the coupons earned on an investment of $100 million in the bond.

The receiver pays interest at a rate of LIBOR+25 bp on $100 million.

Total Return Swaps

- Total return swaps are often used as a financing tool.
- In Example D, we see that they are similar to repos in that they are structured to minimize credit risk when securities are being financed.
  - The receiver wants to invest $100 million in the reference bond.
    - ⋆ Instead of borrowing money at LIBOR+25 bp to buy the bond, it enters into the swap.
  - The payer (likely a financial institution) invests $100 million in the bond.
    - ⋆ The payer retains ownership of the bond for the life of the swap.
    - ⋆ It then faces less credit risk than it would have done if it had lent money to the receiver.
    - ⋆ The bond is used as collateral for the loan.
  - If the receiver defaults the payer does not have the legal problem of trying to realize on the collateral.

Total Return Swaps

- High-cost borrowers who seek financing and leverage, such as hedge funds, are natural receivers.
- Lower cost borrowers, with large balance sheets, are natural payers.
- The spread over LIBOR received by the payer is compensation for bearing the risk that the receiver will default.
  - The payer will lose money if the receiver defaults at a time when the reference bond's price has declined.
  - The spread therefore depends on the credit quality of the receiver, the credit quality of the bond issuer, and the correlation between the two.

Total Return Swaps

- At the end of the life of the swap there is a payment reflecting the change in value of the bond.

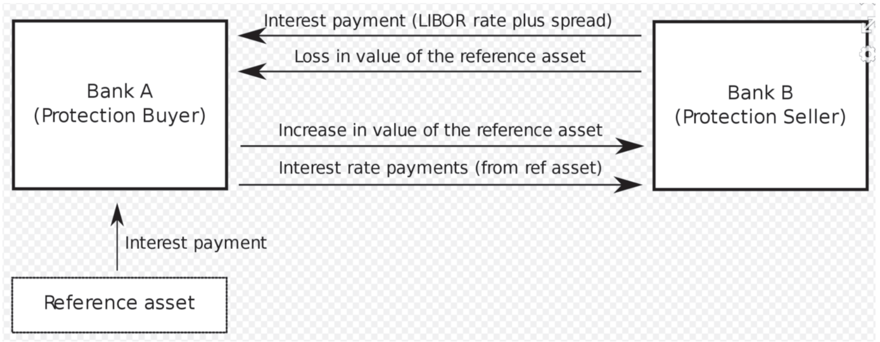
Example (D)

If the bond increases in value by 10% over the life of the swap, the payer is required to pay \$10 million (= 10% $\times$ \$100 million) at the end of the 5 years.

Similarly, if the bond decreases in value by 15%, the receiver is required to pay \$15 million at the end of the 5 years.

If there is a default on the bond, the swap is usually terminated and the receiver makes a final payment equal to the excess of \$100 million over the market value of the bond.

Total Return Swaps



Total Return Swaps

- If the notional principal is added to both sides at the end of the life of the swap, the TRS can be characterized as follows.
  - The receiver pays the cash flows on a \$100 million bond paying LIBOR plus 25 basis points.
  - The payer pays the cash flows on an investment of \$100 million in the corporate bond.
    - If the payer owns the corporate bond, the TRS allows it to pass the credit risk on the bond to the receiver.
    - If it does not own the bond, the TRS allows it to take a short position in the bond.

Chapter 11: Credit Derivatives
Outline

Collateralized debt obligation

Definition

A **collateralized debt obligation (CDO)** is ...

- To be completed...

Chapter 11: Credit Derivatives
Outline

Summary

- *Credit derivatives* enable banks and other financial institutions to actively manage their credit risks.
  - They can be used to transfer credit risk from one company to another and to diversify credit risk by swapping one type of exposure for another.
- The most common credit derivative is a *CDS*.
  - This is a contract where one company buys insurance from another company against a third company (the reference entity) defaulting on its obligations.
    - The payoff is usually the difference between the face value of a bond issued by the reference entity and its value immediately after a default.
  - CDS can be analyzed by calculating the present value of the expected payments and the present value of the expected payoff (in a risk-neutral world).

Summary

- A *forward CDS* is an obligation to enter into a particular CDS on a particular date.
- A *CDS option* is the right to enter into a particular CDS on a particular date.
- Both instruments, forward CDS and CDS option, cease to exist if the reference entity defaults before the date.

Summary

- A TRS is an instrument where the total return on a portfolio of credit sensitive assets is exchanged for LIBOR plus a spread.
  - TRS are often used as financing vehicles.
    - A company wanting to purchase a portfolio of assets will approach a financial institution to buy the assets on its behalf.
    - The financial institution then enters into a TRS with the company where it pays the return on the assets to the company and receives LIBOR plus a spread.
    - The advantage of this type of arrangement is that the financial institution reduces its exposure to a default by the company.