Industrial Organization - Final Exam

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Duration: 90 mn. No document, no calculator allowed.

Exercise 1. Spatial Price Competition with Tariff (9 pts)

Consider a linear country of length one lying on the interval $[0, 1)$, with domestic consumers uniformly distributed along this interval. There are two firms that sell the same physical good. Assume that Firm 1 is located within the country at point $a \in [0, 1)$, and Firm 2 is located at the border, at point 1.

The markets of both domestic Firm 1 and foreign Firm 2 are restricted to the interval $[0, 1)$. Firms compete in price simultaneously. Denote p_i as firm i's price, $i \in \{1, 2\}$. There are no fixed costs, and both firms' marginal costs are equal to a constant $c > 0$.

The market is covered so that all consumers buy exactly one unit of the good. The transportation costs are quadratic, with a parameter $t > 0$ measuring the "intensity" of product differentiation. To purchase from the foreign firm, each consumer has to pay a tariff $\tau \geq 0$.

a) (2 pts) *Denote $\widetilde{x} \in [0, 1)$ as the location of the consumer who is indifferent between buying the good from Firm 1 or Firm 2. Write the firms' demands, denoted as $D_i(p_1, p_2)$ for $i \in \{1, 2\}$.*

b) (2 pts) *Write firm i's profit, denoted as $\pi_i(p_1, p_2)$ for $i \in \{1, 2\}$. Compute firm i's resulting best response, denoted as $p_i^*(p_j)$, where $j \neq i$.*

c) (2 pts) *Solve the Nash equilibrium in prices, denoted as (p_1^N, p_2^N).*

d) (2 pts) *How does each equilibrium price vary with the tariff? Does the tariff benefit the consummers? Give an economic interpretation.*

e) (1 pt) *For each firm, determine whether the tariff is beneficial or detrimental. (Hint: You do not need to compute the entire firm i's profit at equilibrium. The answer can be deduced from the computation of the equilibrium value of $\widetilde{x}$, denoted as $\widetilde{x}^N$).*

Exercise 2. Repeated Monopolistic Competition in Prices (11 pts)

Consider n firms in monopolistic competition for producing goods that are imperfect substitutes. They choose their prices simultaneously. Consumers' demand for the firm i, with $i = 1, ..., n$, is written as $q_i = a - bp_i + \sum_{j \neq i} p_j$, where p_i denotes firm i's price, and $(a, b) \in \mathbb{R}_+^2$, with $b > \frac{n-1}{2}$. We assume production costs are zero.

1) ***(2 pts)*** *What is firm i's optimal price p_i^* given its competitor prices $(p_j)_{j \neq i}$? Solve the Nash equilibrium $(p_i^N)_{i=1,...,n}$ of the stage game. (Hint: Begin by summing the optimal prices chosen by each firm $p_1^* + ... + p_n^*$.)*

2) ***(1 pt)*** *What are the associated quantities q_i^N and profits π_i^N?*

3) ***(2 pts)*** *Find the strategies and joint profit π^c associated with the "cooperative" solution that would maximize the total profit. (Hint: It suffices to solve for a symmetric price, denoted as p^c.) What is the associated firm i's profit, denoted as π_i^c?*

4) ***(1 pt)*** *Consider now the corresponding infinitely repeated game. Let δ_i denote firm i's discount factor. Define a grim trigger strategy that may sustain cooperation at equilibrium.*

5) ***(1 pt)*** *Suppose all firms play this grim trigger strategy. What is the most profitable unilateral deviation for firm i at stage t, denoted as p_i'? What is the corresponding stage profit, denoted as π_i'?*

6) ***(1 pt)*** *What is the condition under which firm i has no profitable unilateral deviation from this grim trigger strategy at stage t?*

7) ***(2 pts)*** *Show that there exist thresholds, denoted as $\bar{\delta}_i$ $(i = 1, ..., n)$, above which any values of δ_i sustain cooperation in every stage as a subgame perfect Nash equilibrium.*

8) ***(1 pt)*** *Numerical application. Compute the corresponding values p_i^N, q_i^N, π_i^N, p^c, π_i^c, p_i', π_i', and $\bar{\delta}_i$ when $a = 100$, and $b = n = 4$.*