Industrial Organization - Solution to the Final Exam

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Duration: 90 mn. No document, no calculator allowed.

Exercise 1. Spatial Price Competition with Tariff (9 pts)

a) ***(2 pts)*** The consumer's decision is based on the total cost of purchase, which includes the product price, the transportation cost, and possibly the tariff. The consumer located at $\widetilde{x} \in [0, 1)$ is indifferent between the two firms, so the total cost of buying from Firm 1 and Firm 2 must be equal:

$$p_1 + t(\widetilde{x} - a)^2 = p_2 + t(1 - \widetilde{x})^2 + \tau$$

which is equivalent to

$$\widetilde{x} = a + \frac{1-a}{2} + \frac{p_2 + \tau - p_1}{2t(1-a)}$$

with the interpretation that: a represents firm 1's turf; $\frac{1-a}{2}$ is half of consumers between firms 1 and 2; and $\frac{p_2+\tau-p_1}{2t(1-a)}$ is the sensitivity of demand of the price differential.

Firm 1's demand is composed from all consumers located to the left of $\widetilde{x}$:

$$D_1(p_1, p_2) = \widetilde{x} = \frac{1+a}{2} + \frac{p_2 + \tau - p_1}{2t(1-a)}$$

Firm 2's demand is composed from all consumers located to the right of $\widetilde{x}$:

$$D_2(p_1, p_2) = 1 - \widetilde{x} = \frac{1-a}{2} + \frac{p_1 - p_2 - \tau}{2t(1-a)}$$

b) ***(2 pts)*** Firm i's profit writes as $\pi_i(p_1, p_2) = (p_i - c)D_i(p_1, p_2)$. Firm 1's profit is then

$$\begin{aligned}\pi_1(p_1, p_2) = (p_1 - c)\widetilde{x} &= (p_1 - c)\left(\frac{1+a}{2} + \frac{p_2 + \tau - p_1}{2t(1-a)}\right)\\ &= -\frac{(p_1)^2}{2t(1-a)} + p_1\left(\frac{1+a}{2} + \frac{p_2 + \tau + c}{2t(1-a)}\right) - c\left(\frac{1+a}{2} + \frac{p_2 + \tau}{2t(1-a)}\right)\end{aligned}$$

Firm 2's profit is

$$\begin{aligned}\pi_2(p_1, p_2) = (p_2 - c)(1 - \widetilde{x}) &= (p_2 - c)\left(\frac{1-a}{2} + \frac{p_1 - p_2 - \tau}{2t(1-a)}\right)\\ &= -\frac{(p_2)^2}{2t(1-a)} + p_2\left(\frac{1-a}{2} + \frac{p_1 - \tau + c}{2t(1-a)}\right) - c\left(\frac{1-a}{2} + \frac{p_1 - \tau}{2t(1-a)}\right)\end{aligned}$$

Firm i's best response writes as $p_i^*(p_j) \in \mathrm{argmax}_{\mathrm{p_i \in \mathbb{R}^+}} \pi_i(p_i, p_j)$.

The F.O.C. give

$$\frac{\partial \pi_1(p_1,p_2)}{\partial p_1} = 0 \iff \frac{2p_1}{2t(1-a)} = \frac{1+a}{2} + \frac{p_2+\tau+c}{2t(1-a)}$$
$$\iff p_1^*(p_2) = \frac{t(1-a^2)+p_2+\tau+c}{2}$$

and

$$\frac{\partial \pi_2(p_1,p_2)}{\partial p_2} = 0 \iff \frac{2p_2}{2t(1-a)} = \frac{1-a}{2} + \frac{p_1-\tau-c}{2t(1-a)}$$
$$\iff p_2^*(p_1) = \frac{t(1-a)^2+p_1-\tau+c}{2}$$

Each S.O.C. is satisfied: $\frac{\partial^2 \pi_i(p_1,p_2)}{\partial p_i^2} = -\frac{1}{t(1-a)} < 0$, so both previous F.O.C are sufficient.

c) ***(2 pts)*** The Nash equilibrium in prices solves

$$p_1^*(p_2^*(p_1)) = p_1$$

which is equivalent to

$$\frac{1}{2}\left(t(1-a^2) + \frac{t(1-a)^2+p_1-\tau+c}{2} + \tau + c\right) = p_1$$

Hence,

$$p_1^N = c + \frac{\tau}{3} + t(1 - \frac{a}{3}(a+2))$$

From $p_2^N = p_2^*\left(p_1^N\right)$, we have

$$p_2^N = \frac{t(1-a)^2+p_1^N-\tau+c}{2} = c - \frac{\tau}{3} + t(1-\frac{a}{3}(4-a))$$

d) ***(2 pts)*** From the previous calculation, it is clear that p_1^N increases with τ, while p_2^N decreases with τ. However, the price paid to firm 2 is $p_2^N + \tau = c + \frac{2\tau}{3} + t(1-\frac{a}{3}(4-a))$, which increases with τ. Therefore, the tariff does not benefit consumers, as they would pay lower prices without it.

While the tariff only applies to the good produced by firm 2, the overall effect is negative for consumers on both prices. Indeed, the tariff reduces competition between the firms, giving firm 1 the opportunity to raise its equilibrium price by $\frac{\tau}{3}$. Firm 2 reduces its equilibrium price by $\frac{\tau}{3}$ only, causing the total price $p_2^N + \tau$ to increase by $\frac{2\tau}{3}$. It is as if the tariff spreads, with one third affecting the domestic price and two thirds affecting the foreign price.

e) ***(1 pt)*** At equilibrium, $\widetilde{x}$ is worth

$$\widetilde{x}^N = \frac{1+a}{2} + \frac{p_2^N+\tau-p_1^N}{2t(1-a)} = \frac{3+a}{6} + \frac{\tau}{6t(1-a)}$$

which increases with τ. Given that firms' profits are worth

$$\pi_1^N = \pi_1(p_1^N,p_2^N) = (p_1^N - c)\widetilde{x}^N \text{ and } \pi_2^N = \pi_2(p_1^N,p_2^N) = (p_2^N-c)(1-\widetilde{x}^N)$$

and that p_1^N (resp. p_2^N) increases (resp. decreases) with τ, we deduce that π_1^N (resp. π_2^N) increases (resp. decreases) with τ. The tariff is then beneficial to firm 1 and detrimental to firm 2.

Exercise 2. Repeated Monopolistic Competition in Prices (11 pts)

1) ***(2 pts)*** For any given competitor prices (p^*_{-i}), firm i's optimal price p_i maximizes the profit:

$$p_i \times q_i(p_i) = p_i \times (a - bp_i + \sum_{j\neq i} p_j) = p_i \times \left(a + \sum_{j\neq i} p_j\right) - bp_i^2$$

This profit function is strictly concave. From the F.O.C., the optimal price p_i^* is such that:

$$\frac{\partial}{\partial p_i}\left(p_i^* \times \left(a + \sum_{j\neq i} p_j\right) - bp_i^{*2}\right) = 0$$

That is:

$$a + \sum_{j\neq i} p_j - 2bp_i^* = 0$$

The optimal price is then

$$p_i^*((p_j)_{j\neq i}) = \frac{a + \sum_{j\neq i} p_j}{2b} \tag{1}$$

A Nash equilibrium solves the system:

$$\left\{ p_i^*((p_j^*)_{j\neq i}) = \frac{a+\sum_{j\neq i} p_j^*}{2b} \quad \forall i \right.$$

By summing the optimal prices we obtain:

$$\sum_{i=1}^{n} p_i^* = \frac{a \times n + (n-1)\sum_{j=1}^{n} p_j^*}{2b}$$

which is equivalent to

$$\sum_{i=1}^{n} p_i^* = \frac{a \times n}{2b - n + 1} \tag{2}$$

From (1), for any $i \in \{1, ..., n\}$ we have:

$$\begin{aligned}(2b+1)p_i^* &= a + \sum_{j=1}^{n} p_j^* \\ &= a + \frac{a \times n}{2b-n+1} = \frac{a(2b+1)}{2b-n+1}\end{aligned}$$

So,

$$p_i^N = \frac{a}{2b-n+1} \tag{3}$$

Therefore, there is a unique solution, given by $p_1^N = ... = p_n^N = \frac{a}{2b-n+1} \equiv p^N$.

2) ***(1 pt)*** At equilibrium, each firm produces

$$q_i = a - bp_i^N + \sum_{j\neq i} p_j^N = a - (b+1)p_i^N + \sum_{j=1}^{n} p_j^N$$

which, from (3), writes as

$$q_i = a - (b+1)\frac{a}{2b-n+1} + \frac{a \times n}{2b-n+1}$$

that is

$$q_i^N = \frac{a \times b}{2b - n + 1} \tag{4}$$

The corresponding profit is

$$\pi_i^N = p_i^N \times q_i^N = \frac{a}{2b - n + 1} \frac{a \times b}{2b - n + 1} = b \left(\frac{a}{2b - n + 1} \right)^2 \tag{5}$$

3) ***(2 pts)*** The sum of the n firms' profits is a symmetric function of $p_1, ..., p_n$. Solving for the maximum can be done by replacing $p_1, ..., p_n$ with a symmetric price p^c. The problem is then to solve

$$\max_{p \geq 0} np(a - p)$$

The solution is given by

$$p^c = \frac{a}{2} \tag{6}$$

The total profit is then

$$\pi^c = np^c(a - p^c) = n\frac{a^2}{4} \tag{7}$$

The associated firm i's profit, writes as

$$\pi_i^c = p^c(a - p^c) = \frac{a^2}{4} \tag{8}$$

Observe that this "cooperative" solution is not immune against unilateral profitable deviation (it is not a Nash equilibrium).

4) ***(1 pt)*** In the infinitely repeated game, consider the following i's trigger strategy:

At $t = 1$, $p_i = p^c = \frac{a}{2}$ ("cooperative" price);

at $t > 1$, $p_i = p^c = \frac{a}{2}$ if $p^c = \frac{a}{2}$ is the only price that has been observed from all firms in the past; otherwise, charge $p_i = p_i^N = \frac{a}{2b-n+1}$.

5) ***(1 pt)*** The most profitable unilateral deviation for firm i at stage t, denoted as p_i', is given by (1) when $p_j = \frac{a}{2}$ for all $j \neq i$:

$$p_i' = p_i^*(\frac{a}{2}, ..., \frac{a}{2}) = \frac{a + (n-1)\frac{a}{2}}{2b} = a\frac{n+1}{4b} \tag{9}$$

$$\pi_i' = -bp_i'^2 + p_i' a\frac{n+1}{2} = -b\left(a\frac{n+1}{4b}\right)^2 + a^2\frac{(n+1)^2}{8b} = a^2\frac{(n+1)^2}{16b} \tag{10}$$

6) ***(1 pt)*** After unilaterally deviating at stage t, firm i obtains at max π_i^N for all subsequent stages. So, the condition under which firm i has no profitable deviation from the grim trigger strategy writes as

$$\pi_i^c \sum_{k=t}^{\infty} \delta_i^k \geq \pi_i' \delta_i^t + \pi_i^N \sum_{k=t+1}^{\infty} \delta_i^k$$

7) ***(2 pts)*** The thresholds, denoted as $\bar{\delta}_i$ $(i = 1, ..., n)$, above which any values of δ_i sustain cooperation in every stage as a subgame perfect Nash equilibrium can be deduced from the previous condition. From (5), (8), and (10), this condition rewrites as

$$\frac{a^2}{4} \sum_{k=t}^{\infty} \delta_i^k \geq a^2\frac{(n+1)^2}{16b}\delta_i^t + b\left(\frac{a}{2b - n + 1}\right)^2 \sum_{k=t+1}^{\infty} \delta_i^k$$

that is

$$\sum_{k=t}^{\infty} \delta_i^k \geq \frac{(n+1)^2}{4b}\delta_i^t + \frac{4b}{(2b-n+1)^2}\sum_{k=t+1}^{\infty} \delta_i^k$$

which is equivalent to

$$\frac{\delta_i^t}{1-\delta_i} \geq \frac{(n+1)^2}{4b}\delta_i^t + \frac{4b}{(2b-n+1)^2}\frac{\delta_i^{t+1}}{1-\delta_i}$$

that is

$$1 \geq \frac{(n+1)^2}{4b}(1-\delta_i) + \frac{4b}{(2b-n+1)^2}\delta_i$$

So,

$$4b \geq (n+1)^2 + \delta_i\left(\frac{16b^2}{(2b-n+1)^2} - (n+1)^2\right)$$

Hence,

$$\delta_i \geq \frac{(n+1)^2 - 4b}{(n+1)^2 - \frac{16b^2}{(2b-n+1)^2}} \equiv \bar{\delta}_i$$

When $\delta_i \geq \bar{\delta}_i$ for all firm i, this equilibrium is perfect. Indeed, we already know that all players adopt an equilibrium behavior in any subgame that belongs to the path of "cooperation". In addition, any subgame that does not belong to the path of "cooperation" triggers a punishment behavior that consists of charging the price of p^N for all firms. Such pricing corresponds to a Nash equilibrium of the stage game.

8) ***(1 pt)*** When $a = 100$, and $b = n = 4$, from (3)–(10), the corresponding values write as

$$p_i^N = \frac{a}{2b-n+1} = \frac{100}{5} = 20;\ q_i^N = \frac{a \times b}{2b-n+1} = \frac{400}{5} = 80$$

$$\pi_i^N = b\left(\frac{a}{2b-n+1}\right)^2 = 4\left(\frac{100}{5}\right)^2 = \frac{40000}{25} = 1600$$

$$p^c = \frac{a}{2} = 50;\ \pi_i^c = \frac{a^2}{4} = \frac{10000}{4} = 2500;\ p_i' = a\frac{n+1}{4b} = \frac{500}{16} = \frac{125}{4} = 31.25$$

$$\pi_i' = a^2\frac{(n+1)^2}{16b} = \frac{250000}{64} = 3906.25$$

and

$$\bar{\delta}_i = \frac{(n+1)^2 - 4b}{(n+1)^2 - \frac{16b^2}{(2b-n+1)^2}} = \frac{25-16}{25 - \frac{16\times 16}{25}} = \frac{25 \times 9}{25 \times 25 - 16 \times 16} = \frac{225}{369} \simeq 0.6097.$$