Derivative Instruments (Produits dérivés) - Solution to the Exam

Université Paris Dauphine-PSL - Master 1 I.E.F. (272)

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Answers can be formulated in English or French.

Solution 1 (1 pt) There is a margin call if more than \$12,000-\$8,000=\$4,000 is lost on one contract.

This happens if the futures price of frozen concentrated orange juice falls by more than $\$4,000/50,000 = 8$ cents, and the new price is then below 84 cents.

Solution 2 (1 pt) (c) is false. Indeed, the maintenance margin ensures that the balance in the margin account never becomes negative.

Solution 3 (3 pts) (a) The no-arbitrage delivery price at date $t = 0$ (today) of a forward contract F that will deliver one share of stock XYZ in exactly one year from now (at date $t = 1$) is

$$F = S_0(1 + r) = 100 \times 1.05 = 105€$$

(b) The no-arbitrage price at date $t = 0.5$-year of a forward contract G that will deliver one share of XYZ in exactly 6-months from that date (with delivery at date $t = 1$ year) is

$$G = S_{0.5}(1 + r)^{1/2} = 92.72 \times (1.05)^{1/2} \simeq 95€$$

(c) To add up our cash flows as of date $T = 1$-year to zero using forward contract G and borrowing or lending at the risk-free rate, at date $t = 0.5$-year it suffices to :

1. Short the forward contract G
   (a) this, together with the other forward, will produce $G - F = 95 - 105 = -10€$ at maturity.
2. Lend $\frac{F-G}{(1+r)^{1/2}} = \frac{10}{(1.05)^{1/2}} \simeq 9.76€$
   (a) this will produce $F - G = 10€$ at maturity.

Exercise 4 (1 pt) The *par yield* for a certain bond maturity is the coupon rate that causes the bond price to equal its par value. (The par value is the same as the principal value.) The value of the bond is equal to its par value of 100 when

$$\frac{c}{2}\left(e^{-0.05\times0.5} + e^{-0.058\times1} + e^{-0.064\times1.5}\right) + \left(100 + \frac{c}{2}\right)e^{-0.068\times2} = 100$$

The two-year par yield writes as

$$c = \frac{(100 - 100d)m}{A}$$

with $m = 2$, $d = e^{-0.068\times 2} = 0.8694$, and $A = e^{-0.05\times 0.5} + e^{-0.06\times 1.0} + e^{-0.065\times 1.5} + e^{-0.07\times 2.0} = 3.6935$. So

$$c = \frac{(100 - 100 \times 0.8694) \times 2}{3.6935} = 6.87$$

The 2-year par yield is therefore 6.87% per annum. This has semiannual compounding because payments are assumed to be made every 6 months. With continuous compounding, the rate is 6.75% per annum.

Solution 5 (1 pt) The bond price is obtained by discounting the cash flows at 7.5%. The price is

$$\frac{\frac{6}{2}}{1 + \frac{0.075}{2}} + \frac{\frac{6}{2}}{\left(1 + \frac{0.075}{2}\right)^2} + \frac{100 + \frac{6}{2}}{\left(1 + \frac{0.075}{2}\right)^3} \simeq 97.909$$

If the 18-month zero rate is r, we must have

$$\frac{\frac{6}{2}}{1 + \frac{0.07}{2}} + \frac{\frac{6}{2}}{\left(1 + \frac{0.07}{2}\right)^2} + \frac{100 + \frac{6}{2}}{\left(1 + \frac{r}{2}\right)^3} \simeq 97.909$$

which gives $r = 7.51\%$ (with continuous compounding, the rate is 7.37%).

Solution 6 (1 pt) The trader sells for 35 cents per pound something that is worth 35.83 cents per pound. The loss is (\$0.3583-\$0.35)×30,000×2=\$498.

Solution 7 (4 pts) a) ABC has an absolute advantage in both markets while XYZ has a comparative advantage in the French market.

b) There is a 1% per annum differential between the euro rates and a 2% per annum differential between the dollar rates. The total gain to all parties from the swap is therefore $2\% - 1\% = 1$ per annum. The bank requires 0.4% per annum, leaving 0.3% per annum for each of ABC and XYZ. The swap should lead to ABC borrowing euro at $6\% - 0.3\% = 5.7$ per annum and to XYZ borrowing US\$ at $5\% - 0.3\% = 4.7$ per annum. The appropriate arrangement is therefore as shown in the following figure. (All foreign exchange risk is borne by the bank.)

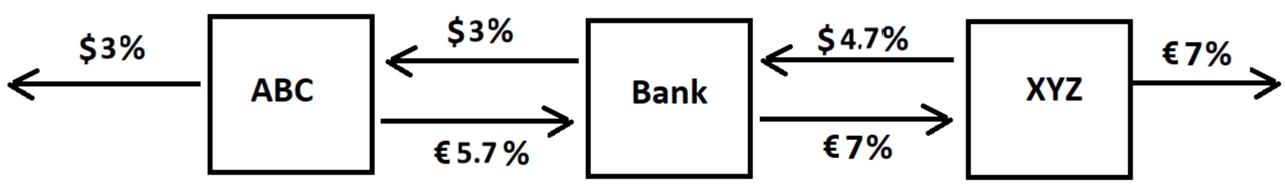

*Figure*1

All foreign exchange risk is assumed by the bank. Indeed, in terms of cash flows, the net payoff is :

- for compagny ABC to pay €5.7% ;
- for compagny XYZ to pay \$4.7% ; and
- for the Bank to receive \$1.7% and pay €1.3%.

c) When the currency risk is taken over by company ABC, compagny XYZ and the Bank have each a net payoff that involves only one currency. The appropriate arrangement is therefore as shown in the following figure.

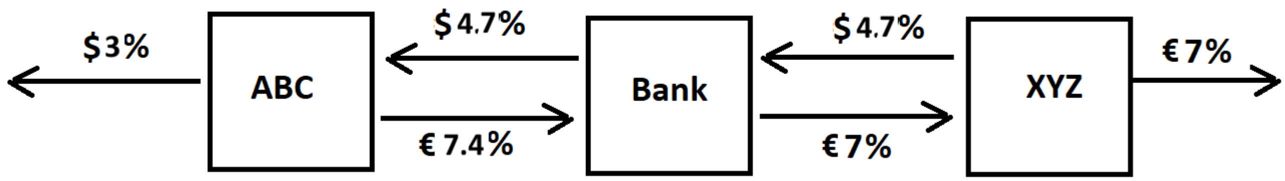

*Figure*2

Indeed, in terms of cashflows, the net payoff is :

- for compagny ABC to receive \$1.7% and to pay €7.4% ;
- for compagny XYZ to pay \$4.7% ; and
- for the Bank to receive \$0.4%.

Solution 8 (1 pt) On the global OTC derivatives market, the rank of the underlying assets by order of importance (from the most to the least in terms of notional amounts outstanding) is : **b)** Interest rate ; **c)** Foreign Exchange ; and **a)** Commodity.

Solution 9 (1 pt) The present value of the storage costs for nine months are

$$0.01 + 0.01e^{-0.095\times 0.25} + 0.01e^{-0.095\times 0.5} = 0.0293$$

The futures price is

$$F_0 = (3.28 + 0.0293)e^{0.095\times 0.75} = 3.5537$$

i.e., it is \$3.55 per ounce.

Solution 10 (1 pt) A straddle is created by buying both the call and the put. This strategy costs \$8. $S_T = 56 > 53$ so the profit is $56 - (53 + 8) = -5$.

Solution 11 (5 pts) (a) The market price of the bond is

$$B = 2e^{-0.05\times(1/2)} + 2e^{-0.05\times 1} + 2e^{-0.05\times(3/2)} + 102e^{-0.05\times 2} \simeq 98.00$$

(b) The equivalent risk-free bond's value is

$$B_{risk-neutral} = 2e^{-0.03\times(1/2)} + 2e^{-0.03\times 1} + 2e^{-0.03\times(3/2)} + 102e^{-0.03\times 2} \simeq 101.88$$

(c) The bond risk premium is

$$B - B_{risk-neutral} = 101.88 - 98.00 \simeq 3.88$$

(d) The loss in case of default at date t is $B^t_{risk-neutral} - rec^t$, for $t \in 1, 2$. With

$$B^1_{risk-neutral} = 2 + 2e^{-0.03\times(1/2)} + 102e^{-0.03\times 1} \simeq 102.96$$

$$B^2_{risk-neutral} = 102$$

and $rec^t = 30$ for any $t \in 1, 2$.

So,

$$B^1_{risk-neutral} - rec^1 \simeq 102.96 - 30 = 72.96$$

and

$$B^2_{risk-neutral} - rec^2 = 102 - 30 = 72$$

(e) The time-invariant default probability $\bar{p}$ then satisfies

$$\bar{p} \sum_{t=1}^{2} (B^t_{risk-neutral} - rec^t)e^{-r\times t} = B_{risk-neutral} - B$$

that is

$$\bar{p} = \frac{B_{risk-neutral} - B}{(B^1_{risk-neutral} - rec^1)e^{-0.03\times 1} + (B^2_{risk-neutral} - rec^2)e^{-0.03\times 2})} = \frac{3.88}{72.96e^{-0.03\times 1} + 72e^{-0.03\times 2}} \simeq 2.80\%$$