Industrial Organization - Final Exam

Paris Dauphine University - Master Quantitative Economics, April 2023

Part A: Jérôme MATHIS (LEDa) - 12 pts

Duration: 75 mn. No document, no calculator allowed.

Bertrand equilibrium with subadditive different costs (Dastidar, Economics Letters, 2011) – 12 pts

Consider a simultaneous move price choice game in a homogeneous product, asymmetric cost duopoly. The cost function for firm i, $i = 1, 2$, when producing quantity $q \in [0; 1]$ is $C_i(q) = \bar{c}_i + c_i q$ if $q > 0$ and zero otherwise, where $\bar{c}_i \in (0; \frac{(1-c_i)^2}{4})$ (resp. $c_i \in (0; 1)$) is the fixed (resp. variable) cost of production, so that the cost function is strictly subadditive (i.e., $C_i(x + y) < C_i(x) + C_i(y)$ for any two quantities x and y).

Firm i chooses which price p_i to quote in the interval $[0, 1]$. There is a demand function for the lowest posted price p, $D(p) = 1 - p$.

In price competition firms have to meet the demand that they face at the posted price. The firm which quotes the lowest price gets all the demand. Any firm which quotes a price higher than its rival gets no demand. If there is a tie at any price, the two firms share the demand equally.

1) (0.75 pts) *How does write $\pi_i(p_i, p_j)$ the profit going to firm i when it quotes price p_i and its competitor charges price p_j?*

2) (0.75 pts) *What price p_i^m would firm i quote if it was a monopoly? Which assumption does guarantee that firm i's monopoly profit is strictly positive?*

3) (0.75 pts) *What price $\tilde{p}_i$ is firm i's* "monopoly breakeven price" *(i.e., the price at which firm i's monopoly profit is zero and just below (resp. above) which it is negative (resp. positive))?*

4) (0.75 pts) *Can we have altogether $\tilde{p}_1 > p_2^m$ and $\tilde{p}_2 > p_1^m$? Why?*

5) (0.75 pts) *Suppose $\tilde{p}_2 \geq p_1^m$. Give a pure strategy Bertrand equilibrium (p_1^*, p_2^*). Prove that there is no unilateral profitable deviation.*

6) (0.75 pts) *Conversely, suppose $\tilde{p}_1 \geq p_2^m$. Give a pure strategy Bertrand equilibrium (p_1^*, p_2^*). (The proof is not required.)*

7) *Suppose $\tilde{p_1} \neq \tilde{p_2}$, $\tilde{p_1} < p_2^m$, and $\tilde{p_2} < p_1^m$. We want to show that there is no pure strategy Bertrand equilibrium (p_1^*, p_2^*). Find a unilateral profitable deviation in each of the following case. (Without loss of generality, when $p_1^* \neq p_2^*$ we will assume that $p_1^* < p_2^*$.)*

7.a) (0.75 pts) *$p_1^* = p_2^* < \tilde{p_i}$ for at least one firm i.*

7.b) (0.75 pts) *$p_1^* = p_2^* > p_i^m$ for at least one firm i.*

7.c) (0.75 pts) *$p_1^* = p_2^* \geq \max\{\tilde{p_1}, \tilde{p_2}\}$.*

7.d) (0.75 pts) *$\tilde{p_2} < p_1^* < p_2^*$.*

7.e) (0.75 pts) *$p_1^* < p_2^*$ and $p_1^* \leq \tilde{p_2}$.*

8) *Suppose again $\tilde{p_1} \neq \tilde{p_2}$, $\tilde{p_1} < p_2^m$, and $\tilde{p_2} < p_1^m$. Without loss of generality, assume $\tilde{p_1} < \tilde{p_2}$. We want to show that there is a Bertrand equilibrium (p_1^*, p_2^*) that relies on a firm 2's mixed strategy that consists in randomizing uniformly on the interval $[\tilde{p_2}; \tilde{p_2} + a]$, with $a > 0$ small enough.*

8.a) (0.75 pts) *Does $p_1^* \geq p_2 \tilde{+} a$? Explain.*

8.b) (0.75 pts) *Does $p_1^* < \tilde{p_2}$? Explain.*

8.c) (1.5 pts) *Does $p_1^* \in (\tilde{p_2}; \tilde{p_2} + a)$? Explain. (Hint: assume $p_1^* \in (\tilde{p_2}; \tilde{p_2} + a)$, compute firm 1's expected payoff and show that it has a unilateral profitable deviation when the parameter a is small enough.)*

8.d) (0.75 pts) *So what is firm 1's best response?*

Vertical differentiation with costless quality – 1.5 bonus pts

1) (0.5 pts) *What is the unique pure-strategy Nash equilibrium in the duopoly vertical differentiation model of Chapter 2 where firms simultaneously choose a costless quality then compete in prices given these qualities?*

2) (0.5 pts) *What would be the leader and the follower respective choices in the Stackelberg (sequential) version of this model?*

3) (0.5 pts) *Give a practical example of such a result.*