Industrial Organization - Solution to the Final Exam

Paris Dauphine University - Master Quantitative Economics, April 2023

Part A : Jérôme MATHIS (LEDa) - 12 pts

Duration: 75 mn. No document, no calculator allowed.

Bertrand equilibrium with subadditive different costs (Dastidar, Economics Letters, 2011) – 12 pts

1) (0.75 pts) When firm i quotes price p_i and its competitor charges price p_j, its profit writes as

$$\pi_i(p_i,p_j)=\begin{cases} p_iD(p_i)-C_i(D(p_i)) & \text{if} \quad p_i<p_j \\ p_i\frac{D(p_i)}{2}-C_i(\frac{D(p_i)}{2}) & \text{if} \quad p_i=p_j \\ 0 & \text{if} \quad p_i>p_j \end{cases}$$
$$=\begin{cases} p_i(1-p_i)-(\bar{c}_i+c_i(1-p_i)) & \text{if} \quad p_i<p_j \\ p_i\frac{1-p_i}{2}-(\bar{c}_i+c_i\frac{1-p_i}{2}) & \text{if} \quad p_i=p_j \\ 0 & \text{else} \end{cases}$$
$$=\begin{cases} -p_i^2+p_i(1+c_i)-(\bar{c}_i+c_i) & \text{if} \quad p_i<p_j \\ -\frac{p_i^2}{2}+\frac{p_i}{2}(1+c_i)-(\bar{c}_i+\frac{c_i}{2}) & \text{if} \quad p_i=p_j \\ 0 & \text{else} \end{cases}$$

2) (0.75 pts) If it was a monopoly, firm i would quote the price p_i^m that solves

$$\max_{p_i\in[0;1]}\pi_i^m(p_i)=\max_{p_i\in[0;1]}p_iD(p_i)-C_i(D(p_i))=\max_{p_i\in[0;1]}-p_i^2+p_i(1+c_i)-(\bar{c}_i+c_i)$$

The first order condition $\frac{\partial\pi_i^m(p_i)}{\boldsymbol{\partial p_i}}=0$ writes as $-2p_i+1+c_i=0$. That is,

$$p_i^m=\frac{1+c_i}{2}\in(1/2;1)$$

(Clearly, $\frac{\partial^2\pi_i^m(p_i)}{\boldsymbol{\partial p_i}^2}=-2<0$, so the first order condition is sufficient.)
At this price, the monopoly profit writes as:

$$\pi_i^m(\frac{1+c_i}{2})=-(\frac{1+c_i}{2})^2+(\frac{1+c_i}{2})(1+c_i)-(\bar{c}_i+c_i)$$
$$=\frac{(1+c_i)^2-4(\bar{c}_i+c_i)}{4}=\frac{(1-c_i)^2}{4}-\bar{c}_i$$

which is strictly positive from the assumption $\bar{c}_i < \frac{(1-c_i)^2}{4}$.

3) (0.75 pts) Firm i's "*monopoly breakeven price*" solves $\min\{p \in [0;1] | \pi_i^m(p_i) = 0\}$. The equality $\pi_i^m(p_i) = 0$ is equivalent to $-p_i^2 + p_i(1+c_i) - (\bar{c}_i + c_i) = 0$. We have $\Delta = (1+c_i)^2 - 4(\bar{c}_i + c_i) = (1-c_i)^2 - 4\bar{c}_i$, which from $\bar{c}_i < \frac{(1-c_i)^2}{4}$, is strictly positive, so this second degree polynomial has two solutions :

$$p_{i,1} = \frac{1 + c_i - \sqrt{\Delta}}{2} \text{ and } p_{i,2} = \frac{1 + c_i + \sqrt{\Delta}}{2}$$

From $\Delta = (1+c_i)^2 - 4(\bar{c}_i + c_i) < (1+c_i)^2$ the numerator of $p_{i,1}$ is positive. From $c_i < 1$ we then have $p_{i,1} = \frac{1+c_i-\sqrt{(1+c_i)^2-4(\bar{c}_i+c_i)}}{2} \in (0;1)$ which means that firm i's "*monopoly breakeven price*" writes as $\tilde{p_i} = p_{i,1}$.

4) (0.75 pts) No, we cannot have altogether $\tilde{p_1} > p_2^m$ and $\tilde{p_2} > p_1^m$ because by definition we have $\tilde{p_i} \leq p_i^m$, so $\tilde{p_1} > p_2^m$ implies

$$p_1^m \geq \tilde{p_1} > p_2^m \geq \tilde{p_2}$$

which would contradict $\tilde{p_2} > p_1^m$.

5) (0.75 pts) A pure strategy Bertrand equilibrium is given by $(p_1^* = p_1^m, p_2^*)$ with any $p_2^* > p_1^m$.

Let us prove that there is no unilateral profitable deviation. It is obvious that firm 1 does not gain by deviating. Firm 2 gets zero in equilibrium. If it quotes any price $p_2 > p_1^m$ its profit stays at zero. If it quotes p_1^m (resp. any price $p_2 < p_1^m$) it gets $\pi_2(p_1 = p_1^m, p_2 = p_1^m)$ (resp. $\pi_2^m(p_2)$) which from $\tilde{p_2} \geq p_1^m$ and the strict concavity of $\pi_2(p_1, p_2 = p_1)$ (resp. $\pi_2^m(.)$) gives a zero or negative profit. Hence the stated strategy profile constitutes a pure strategy Bertrand equilibrium.

6) (0.75 pts) A pure strategy Bertrand equilibrium is given by $(p_1^*, p_2^* = p_2^m)$ with any $p_1^* > p_2^m$.

7)

7.a) (0.75 pts) From $p_1^* = p_2^* < \tilde{p_i}$, firm i makes a negative profit. A unilateral profitable deviation is given by $p_i' = \tilde{p_i}$.

7.b) (0.75 pts) From $p_1^* = p_2^* > p_i^m$, a unilateral profitable deviation is given by $p_i' = p_i^m$.

7.c) (0.75 pts) From $p_1^* = p_2^* \geq \max\{\tilde{p_1}, \tilde{p_2}\}$, a unilateral profitable deviation is given by $p_i' = p_i^* - \epsilon$ for ϵ small enough.

7.d) (0.75 pts) From $\tilde{p_2} < p_1^* < p_2^*$, a unilateral profitable deviation is given by $p_2' = p_1^* - \epsilon$ with $\epsilon \in (0; p_1^* - \tilde{p_2})$.

7.e) (0.75 pts) From $p_1^* < p_2^*$ and $p_1^* \leq \tilde{p_2}$, a unilateral profitable deviation is given by $p_1' = p_1^* + \epsilon$ with $\epsilon \in (0; p_2^* - p_1^*)$.

8)

8.a) (0.75 pts) No! A unilateral profitable deviation is given by $p_1' = \tilde{p_2}$.

8.b) (0.75 pts) No! A unilateral profitable deviation is given by $p_1' = \tilde{p_2}$.

8.c) (1.5 pts) Assume $p_1^* \in (\tilde{p_2}; \tilde{p_2} + a)$ and let us show that a unilateral profitable deviation is given by $p_1' = \tilde{p_2}$. Let $F(p_1^*)$ denote the probability that firm 2 charges a price below p_1^* when it uses a mixed strategy that consists in randomizing uniformly on the interval $[\tilde{p_2}; \tilde{p_2} + a]$. Firm 1's expected payoff writes then

$$\begin{aligned} E[\pi_1(p_1^*, p_2^*)] &= F(p_1^*) \times 0 + (1 - F(p_1^*)) \times \pi_1^m(p_1^*) \\ &= (1 - \frac{p_1^* - \tilde{p_2}}{a}) \times \pi_1^m(p_1^*) \end{aligned}$$

So,

$$\frac{dE[\pi_1(p_1^*, p_2^*)]}{dp_1^*} = -\frac{\pi_1^m(p_1^*)}{a} + (1 - \frac{p_1^* - \tilde{p_2}}{a}) \times \frac{d\pi_1^m(p_1^*)}{dp_1^*}$$

From $p_1^* \in (\tilde{p_2}; \tilde{p_2} + a)$, we have $(1 - \frac{p_1^* - \tilde{p_2}}{a}) \in [0; 1)$. From $\tilde{p_2} < p_1^m$, for a small enough, $p_1^* < p_1^m$ and the term $\frac{d\pi_1^m(p_1^*)}{dp_1^*}$ is positive. For a small enough, it is bounded above by $\frac{\pi_1^m(p_1^*)}{a}$, so $\frac{dE[\pi_1(p_1^*, p_2^*)]}{dp_1^*} < 0$. This demonstrates that firm 1 does not prefer posting a price in $(\tilde{p_2}; \tilde{p_2} + a)$. Therefore, the proposed mixed strategy profile constitutes a Bertrand equilibrium.

8.d) (0.75 pts) Firm 1's best response is then $p_1^* = \tilde{p_2} < p_1^m$.

For a generalization of these results, see Dastidar, K. G. (2011). Bertrand equilibrium with subadditive costs. *Economics Letters*, 112(2), 202-204.

Vertical differentiation with costless quality – 1.5 bonus pts

1) (0.5 pts)

In the duopoly vertical differentiation model of Chapter 2 where firms simultaneously choose a costless quality then compete in prices given these qualities, the unique pure-strategy Nash equilibrium exhibits maximal differentiation: one firm chooses the lowest quality while its competitor chooses the highest quality.

2) (0.5 pts) In the Stackelberg (sequential) version of this model, the leader (resp. follower) would choose the highest (resp. lowest) quality.

3) (0.5 pts) A practical example of this result is given by ...