

# Industrial Organization - Exercises Chapter 3

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## Exercise 1. Collusion in a quantity oligopoly

Consider a repeated game where in each period  $t \in 1, 2, \dots, T$ ,  $n$  firms (the same discount factor  $\delta \in [0, 1]$ ) independently choose the quantity of an identical good they produce. We denote

- $N = \{1, 2, \dots, n\}$ : the set of  $n$  firms;
- $C(q) = cq$ : identical production cost function for each firm, with  $c \in [0, 1]$ ;
- $\pi_i(p, q) = (p - c)q$ : firm  $i$ 's profit when producing  $q$  units and selling it at price  $p$ ;
- $q_i(t)$ : the quantity produced by firm  $i$  at period  $t$ ; and
- $Q(t) = \sum_{i=1}^N q_i(t)$ : aggregate output at period  $t$ .

Once the quantities produced have been observed, all firms sell the good at market price  $p(t)$  which is established at  $1 - Q(t)$  if  $Q(t) \leq 1$  and zero otherwise.

- a) *In the stage game, determine the aggregate quantity that maximizes the aggregate profit that the firms can be led to share within a cartel (hint: think as if the cartel were a monopoly).*
- b) *Determine the unique pure strategies Nash equilibrium payoff of the stage game.*
- c) *Is collusion sustainable in equilibrium when the game is finitely repeated?*
- d) *In the infinitely repeated game define a grim-trigger strategy devoted to sustain collusion.*
- e) *For which discount factors is the corresponding strategy profile a Nash equilibrium of the repeated game? A subgame perfect Nash equilibrium? What is the influence of the number of firms?*
- f) *Is collusion on another quantity sustainable in equilibrium? Under what conditions?*

## Problem. The OECD's solution to tax multinational enterprises' income

Over the past decades, digitization and globalization have had a profound impact on economies, challenging the rules for taxing the international companies' income. New business models that rely heavily on intellectual property have made it easier for multinational enterprises to shift their profits to low-tax jurisdictions. Many multinationals have taken the opportunity to avoid paying their fair share of tax despite the enormous profits they have garnered as the winners of globalization. Over the last couple of years the Organisation for Economic Co-operation and Development (OECD) ramped up its efforts to address these challenges in response to growing public and political concerns about tax avoidance by large multinationals. In 2021, after intense debate and negotiations, the OECD proposed a two-pillar solution for the taxation of international business income. A first pillar sets a floor on tax competition by establishing an overall minimum corporate tax at the rate of 15%. A second pillar aims to reallocate the distribution of the profits of the largest and most profitable businesses between countries. How to encourage countries to ratify the new rules?

### Part A. Tax competition between two countries : implementing a minimum corporate tax

The objective of this part is to study the strategic interaction of two countries which initially have opposite tax preferences due to the difference in size of their economies. Consider two countries,  $A$  and  $B$ , which each have to choose between two levels of corporate tax rates : low ( $L$ ) and high ( $H$ ), with  $L < H$  and  $(L, H) \in (0, 1)^2$ . The larger companies of the two countries choose to pay their taxes in the country with the lower rate. In the case of similar tax rates, each company pays taxes in its home country. Let  $\alpha^A$  (resp.  $\alpha^B$ ) be the proportion of revenue generated by companies in country  $A$  (resp.  $B$ ) which are large enough

to have the possibility of paying taxes abroad, with  $(\alpha^A, \alpha^B) \in (0, 1)^2$ . Let  $Y^A$  (resp.  $Y^B$ ) be the taxable profit of companies originating in country  $A$  (resp.  $B$ ). Suppose country  $A$  has both a higher proportion of revenues generated by multinational firms (i.e.,  $\alpha^A > \alpha^B$ ) and a higher taxable positive profit (i.e.,  $Y^A > Y^B > 0$ ). More generally, assume that

$$\frac{H - L Y^B}{L Y^A} < \alpha^B < \alpha^A < H - L < \frac{H - L Y^A}{L Y^B}$$

Both countries can be thought as being members of the European Union, where country  $A$  being a medium-sized economy with a combined corporate income tax rate above 25% (e.g., France, Germany, Italy, Spain) and country  $B$  as a small economy with a combined corporate income tax rate below 15% (e.g., Hungary, Ireland, Lithuania). Suppose each country aims to maximize its tax revenue.

**A.1)** *Depict the corresponding strategic interaction in a context of simultaneous moves with a  $2 \times 2$  payoff matrix where  $A$  (resp.  $B$ ) is the row (resp. column) player.*

**A.2)** *What are the countries' best response correspondences? Does a country have a dominant strategy?*

**A.3)** *What is the set of (pure) strategy Nash equilibrium? What is the set of Pareto optima?*

Suppose country  $A$  has a new law according to which domestic companies that are taxed at a lower rate abroad have to pay the difference in tax to country  $A$ . We assume that companies have not yet had time to change tax location.

**A.4)** *What is the set of (pure) strategy Nash equilibrium?*

**A.5)** *Is country  $A$  better off here compared to the situation described in the previous question? Does the corresponding equilibrium outcome Pareto-dominate the previous one?*

**A.6)** *Assume that now that country  $A$  has implemented this new law, all companies which are originally from country  $A$  fully pay their taxes in country  $A$ . What does this change for country  $B$ ? What is the new set of (pure) strategy Nash equilibrium?*

## Part B. Using threat to achieve tax cooperation between the EU and the US

After two decades of a disconnected and unfair international tax system whereby digital companies pay taxes in countries where their algorithms were developed instead of where their customers are located, roughly half of all European countries (EU) recently announced, proposed, or implemented a domestic digital services tax. In response, the United States (US), home to many big tech companies (e.g., Amazon, Apple, Google, Facebook, Microsoft), has consequently dedicated significant resources and attached substantial political imperative to finding a timely resolution of the issues at stake. The US is ready to reallocate some taxing rights on the largest multinational enterprise from their home country to the markets where they do business and make profits on condition that the EU countries withdraw their existing (and planned) domestic digital services taxes. Because the EU demands that the US first ratifies OECD's corresponding pillar solution, the US warns that it will start a trade war with the EU if it does not remove the digital taxes afterward.

Consider the following strategic game where the EU and the US interact sequentially. The US chooses whether or not to ratify the OECD tax solution. Before that, the EU chooses whether or not to implement a digital tax. If the US ratifies, then the EU chooses whether or not to cancel its digital taxes. If the EU does not, the US chooses whether or not to start a trade war.

Players' preferences satisfy the following assumptions :

1. The EU prefers the US to ratify the OECD suggested tax rather than having no tax applied.
2. The EU also prefers to avoid a trade war.
3. In the event that no digital tax is implemented, the US prefers not to ratify the tax suggested by the OECD.
4. However, the US prefers the OECD suggested tax to the EU digital taxes.
5. For reasons of reputation, the US prefers to start a trade war rather than renounce its threat.

**B.1)** *Draw the corresponding game tree and solve for its subgame-perfect Nash equilibrium.*

## Part C. Minimum tax in repeated interaction

The recommendation for a minimum tax is based on the idea that countries, engaged in a competition for mobile resources like capital investment, are forced to lower their corporate tax rates to suboptimal levels. A minimum tax, according to this view, could stop the « race to the bottom » and thus improve the situation of all countries. Is a minimum tax Pareto improving if tax competition occurs repeatedly rather than as a one-shot interaction ?

Consider an economy with an infinite time horizon with periods  $t = 1, 2, \dots$ . There are two ex-ante identical countries. In each period, each country takes a single action, setting a tax rate on a mobile tax base (capital) at source. The tax rate set by country  $i \in \{1, 2\}$  in period  $t$  is  $\tau_{i,t}$ , taken from the interval  $[0; 1]$ . Let the one-period payoff of country  $i$  be  $g_i(\tau_1, \tau_2)$ , and satisfy the following :

- It is twice continuously differentiable (and strictly quasi-concave in all tax rates, so that the iso-payoff curves are convex to the origin) ;
- It increases with  $\tau_j$ , so that the so-called *tax base effect* holds : if a country increases its tax rate, leaving the tax rates in the other country unchanged, some (but not all) of its capital relocates to the other country ;
- Among the symmetric tax rates, there is a unique rate  $\tau^*$  that maximizes the one-period payoff, and this rate is 25%. (This does not exclude that a pair of asymmetric tax rates may offer a country a higher payoff.) ; and
- Player  $i$ 's best-response writes as :  $BR^i(\tau_j) = \frac{\tau_j}{2} + \frac{1}{20}$ , with  $i \neq j$ .

Countries discount the future by a common discount factor  $\delta \in (0; 1)$ .

**C.1)** Consider the one-shot interaction. Is the tax rate  $\tau^*$  sustainable ? Why ? What is the one-shot Nash equilibrium  $(\tau_1^N, \tau_2^N)$  ? In case this equilibrium is symmetric, denote it as  $(\tau^N, \tau^N)$ .

**C.2)** What is the minimum discount factor  $\bar{\delta}$  above which the symmetric tax rate  $\tau^*$  can be supported by grim-trigger strategies as a subgame-perfect Nash equilibrium of the infinitely repeated game ?

**C.3)** Suppose now a minimum corporate tax rate is set internationally at level  $\underline{\tau} \in (\tau^N, \tau^*)$ . This level satisfies  $g_i(\underline{\tau}, \underline{\tau}) > g_i(\tau^N, \tau^N)$  for each country  $i \in \{1, 2\}$ . Is the minimum discount factor  $\bar{\delta}$  still valid ? Conclude.