Introduction to Derivative Instruments

Paris Dauphine University - Master I.E.F. (272)
Autumn 2024

Jérôme MATHIS

www.jeromemathis.fr/Derivatives
password: 272-Derivatives

Slides on book: John C. Hull, “Options, Futures, and Other Derivatives”, Pearson ed.

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Chapter 6

Chapter 6: Interest Rate Futures
Outline

Motivation
Motivation

- So far we have covered futures contracts on:
  - commodities;
  - stock indices; and
  - foreign currencies.
- This chapter deals with interest rate futures.
  - We shall see how interest rate futures contracts, when used in conjunction with the duration measure introduced in Chapter 5, can be used to hedge a company’s exposure to interest rate movements.
- Many of the interest rate futures contracts throughout the world have been modeled on *Treasury bond* and *Eurodollar* futures contracts that trade in the United States.
  - We shall study both of these popular contracts.

Chapter 6: Interest Rate Futures
Outline

Day Counts and Quotation Conventions
Day Counts

Definition

The **day count** is the way in which interest accrues over time. It is usually expressed as X/Y where X denotes the way in which the number of days between the two considered dates is calculated and Y denotes the way in which the total number of days in the reference period is measured.

- Three day count conventions are commonly used.
- 1. Actual/actual (in period)
  - It is used for Treasury bonds in U.S. and E.U.
  - The interest earned between two dates is based on the ratio of the actual days elapsed to the actual number of days in the period between coupon payments.

Day Counts and Quotation Conventions
Day Counts

Example

Assume that the bond principal is \$100, coupon payment dates are April 1 and October 1, and the coupon rate is 6% per annum.

Suppose that we wish to calculate the interest earned between April 1 and June 3.

There are 183 (actual) days in the reference period (from April 1 to October 1), and interest of \$3 is earned during the period.

There are 63 (actual) days between April 1 and June 3.

The interest earned between April 1 and June 3 is therefore

$$\frac{63}{183} \times 3 = 1.0328.$$

Day Counts and Quotation Conventions
Day Counts

- 2. 30/360
  - It is used for corporate and municipal bonds.
  - This means that we assume 30 days per month (to compute the distance between the two considered dates) and 360 days per year (to compute the reference period).
  - The total number of days between April 1 and June 3 is assumed to be $(2 \times 30) + 2 = 62$.
  - The total number of days between February 28 and March 1 is assumed to be 3!
  - On a 360 basis, the semiannual compounding accounts as 180 days.

Example

In the previous example the interest earned between April 1 and June 3 here writes as

$$\frac{62}{180} \times 3 = 1.0333$$

Day Counts and Quotation Conventions
Day Counts

- 3. Actual/360
  - It is used for many other money market instruments

Example

The interest earned between April 1 and June 3 here writes as

$$\frac{63}{180} \times 3 = 1.05$$

- Treasury bond prices are quoted in dollars and thirty-seconds (1/32) of a dollar.
  - The quoted price is for a bond with a face value of $100.
  - Thus, a quote of 87-11 indicates that the quoted price for a bond with a face value of $100,000 is $(87 + \frac{11}{32}) \times 1,000 = \$87,343.75$.

Definition

The **quoted price** corresponds to the price of the bond as quoted in the paper.

Definition

The **accrued interest** corresponds to the amount of coupon earned on a bond since the last coupon payment. It writes as

$$\frac{\text{days since last coupon}}{\text{total days in period}} \times \text{coupon paid during period.}$$

Definition

Cash price =Quoted price + Accrued interest.

Question

Suppose that it is March 5 and a bond with $100 face value and 8% coupon bond with semiannually compounding and maturing on July 10 has a quoted price of 89-08 or $89.25.

What is the cash price?

Solution

The most recent coupon date is January 10 and the next coupon date is July 10.

The number of days between January 10 and March 5 is 54.

The number of days between January 10 and July 10 is 181.

On a bond with $100 face value, the coupon payment is $4 on January 10 and July 10.

Solution

The accrued interest on March 5 is the share of the July 10 coupon accruing to the bondholder on March 5.

Because actual/actual in period is used for Treasury bonds in the United States, this is

$$\frac{54}{181} \times \$4 = \$1.19$$

The cash price per $100 face value for the bond is therefore

$$\$89.25 + \$1.19 = \$90.44$$

Chapter 6: Interest Rate Futures
Outline

Treasury Bond Futures
Conversion Factors

- The bonds that can be delivered are standardized through a system of conversion factors used to equalize coupon and accrued interest differences of all delivery bonds.

Definition

A **conversion factor** for a bond is the approximate price the bond would have per dollar of principal (i.e., $1 par) if it had a 6% yield-to-maturity.

- In practice, the conversion factor for a bond is set equal to the quoted price the bond would have per dollar of principal on the first day of the delivery month on the assumption that the interest rate for all maturities equals 6% per annum (with semiannual compounding).

Treasury Bond Futures
Conversion Factors

- If a contract specifies that a bond has a notional coupon of 6%, the conversion factor will be less than 1 for bonds with a coupon less than 6%, and higher than 1 for bonds with a coupon higher than 6%.
  - A conversion factor of 0.93 means that a bond is approximately valued at 93% as much as a 6% coupon security.
- It is a parameter that defines the price received for the bond by the party with the short position.
- The cash received for each $100 face value of the bond delivered is:

  (Most recent settlement price x Conversion factor) + Accrued interest

Treasury Bond Futures
Conversion Factors

Example

Each contract is for the delivery of $100,000 face value of bonds.

Suppose that the most recent settlement price is 90-00, the conversion factor for the bond delivered is 1.3800, and the accrued interest on this bond at the time of delivery is $3 per $100 face value.

The cash received by the party with the short position (and paid by the party with the long position) is then

$$(1.3800 \times 90.00) + 3.00 = \$127.20$$

per $100 face value.

A party with the short position in one contract would deliver bonds with a face value of $100,000 and receive $\$127,200$.

Treasury Bond Futures
Cheapest-to-Deliver Bond

- At any given time during the delivery month, there are many bonds that can be delivered in the Treasury bond futures contract.
- The party with the short position can choose which of the available bonds is "cheapest" to deliver.
- What is the cheapest-to-deliver bond?
  - The party with the short position receives:

    (Most recent settlement price x Conversion factor) + Accrued interest

  - The cost of purchasing a bond is

    Quoted bond price + Accrued interest

  - The cheapest-to-deliver bond is then the one for which

    Quoted bond price - (Most recent settlement price x Conversion factor)

    is least.

Treasury Bond Futures
Cheapest-to-Deliver Bond

Question

The party with the short position has decided to deliver and is trying to choose between the three bonds in the table below.

Assume the most recent settlement price is 92-08, or 92.25.

Bond	*Quoted bond price ($)*	*Conversion factor*
1	99.50	1.0382
2	143.50	1.5188
3	119.75	1.2615

What is the cheapest-to-deliver bond?

Treasury Bond Futures
Cheapest-to-Deliver Bond

Solution

The cost of delivering each of the bonds is as follows.

Bond 1:

$$99.50 - (92.25 \times 1.0382) = \$3.73$$

Bond 2:

$$143.50 - (92.25 \times 1.5188) = \$3.39$$

Bond 3:

$$119.75 - (92.25 \times 1.2615) = \$3.37$$

The cheapest-to-deliver bond is Bond 3.

Chapter 6: Interest Rate Futures
Outline

Eurodollar Futures

Definition

A **Eurodollar** is a dollar deposited in a bank outside the United States.

- The name eurodollars was derived from the fact that initially dollar-denominated deposits were largely held in European banks.
  - At first these deposits were known as eurobank dollars.
  - However, U.S. dollar-denominated deposits are now held in financial centers across the globe and referred to as eurodollars.
- Similarly, the term **eurocurrency** is used to describe currency deposited in a bank that is not located in the home country where the currency was issued.
  - E.g., Japanese yen deposited at a bank in Brazil would be defined as eurocurrency.

Eurodollar Futures

- The most popular interest rate futures contract is the 3-month Eurodollar futures contract traded by the CME Group.
- A three-month Eurodollar futures is a contract on the interest that will be paid on $1 million for a future three-month period.
  - One contract is on the rate earned on $1 million as quoted on the 3-month Eurodollar deposit rate (same as 3-month LIBOR rate).
  - It is settled in cash and when it expires (on the third Wednesday of the delivery month) the final settlement price is: 100−(3-month Eurodollar deposit rate).

Eurodollar Futures

Example

Suppose on November 1 you have $1 million to invest on for three months on Dec 21. The quoted price is $\$97.12$ on November 1.

The contract then locks in a rate of $100 - 97.12 = 2.88\%$.

The total profit on your $1 million according to the contract is then $\$1{,}000{,}000 \times \frac{3}{12} \times 2.88\% = \$7{,}200$.

If the quoted price turns out to be $\$97.42$ on December 21. We make a gain in the futures contract of $(2.88\% - 2.58\%) \times \frac{3}{12} \times \$1,000,000 = \$750.$.

Eurodollar Futures

- A one-basis-point change in the futures quote corresponds to a 0.01% change in the futures interest rate and to a contract price change of $25.
  - 25 comes from the fact that the 3-month rate of $x\%$ annual rate is $\frac{3}{12} \times x = 0.25 \times x$.
  - So, let Δx denote the change in the interest rate. The variation on the 1 million amounts to

$$0.25 \times \Delta x \times \$1,000,000 = \Delta x \times \$250,000$$

  - Now, for $\Delta x = 0.01\%$ we obtain

$$\Delta x \times \$250,000 = 0.01\% \times \$250,000 = 0.0001 \times \$250000 = \$25$$

Eurodollar Futures

Example

A settlement price changes from 81.265 to 81.325.

Traders with long (resp. short) positions gain (resp. lose)

$$\begin{aligned} & \$1,000,000 \times 0.01\% \times (81.325 - 81.265) \times 25 \\ = & \$100 \times (81.325 - 81.265) \times 25 \\ = & \$100 \times 0.06 \times 25 = \$150 \end{aligned}$$

per contract.

Eurodollar Futures

- If Q is the quoted price of a Eurodollar futures contract, the value of one contract is

$$10,000[100 - 0.25(100 - Q)]$$

- This corresponds to the $25 per basis point rule.

Eurodollar Futures

Forward rate vs. Eurodollar futures

- Futures is settled daily whereas forward is settled once.
- Futures is settled at the beginning of the underlying three-month period while FRA is settled at the end of the underlying three-month period.
- So for Eurodollar futures lasting beyond two years we cannot assume that the forward rate equals the futures rate.
- Analysts make what is known as a *convexity adjustment* to account for the total difference between the two rates.

Eurodollar Futures

Forward rate vs. Eurodollar futures

- One popular adjustment is

$$\text{Forward Rate} = \text{Futures Rate} - \frac{\sigma^2}{2} T_1 T_2$$

where:

- T_1 is the start of period covered by the forward/futures rate;
- T_2 is the end of period covered by the forward/futures rate (90 days later that T_1);
- σ is the standard deviation of the change in the short rate per year (often assumed to be about 1.2%); and
- Both rates are expressed with continuous compounding.

Eurodollar Futures
Forward rate vs. Eurodollar futures

Example

Consider the situation where $\sigma = 1.2\%$ and we wish to calculate the forward rate when the 10-year Eurodollar futures price quote is 89.

In this case $T_1 = 10$, $T_2 = 10.25$, and the convexity adjustment is

$$\frac{\sigma^2}{2} T_1 T_2 = \frac{0.012^2}{2} 10 \times 10.25 = 0.00738$$

or 0.738% (73.8 basis points).

The futures rate is $100 - 89 = 11\%$ per annum on an actual/360 basis with quarterly compounding.

Eurodollar Futures
Forward rate vs. Eurodollar futures

Example

This corresponds to $\frac{11}{4} = 2.75\%$per 90 days or an annual rate of

$$(365/90) \ln 1.0275 = 11.002\%$$

with continuous compounding and an actual/365 day count.

The estimate of the forward rate is then

$$11.002 - 0.738 = 10.264\%$$

per annum with continuous compounding.

Eurodollar Futures
Forward rate vs. Eurodollar futures

- Observe that the size of the adjustment is roughly proportional to the square of the time to maturity of the futures contract.
  - Indeed, from $T_2 = T_1 + 0.25 \sim T_1$ the convexity adjustment is close to $\frac{\sigma^2}{2}(T_1)^2$.
  - So, when the maturity doubles from 2 to 4 years, the size of the convexity approximately quadruples.

Eurodollar Futures
Using Eurodollar Futures to Extend the LIBOR Zero Curve

- LIBOR deposit rates define the LIBOR zero curve out to one year.
- Eurodollar futures can be used to determine forward rates and the forward rates can then be used to bootstrap the zero curve.

Eurodollar Futures
Using Eurodollar Futures to Extend the LIBOR Zero Curve

- Suppose that the $i-$th Eurodollar futures contract matures at time T_i, with $i = 1, 2, ...$.
  - It is usually assumed that the forward interest rate calculated from the $i-$th futures contract applies to the period T_i to T_{i+1}.
  - This enables a bootstrap procedure to be used to determine zero rates.
  - Suppose that F_i is the forward rate calculated from the $i-$th Eurodollar futures contract and R_i is the zero rate for a maturity T_i.
  - From slides 45 of our previous Chapter we have

$$F_i = \frac{R_{i+1}T_{i+1} - R_iT_i}{T_{i+1} - T_i}$$

so that

$$R_{i+1} = \frac{F_i\left(T_{i+1} - T_i\right) + R_iT_i}{T_{i+1}}.$$

Eurodollar Futures
Using Eurodollar Futures to Extend the LIBOR Zero Curve

- Other Euro rates such as Euroswiss, Euroyen, and Euribor are used in a similar way.

Example

If the 400-day LIBOR zero rate has been calculated as 4.80% and the forward rate for the period between 400 and 491 days is 5.30% the 491 day rate is

$$R_{491} = \frac{0.053 \times 91 + 0.048 \times 400}{491} = 4.893\%.$$

Exercise (4)

The 350-day LIBOR rate is 3% with continuous compounding and the forward rate calculate from a Eurodollar futures contract that matures in 350 days is 3.2% with continuous compounding.

Estimate the 440-day zero rate.

Solution (4)

We have

$$R_2 = \frac{F_1\left(T_2 - T_1\right) + R_1T_1}{T_2}.$$

with $F_1 = 3.2\%$, $T_1 = 350$, $T_2 = 440$, *and* $R_1 = 3\%$. *So*

$$R_2 = \frac{3.2\% \times 90 + 3\% \times 350}{440} = 3.0409\%.$$

Chapter 6: Interest Rate Futures
Outline

- Duration-based hedging involves hedging against interest rate risk by matching the durations of assets and liabilities.
- As we discussed in the previous chapter, it provides protection against small parallel shifts in the zero curve.

- The number of contracts required to hedge against an uncertain change in the yield, Δy, is

$$N^* = \frac{PD_P}{V_F D_F}$$

where:

  - V_F: Contract price for one interest rate futures contract
  - D_F: Duration of the asset underlying the futures contract at the maturity of the futures contract
  - P: Forward value of the portfolio being hedged at the maturity of the hedge
    - in practice, this is usually assumed to be the same as the value of the portfolio today
  - D_P: Duration of the portfolio at the maturity of the hedge

Example

It is August. A fund manager has $10 million invested in a portfolio of government bonds with a duration of 6.80 years and wants to hedge against interest rate moves between August and December.

The manager decides to use December T-bond futures.

The futures price is 93-02 or 93.0625 and the duration of the cheapest to deliver bond is 9.2 years.

The number of contracts (one contract is on 1,000 futures) that should be shorted is

$$N^* = \frac{PD_P}{V_F D_F} = \frac{10,000,000 \times 6.8}{93,062.50 \times 9.2} \simeq 79.423 \simeq 79$$

Exercise (8)

On August 1 a portfolio manager has a bond portfolio worth $10 million. The duration of the portfolio in October will be 7.1 years. The December Treasury bond futures price is currently 91-12 and the cheapest-to-deliver bond will have a duration of 8.8 years at maturity. One contract is on 1,000 futures.

a) How should the portfolio manager immunize the portfolio against changes in interest rates over the next two months?

b) How can the portfolio manager change the duration of the portfolio to 3.0 years?

Solution (8)

a) The treasurer should short Treasury bond futures contract. If bond prices go down, this futures position will provide offsetting gains. The number of contracts that should be shorted is

$$\frac{10,000,000}{91,375}\frac{7.1}{8.8} = 88.30$$

Exercise (8)

On August 1 a portfolio manager has a bond portfolio worth $10 million. The duration of the portfolio in October will be 7.1 years. The December Treasury bond futures price is currently 91-12 and the cheapest-to-deliver bond will have a duration of 8.8 years at maturity.

b) How can the portfolio manager change the duration of the portfolio to 3.0 years?

Solution (8)

Rounding to the nearest whole number 88 contracts should be shorted.
b) In a) the problem is designed to reduce the duration to zero. To reduce the duration from 7.1 to 3.0 instead of from 7.1 to 0, the treasurer should short

$$\frac{4.1}{7.1} \times 88.30 = 50.99$$

Duration-Based Hedging
Limitations of Duration-Based Hedging

- Duration matching does not immunize a portfolio against nonparallel shifts in the zero curve.
  - This is a weakness of the approach.
  - In practice, short-term rates are usually more volatile than, and are not perfectly correlated with, long-term rates.
  - Sometimes it even happens that, short- and long-term rates move in opposite directions to each other.

Duration-Based Hedging
Limitations of Duration-Based Hedging

- A bank is still subject to liquidity risk.
  - If a bank funds long term assets with short term liabilities such as commercial paper, it can use FRAs, futures, and swaps to hedge its interest rate exposure.
  - But it still has a liquidity exposure.
  - It may find it impossible to roll over the commercial paper if the market loses confidence in the bank
  - Northern Rock is an example.

Chapter 6: Interest Rate Futures
Outline

Summary

- Two very popular interest rate contracts are the Treasury bond and Eurodollar futures contracts.
- In the Treasury bond futures contracts, the party with the short position has a number of interesting delivery options (Delivery can be made on any day during the delivery month, alternative bonds can be delivered) that tends to reduce the futures price.
- The Eurodollar futures contract is a contract on the 3-month rate on the third Wednesday of the delivery month.
  - Eurodollar futures are frequently used to estimate LIBOR forward rates for the purpose of constructing a LIBOR zero curve.
  - When long-dated contracts are used in this way, it is important to make what is termed a convexity adjustment to allow for the marking to market in the futures contract.

Summary

- Changing the duration enables a hedger to assess the sensitivity of:
  - a bond portfolio to small parallel shifts in the yield curve; and
  - an interest rate futures price to small changes in the yield curve.
- The key assumption underlying duration-based hedging is that all interest rates change by the same amount.
  - This means that only parallel shifts in the term structure are allowed for.
  - In practice, short-term interest rates are generally more volatile than are long-term interest rates, and hedge performance is liable to be poor if the duration of the bond underlying the futures contract differs markedly from the duration of the asset being hedged.