Introduction to Derivative Instruments

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Slides on book: John C. Hull, “Options, Futures, and Other Derivatives”, Pearson ed.

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Chapter 4

Chapter 4: Hedging Strategies Using Futures
Outline

Motivation

- Many of the participants in futures markets are hedgers.
  - Their aim is to use futures markets to reduce a particular risk that they face.
    - ⋆ This risk might relate to fluctuations in the price of oil, a foreign exchange rate, the level of the stock market, or some other variable.
- A study of hedging using futures contracts is a study of the ways in which hedges can be constructed so that they eliminates the risk as much as possible.

Motivation

- Assuming that no attempt is made to adjust the hedge once it has been put in place we will answer the following questions:
  - When is a short futures position appropriate?
  - When is a long futures position appropriate?
  - Which futures contract should be used?
  - What is the optimal size of the futures position for reducing risk?
- The chapter initially treats futures contracts as forward contracts
  - I.e., it ignores daily settlement.
- Later it explains an adjustment known as “tailing” that takes account of the difference between futures and forwards.

Chapter 4: Hedging Strategies Using Futures
Outline

Short & Long Hedges
Example of Short Hedge

- Assume that it is May 15 today and that an oil producer has just negotiated a contract to sell 1 million barrels of crude oil.
  - It has been agreed that the price that will apply in the contract is the market price on August 15.
  - Suppose that on May 15 the spot price is \$80 per barrel and the crude oil futures price for August delivery is \$79 per barrel.
  - Because each futures contract is for the delivery of 1,000 barrels, the company can hedge its exposure by shorting (i.e., selling) 1,000 futures contracts.
  - If the oil producer closes out its position on August 15, the effect of the strategy should be to lock in a price close to \$79 per barrel.

Short & Long Hedges
Example of Short Hedge

- To illustrate what might happen, suppose that the spot price on August 15 proves to be \$75 per barrel.
  - The company realizes \$75 million for the oil under its sales contract.
  - Because August is the delivery month for the futures contract, the futures price on August 15 should be very close to the spot price of \$75 on that date.
  - The company therefore gains approximately $\$79 - \$75 = \$4$ per barrel, or \$4 million in total from the short futures position.
  - The total amount realized from both the futures position and the sales contract is therefore approximately \$79 per barrel, or \$79 million in total.

Short & Long Hedges
Example of Short Hedge

- For an alternative outcome, suppose that the price of oil on August 15 proves to be \$85 per barrel.
  - The company realizes \$85 per barrel for the oil and loses approximately $\$85 - \$79 = \$6$ per barrel on the short futures position.
  - Again, the total amount realized is approximately \$79 million.
  - It is easy to see that in all cases the company ends up with approximately \$79 million.

Short & Long Hedges

- A *short futures hedge* is then appropriate when you know you will *sell* an asset in the future and want to lock in the price.
- Similarly, a *long futures hedge* is appropriate when you know you will *purchase* an asset in the future and want to lock in the price.

Chapter 4: Hedging Strategies Using Futures
Outline

Arguments in Favor of Hedging

- Companies should focus on the main business they are in and take steps to minimize risks arising from interest rates, exchange rates, and other market variables.
  - Most companies are in the business of manufacturing, or retailing or wholesaling, or providing a service.
  - They have no particular skills or expertise in predicting variables such as interest rates, exchange rates, and commodity prices.
  - By hedging, they avoid unpleasant surprises such as sharp rises in the price of a commodity that is being purchased.

Arguments against Hedging

- Shareholders are usually well diversified and can make their own hedging decisions.
  - For example, in addition to holding shares in a company that uses copper, a well-diversified shareholder may hold shares in a copper producer, so that there is very little overall exposure to the price of copper.
- It may increase risk to hedge when competitors do not.
  - Competitive pressures within the industry may be such that the prices of the goods and services produced by the industry fluctuate to reflect raw material costs, interest rates, exchange rates, and so on.
    - A company that does not hedge can expect its profit margins to be roughly constant.
    - However, a company that does hedge can expect its profit margins to fluctuate!

Arguments against Hedging

- Explaining a situation where there is a loss on the hedge and a gain on the underlying can be difficult.
  - E.g., consider the previous example of short hedge where the company realizes \$85 per barrel for the oil and loses approximately $\$85 - \$79 = \$6$ per barrel on the short futures position.

Chapter 4: Hedging Strategies Using Futures
Outline

Closing out the futures contract before delivery date

- Taking delivery can be expensive and inconvenient.
  - Futures prices are in some instances quite erratic during the delivery month.
- Long hedgers normally prefer to close out the futures contract and buy the asset from their usual suppliers.
  - A long hedger does not like to run the risk of having to take delivery of the physical asset if the contract is held during the delivery month.
- A good rule of thumb is therefore to choose a delivery month that is as close as possible to, but later than, the expiration of the hedge.
  - Suppose delivery months are March, June, September, and December for a futures contract on a particular asset.
    - For hedge expirations in December, January, and February, the March contract will be chosen;
    - for hedge expirations in March, April, and May, the June contract will be chosen; and so on.

Closing out the futures contract before delivery date
Example 1

- It is March 1. A US company expects to receive 50 million Japanese yen at the end of July.
  - Yen futures contracts on the CME Group have delivery months of March, June, September, and December.
  - One contract is for the delivery of 12.5 million yen.
  - The company therefore shorts four September yen futures contracts on March 1.
  - When the yen are received at the end of July, the company closes out its position (i.e., exchanges yen for dollars, and buy four long September yen futures contracts).
  - We suppose that the futures price on March 1 in cents per yen is 0.7800 ($F_1 = 0.78$) and that the spot and futures prices when the contract is closed out are 0.7200 ($S_2 = 0.72$) and 0.7250 ($F_2 = 0.725$), respectively.

Question (Example 1)

What is the effective price obtained in cents per yen?

Closing out the futures contract before delivery date
Example 1

Solution

Closing out the futures contract before delivery date
Example 2

- It is June 8 and a company knows that it will need to purchase 20,000 barrels of crude oil at some time in October or November.
  - Oil futures contracts are currently traded for delivery every month on the NYMEX division of the CME Group and the contract size is 1,000 barrels.
  - The company therefore decides to use the December contract for hedging and takes a long position in 20 December contracts.
  - The futures price on June 8 is \$68.00 per barrel, $F_1 = 68$.
  - The company finds that it is ready to purchase the crude oil on November 10 and therefore closes out its futures contract on that date.
  - The spot price and futures price on November 10 are \$70.00 per barrel and \$69.10 per barrel, $S_2 = 70$ and $F_2 = 69.10$.

Question (Example 2)

What is the effective price paid (in dollars per barrel)?

Closing out the futures contract before delivery date
Example 1

Solution

Closing out the futures contract before delivery date
Example 2

Solution

Chapter 4: Hedging Strategies Using Futures
Outline

Basis Risk

- Observe that in both previous examples the effective price writes as

$$S_2 + F_1 - F_2$$

- Why is the formulae the same?
  - The first example considers a *short hedge for sale* of an asset
  - The second example considers a *long hedge for purchase* of an asset
  - In the first example the computed effective price is the price that the company *obtain*.
  - In the second example the computed effective price is the price that the company *pay*.

Basis Risk

- The **basis** in a hedging situation is defined as the spot price of asset to be hedged minus the futures price of contract used.
  - The basis b_t at time t writes as $S_t - F_t$.
  - If t is the delivery date then the spot price and the futures price are the same, and the basis is zero.
- In the two previous examples the effective price that is obtained for the asset with hedging is

$$S_2 + F_1 - F_2 = F_1 + S_2 - F_2 = F_1 + b_2.$$

- The value of F_1 is known at time t_1 and if b_2 were known at this time, a perfect hedge would result.
- The hedging risk is the uncertainty associated with b_2 and is known as **basis risk**.

Basis Risk

- The basis risk can lead to an improvement or a worsening of a hedger's position.
  - Consider a short hedge.
    - ⋆ If the basis strengthens (i.e., increases) unexpectedly, the hedger's position improves;
    - ⋆ if the basis weakens (i.e., decreases) unexpectedly, the hedger's position worsens.
  - For a long hedge, the reverse holds.
    - ⋆ If the basis strengthens unexpectedly, the hedger's position worsens;
    - ⋆ if the basis weakens unexpectedly, the hedger's position improves.

Chapter 4: Hedging Strategies Using Futures
Outline

Cross Hedging

- In the two previous examples, the asset underlying the futures contract was the same as the asset whose price is being hedged.
- **Cross hedging** occurs when the two assets are different.
  - Consider, for example, an airline that is concerned about the future price of jet fuel.
    - Because jet fuel futures are not actively traded, it might choose to use heating oil futures contracts to hedge its exposure.

Cross Hedging

- The **hedge ratio** is the ratio of the size of the position taken in futures contracts to the size of the exposure.
  - When the asset underlying the futures contract is the same as the asset being hedged, it is natural to use a hedge ratio of 1
    - This is the hedge ratio we have used in the examples considered so far.
    - For instance, in the previous example, the hedger's exposure was on 20,000 barrels of oil, and futures contracts were entered into for the delivery of exactly this amount of oil.
- When cross hedging is used, setting the hedge ratio equal to 1.0 is not always optimal.
  - The hedger should choose a value for the hedge ratio that minimizes the variance of the value of the hedged position.

Cross Hedging
Calculating the Minimum Variance Hedge Ratio

- The **minimum variance hedge ratio** depends on the relationship between changes in the spot price and changes in the futures price.
  - Assume a portfolio with underlying asset S is hedged with derivative instrument F according to the ratio h.
  - At time t, the valuation of the hedged portfolio X writes as

$$X_t = S_t + hF_t$$

  - Its variation writes as

$$\Delta X = \Delta S + h\Delta F$$

- What is the hedge ratio that minimizes the variance of the variation of the portfolio?

Cross Hedging
Calculating the Minimum Variance Hedge Ratio

- Define:
  - ΔS: change in spot price, S, during a period of time equal to the life of the hedge;
  - ΔF: change in futures price, F, during a period of time equal to the life of the hedge;
  - σ_S (resp. σ_F) the standard deviation of ΔS (resp. ΔF); and
  - ρ the coefficient of correlation between ΔS and ΔF.
- The variance of the variation of the portfolio writes as

$$\sigma^2_{\Delta X} := VAR(\Delta X) = VAR(\Delta S + h\Delta F)$$

- Using the formulae

$$Var[aY + bZ] = a^2\sigma^2_Y + b^2\sigma^2_Z + 2ab(corr(Y,Z) \times \sigma_Y\sigma_Z).$$

- We obtain

$$\sigma^2_{\Delta X} = \sigma^2_S + h^2\sigma^2_F + 2h(\rho\sigma_S\sigma_F)$$

Cross Hedging
Calculating the Minimum Variance Hedge Ratio

- The minimum variance hedge ratio h^* solves

$$h^* \in \arg\min_h [\sigma^2_S + h^2\sigma^2_F + 2h(\rho\sigma_S\sigma_F)]$$

- The first-order condition writes as

$$2h\sigma^2_F + 2(\rho\sigma_S\sigma_F) = 0$$

- The second-order condition is strictly positive:

$$2\sigma^2_F > 0$$

Cross Hedging
Calculating the Minimum Variance Hedge Ratio

- So that, the minimum variance hedge ratio, h^*, satisfies

$$h^* = -\rho\frac{\sigma_S\sigma_F}{\sigma^2_F} = -\rho\frac{\sigma_S}{\sigma_F}$$

  with the interpretation that $\rho\frac{\sigma_S}{\sigma_F}$ should be shorted to hedge our risk.
- In most books, h^* is defined as a positive number (absolute value), so that

$$h^* = \rho\frac{\sigma_S}{\sigma_F}$$

Cross Hedging
Calculating the Minimum Variance Hedge Ratio

- For instance:
  - If $\rho = 1$ and $\sigma_F = \sigma_S$, the hedge ratio, h^*, is 1.
    - ⋆ This result is to be expected, because in this case the futures price mirrors the spot price perfectly.
  - If $\rho = 1$ and $\sigma_F = 2\sigma_S$, the hedge ratio, h^*, is 0.5.
    - ⋆ This result is also as expected, because in this case the futures price always changes by twice as much as the spot price.

Cross Hedging
Optimal Number of Contracts

- To calculate the number of contracts that should be used in hedging, define:
  - Q_A: size of position being hedged (units);
  - Q_F: size of one futures contract (units); and
  - N^*: optimal number of futures contracts for hedging.
- The futures contracts should be on $h^* Q_A$ units of the asset.
- The number of futures contracts required is therefore given by

$$N^* = \frac{h^* Q_A}{Q_F}$$

Cross Hedging
Optimal Number of Contracts

Example

Airline will purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil futures for hedging.

From historical data $\sigma_F = 0.0313$, $\sigma_S = 0.0263$, *and* $\rho = 0.928$

$$h^* = \rho \frac{\sigma_S}{\sigma_F} = 0.928 \frac{0.0263}{0.0313} \simeq 0.7777$$

Each heating oil contract traded on NYMEX is on 42,000 gallons of heating oil. The optimal number of contracts is then

$$N^* = \frac{h^* Q_A}{Q_F} = 0.7777 \frac{2\ 000\ 000}{42\ 000} \simeq 37.03$$

or, rounding to the nearest whole number, 37.

Cross Hedging
Tailing the Hedge

- When futures are used for hedging, a small adjustment, known as **tailing the hedge**, can be made to allow for the impact of daily settlement.
- In practice this means that the optimal number of contracts writes as

$$N^* = h^* \frac{V_A}{V_F}$$

where V_A is the dollar value of the position being hedged and V_F is the dollar value of one futures contract (the futures price times Q_F).

Cross Hedging
Tailing the Hedge

Example

Suppose that in the previous example the spot price is 1.94 *and the futures price is* 1.99 *(both dollars per gallon).*

We then have

$$\begin{aligned} V_A &= 1.94 \times 2\ 000\ 000 = 3\ 880\ 000 \\ V_F &= 1.99 \times 42\ 000 = 83\ 580 \end{aligned}$$

So the optimal number of contracts after tailing is

$$0.7777 \times \frac{3\ 880\ 000}{83\ 580} \simeq 36.103$$

Chapter 4: Hedging Strategies Using Futures
Outline

Hedging Using Index Futures

- A **stock index** tracks changes in the value of a hypothetical portfolio of stocks.
  - E.g.,
    - *Standard & Poor's 500* (S&P 500) *Index* is based on a portfolio of 500 different stocks (400 industrials, 40 utilities, 20 transportation companies, and 40 financial institutions).
    - *Nasdaq-100* is based on 100 stocks using the National Association of Securities Dealers Automatic Quotations Service.
  - As mentioned in Chapter 2, futures contracts on stock indices are settled in cash, not by delivery of the underlying asset.

Hedging Using Index Futures

- Stock index futures can be used to hedge a well-diversified equity portfolio.
- Define:
  - V_A: current value of the portfolio; and
  - V_F: current value of one futures contract (the futures price times the contract size).
- If the portfolio mirrors the index, the optimal hedge ratio, h^*, equals 1 and the number of futures contracts that should be shorted is

$$N^* = \frac{V_A}{V_F}.$$

Hedging Using Index Futures

Example

A portfolio worth \$5,050,000 mirrors the S&P 500.

The index futures price is 1,010 and each futures contract is on \$250 times the index.

In this case $V_A = 5\,050\,000$ *and* $V_F = 1\,010 \times 250 = 252\,500$.

So that $N^* = \frac{5050000}{252500} = 20$ *contracts should be shorted to hedge the portfolio.*

Hedging Using Index Futures

- When the portfolio does not exactly mirror the index, we can use a model known as the *capital asset pricing model*.
  - In this model, there is a parameter beta (β) which is the slope of the best-fit line obtained when excess return on the portfolio over the risk-free rate is regressed against the excess return of the index over the risk-free rate.
    - ⋆ when $\beta = 1$, the return on the portfolio tends to mirror the return on the index;
    - ⋆ when $\beta = 2$, the excess return on the portfolio tends to be twice as great as the excess return on the index;
    - ⋆ when $\beta = 0.5$, it tends to be half as great; and so on.

Hedging Using Index Futures

- A portfolio with a beta of 2 is twice as sensitive to movements in the index as a portfolio with a $\beta = 1$.
  - It is therefore necessary to use twice as many contracts to hedge the portfolio.
- Similarly, a portfolio with a beta of 0.5 is half as sensitive to market movements as a portfolio with a beta of 1 and we should use half as many contracts to hedge it.
- In general, the optimal number of contracts writes as

$$N^* = \beta \frac{V_A}{V_F}.$$

Hedging Using Index Futures

- Comparing this number, $N^* = \beta \frac{V_A}{V_F}$, with the optimal number of contracts we obtained in the previous section when tailing the hedge, $N^* = h^* \frac{V_A}{V_F}$, we see that $h^* = \beta$.
- This is not surprising.
  - The hedge ratio h^* is in fact the slope of the best-fit line when changes in the portfolio are regressed against changes in the futures price of the index.
  - Beta (β) is the slope of the best-fit line when the return from the portfolio is regressed against the return for the index.

Hedging Using Index Futures

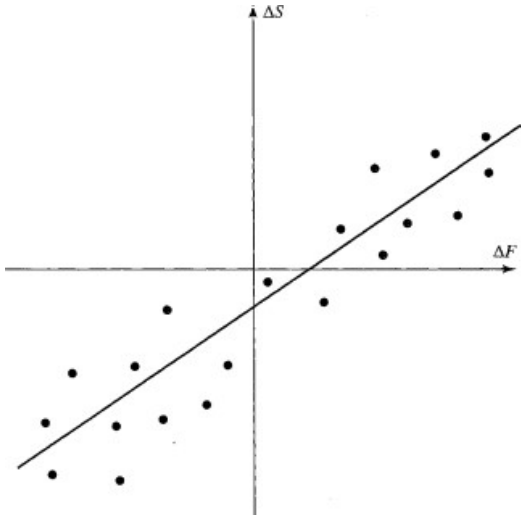

Figure: h^* : Regression of change in spot price against change in futures price.

Hedging Using Index Futures

Example

S&P 500 futures price =1,010 and each futures contract is on $250 times the index; Value of portfolio = $5,050,000; and Beta of portfolio = 1.5

Question

What position in futures contracts on the S&P 500 is necessary to hedge this portfolio?

Solution

Hedging Using Index Futures
Changing Beta

- Sometimes futures contracts are used to change the beta of a portfolio.
- To reduce the beta of the previous portfolio from 1.5 to 0.75, the number of contracts shorted should be 15 rather than 30;
  - to increase the beta of the portfolio to 2, a long position in 10 contracts should be taken; and so on.
- In general, to change the beta of the portfolio from β to β^*:
  - when $\beta > \beta^*$, a short position in

$$(\beta - \beta^*)\,\frac{V_A}{V_F}$$

    contracts is required.
  - when $\beta < \beta^*$, a long position in

$$(\beta^* - \beta)\,\frac{V_A}{V_F}$$

    contracts is required.

Exercise (4)

On July 1, an investor holds 50,000 shares of a certain stock. The market price is $30 per share. The investor is interested in hedging against movements in the market over the next month and decides to use the September Mini S&P500 futures contract. The index is currently 1,500 and one contract is for delivery of $50 times the index. The beta of the stock is 1.3.

What strategy should the investor follow?

Under what circumstances will it be profitable?

Solution (4)

Exercise (2)

A company has a $20 million portfolio with a beta of 1.2. It would like to use futures contracts on the S&P 500 to hedge its risk. The index futures is currently standing at 1080, and each contract is for delivery of $250 times the index.

a) What is the hedge that minimizes risk?

b) What should the company do if it wants to reduce the beta of the portfolio to 0.6?

Solution (2)

Exercise (2)

A company has a $20 million portfolio with a beta of 1.2. It would like to use futures contracts on the S&P 500 to hedge its risk. The index futures is currently standing at 1080, and each contract is for delivery of $250 times the index.

a) What is the hedge that minimizes risk?

b) What should the company do if it wants to reduce the beta of the portfolio to 0.6?

Solution (2)

Chapter 4: Hedging Strategies Using Futures
Outline

Stack and Roll

- Sometimes the expiration date of the hedge is later than the delivery dates of all the futures contracts that can be used.
- The hedger must then roll the hedge forward by closing out one futures contract and taking the same position in a futures contract with a later delivery date.
  - Hedges can be rolled forward many times.
  - The procedure is known as **stack and roll**.
- In practice, a company usually has an exposure every month to the underlying asset and uses a 1-month futures contract for hedging because it is the most liquid.
  - Initially it enters into (“stacks”) sufficient contracts to cover its exposure to the end of its hedging horizon.
  - One month later, it closes out all the contracts and “rolls” them into new 1-month contracts to cover its new exposure, and so on.

Chapter 4: Hedging Strategies Using Futures
Outline

Liquidity Issues

- In any hedging situation there is a danger that losses will be realized on the hedge while the gains on the underlying exposure are unrealized.
  - This is due to a mismatch between the timing of the cash flows on hedge and the timing of the cash flows from the position being hedged.
  - This might led to liquidity problems that could not be handled.
  - E.g., in the early 1990s, the German company *Metallgesellschaft* sold a huge volume of 5- to 10-year heating oil and gasoline fixed-price supply contracts to its customers at 6 to 8 cents above market prices. It hedged its exposure with long positions in short-dated futures contracts that were rolled forward. As it turned out, the price of oil fell and there were margin calls on the futures positions. Considerable short-term cash flow pressures were placed on MG. As a result, the company closed out all the hedge positions and incurred a loss of $1.33 billion.

Chapter 4: Hedging Strategies Using Futures
Outline

Summary

- This chapter has discussed various ways in which a company can take a position in futures contracts to offset an exposure to the price of an asset.
  - If the exposure is such that the company gains when the price of the asset increases and loses when the price of the asset decreases, a short hedge is appropriate.
  - If the company gains when the price of the asset decreases and loses when the price of the asset increases, a long hedge is appropriate.

Summary

- Hedging is a way of reducing risk. In reality, there are a number of theoretical and practical reasons why companies do not hedge.
  - On a theoretical level, we can argue that shareholders, by holding well-diversified portfolios, can eliminate many of the risks faced by a company.
    - ⋆ They do not require the company to hedge these risks.
  - On a practical level, a company may find that it is increasing rather than decreasing risk by hedging if none of its competitors does so.
    - ⋆ Also, a treasurer may fear criticism from other executives if the company makes a gain from movements in the price of the underlying asset and a loss on the hedge.

Summary

- An important concept in hedging is basis risk.
  - The basis is the difference between the spot price of an asset and its futures price.
  - Basis risk arises from uncertainty associated to the difference between both prices.
  - Basis risk is reinforced when:
    - The asset whose price is to be hedged is not exactly the same as the asset underlying the futures contract;
    - The hedger is uncertain as to the exact date when the asset will be bought or sold;
    - The hedge requires the futures contract to be closed out before its delivery month.

Summary

- The hedge ratio is the ratio of the size of the position taken in futures contracts to the size of the exposure.
  - It is not always optimal to use a hedge ratio of 1.
    - If the hedger wishes to minimize the variance of a position, a hedge ratio different from 1 may be appropriate.
    - The optimal hedge ratio is the slope of the best-fit line obtained when changes in the spot price are regressed against changes in the futures price.
- Stock index futures can be used to hedge the systematic risk in an equity portfolio.
  - The number of futures contracts required is the beta of the portfolio multiplied by the ratio of the value of the portfolio to the value of one futures contract.
  - Stock index futures can also be used to change the beta of a portfolio without changing the stocks that make up the portfolio.

Summary

- When there is no liquid futures contract that matures later than the expiration of the hedge, a strategy known as stack and roll may be appropriate.
  - This involves entering into a sequence of futures contracts.
    - When the first futures contract is near expiration, it is closed out and the hedger enters into a second contract with a later delivery month.
    - When the second contract is close to expiration, it is closed out and the hedger enters into a third contract with a later delivery month; and so on.
  - The result of all this is the creation of a long-dated futures contract by trading a series of short-dated contracts.