Derivative Instruments (Produits dérivés) - Solution to the Exam

Université Paris Dauphine-PSL - Master 1 I.E.F. (272)

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Answers can be formulated in English or French.

Solution 1 (1 pt) (a) The no-arbitrage forward price for this contract is

$$F_0 = (150 - 2.5 \times 1.036^{-\frac{9}{12}}) \times 1.036 \simeq 152.88.$$

(b) The value at $t = 0$ of the forward contract enterred a year ago is

$$\frac{F_0 - 151}{1 + r} = \frac{152.88 - 151}{1.03} \simeq 1.81.$$

Solution 2 (1 pt) The value of a contract is

$$(103 + \frac{22}{32}) \times 1,000 = 103,690$$

The number of contracts that should be shorted is

$$\frac{7,500,000}{103,690}\frac{7.4}{6.4} \simeq 83.63$$

Rounding to the nearest whole number, 84 contracts should be shorted.

Solution 3 (1 pt) According to the Bank for International Settlements :

(a) Notional amounts outstanding are larger than gross market values.

(b) The two most widely used underlying assets in terms of notional amounts outstanding in the past ten years are **interest rate** and **foreign exchange**.

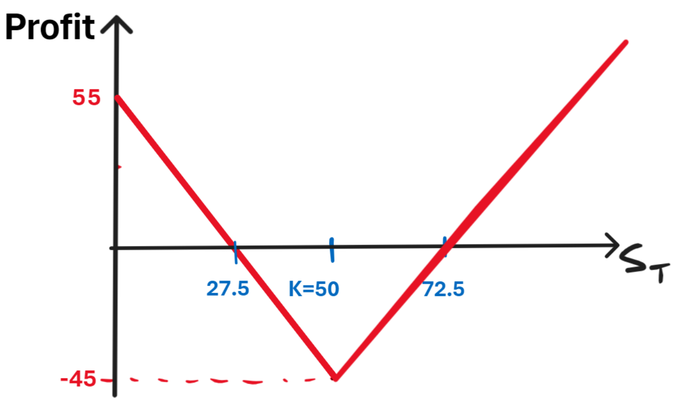
Solution 4 (4 pts) (a) The current price of the underlying asset such that there is no arbitrage opportunity is given by the Put-Call Parity. We must have

$$c - p = S_0 - K$$

that is

$$S_0 = c - p + K = 5 - 15 + 50 = 40$$

(b) The profit diagram of derivative Z as a function of S_T draws as



(c) Yes, there is an arbitrage opportunity. The derivative Z is the combination of two puts and two calls, that is, Z is twice a long straddle. Z costs 45, whereas the portfolio of two puts and two calls would cost $2 \times 5 + 2 \times 15 = 40$. An arbitrage opportunity is the following.

At date $t = 0$:

– Sell Z ($Z_0 = 45$€) ;

– Buy two European call ($-2c = -10$€) ;

– Buy two European put ($-2p = -30$€) ; and

– Invest the amount of $Z_0 - 2p - 2c = 5$€ on the money market so this strategy costs nothing out-of-pocket.

At date $t = T$:

– Exercise the two calls if $S_T > K$, otherwise exercise the two puts, and receive the absolute value of $2 \times (S_T - K)$€.

– Deliver the absolute value of $2 \times (S_T - K)$€ to the holder of derivative Z ; and

– Receive $Z_0 - 2p - 2c = 5$€ from the money market.

This leads to a positive profit of 5€ in all circumstances.

Solution 5 (4 pts) (a) From the Put-Call Parity, we must have

$$c - p = S_0 - Ke^{-rT}$$

So,

$$K = (S_0 + p - c)e^{rT} = (100 + 3 - 3)e^{2.44\% \times 2} \simeq \$105$$

(b) Yes, this price leads to an arbitrage opportunity. The American call should have the same market price than the European call (\$3 instead of \$3.5) because it is never optimal to exercise an American call on a non-dividend paying stock before its maturity.

(c) The arbitrage strategy consists in the following.

(c_1) At date $t = 0$:

– Sell the American call ($C = \$3.5$) ;

– Buy the stock ($-S_0 = -\$100$) ;

– Buy the European put ($-p = -\$3$) ; and

– Borrow the amount of $|C - S_0 - p| = |-\$99.5| = \$99.5$ from the money market so this strategy costs nothing out-of-pocket.

(c_2) In case of no early exercise of the American call, the profit at date $T = 2$ is

$$max\{K - S_T; 0\} - max\{S_T - K; 0\} + S_T + (C - S_0 - p)e^{rT} = K + (C - S_0 - p)e^{rT}$$
$$= 105 - 99.5e^{2.44\% \times 2}$$
$$\simeq 105 - 104.48 = 0.52 > 0$$

(c_3) In case of an (accidental) early exercise of the American call at date $t \in (0; T)$, we have the following.

At date t :

– Sell the stock at strike ($K = 105$) to the holder of the American call ; and

– Invest this amount on the money market.

The profit at date $T = 2$ is then

$$max\{K - S_T; 0\} + (C - S_0 - p)e^{rT} + Ke^{r(T-t)}$$

which is strictly positive from $(C - S_0 - p)e^{rT} > (c - S_0 - p)e^{rT} = -K$ and $e^{r(T-t)} \geq 1$. More precisely, we have

$$max\{105 - S_T; 0\} - 104.48 + 105e^{2.44\% \times (2-t)} > 0.$$

Solution 6 (1 pt) The bond price is obtained by discounting the cash flows at 7.5%. The price is

$$B = 3e^{-7.5\% \times \frac{6}{12}} + 3e^{-7.5\% \times \frac{12}{12}} + 103e^{-7.5\% \times \frac{18}{12}} \simeq 97.71$$

If the 18-month zero rate is r, we must have

$$B = 3e^{-7\% \times \frac{6}{12}} + 3e^{-7\% \times \frac{12}{12}} + 103e^{-r\% \times \frac{18}{12}}$$

which gives

$$r = -\frac{12}{18} ln(\frac{B - (3e^{7\% \times \frac{6}{12}} + 3e^{7\% \times \frac{12}{12}})}{103}) \simeq 7.52\%.$$

Solution 7 (2 pts) According to the swap, it is immediate that the market offers :

– A floating rate of $LIBOR - 0.1\%$ to company A ; and

– A fixed short-term rate of 2.1% to company B.

This swap leads to A borrowing at $(LIBOR - 0.1) - LIBOR + 1.925 = 1.825\%$ and to B borrowing at $2.1 - 1.825 + LIBOR = LIBOR + 0.275\%$.

The total gain to all parties from the swap is 0.25% per annum. The bank, acting as an intermediary, receives 0.1%. Therefore, each company must be $\frac{0.25-0.1}{2} = 0.075\%$ better off thanks to the swap. This means that the market offers :

– A fixed short-term rate of $1.825 + 0.075 = 1.9\%$ to company A ; and

– A floating rate of $LIBOR + 0.275 + 0.075 = LIBOR + 0.35\%$ to company B.

Overall, we then obtain the following table :

	Fixed short-term rate	*Floating rate*
Company A	1.9%	$LIBOR - 0.1\%$
Company B	2.1%	$LIBOR + 0.35\%$

Solution 8 (1 pt) (a) Currency swaps and foreign exchange swaps are different to one another in that a foreign exchange swap involves only 2 transactions (sell or purchase at the spot rate, and repurchase or resell at forward rate), whereas a currency swap exchanges a series of cash flows (interest payments and principles).

(b) According to the Bank for International Settlements, the most widely used is **foreign exchange swap**.

Solution 9 (1 pt) An investor can create a butterfly spread by buying both call options with strike prices of \$8 and \$19 and selling two call options with strike prices of \$13. The initial investment is $3+0.7-2\times 1.5 = 0.7$. If $S_T = 9.5 \in [8, 13]$ the profit of the corresponding butterfly spread is $(9.5-8) - 0.7 = \$0.8$.

Solution 10 (4 pts) (a) The market price of the bond is

$$B = 1.5(e^{-0.028\times 0.5} + e^{-0.028\times 1.0} + e^{-0.028\times 1.5}) + 101.5e^{-0.028\times 2.0} \simeq 100.35$$

(b) The equivalent risk-free bond's value is

$$B_{risk-neutral} = 1.5(e^{-0.02\times 0.5} + e^{-0.02\times 1.0} + e^{-0.02\times 1.5}) + 101.5e^{-0.02\times 2.0} \simeq 101.93$$

(c) The bond risk premium is

$$B - B_{risk-neutral} \simeq 101.93 - 100.35 = 1.58$$

(d) The loss in case of default at date t is $B^t_{risk-neutral} - rec^t$, for $t \in 1, 2$. With

$$B^1_{risk-neutral} = 1.5 + 1.5e^{-0.02\times 0.5} + 101.5e^{-0.02\times 1} \simeq 102.48$$

$$B^2_{risk-neutral} = 101.5$$

and $rec^t = 35$ for any $t \in 1, 2$.

So,

$$B^1_{risk-neutral} - rec^1 \simeq 102.48 - 35 = 67.48$$

and

$$B^2_{risk-neutral} - rec^2 = 101.5 - 35 = 66.5$$

(e) The time-invariant default probability $\bar{p}$ then satisfies

$$\begin{aligned}\bar{p} &= \frac{B - B_{risk-neutral}}{\sum_{t=1}^{2}(B^t_{risk-neutral} - rec^t)e^{-0.02\times t}} \\ &= \frac{1.58}{67.48e^{-0.02\times 1.0} + 66.5e^{-0.02\times 2.0}} \simeq 1.22\%.\end{aligned}$$