Derivative Instruments (Produits dérivés) - Exam

Université Paris Dauphine-PSL - Master 1 I.E.F. (272)

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Exercise 1 (1 pt) Consider a forward contract on one share of stock ABC with maturity in exactly one year from now. Today the spot price for one share of ABC is 150 € and the one-year risk-free interest rate is 3.6% (compounded annually). In 9 months from now, ABC will pay a dividend of 2.5€ per share.

(a) What is the no-arbitrage forward price for this contract ?

(b) Suppose that one year ago the forward price (which was then a two-year contract) was 151€. What is the value today of the forward contract enterred a year ago ?

Exercise 2 (1 pt) It is January 30. You are managing a bond portfolio worth \$7.5 million. The duration of the portfolio in six months will be 7.4 years. The September Treasury bond futures price is currently $103 - 22$, and the cheapest-to-deliver bond will have a duration of 6.4 years in September.

How many contracts should be shorted to hedge against changes in interest rates over the next six months ?

Exercise 3 (1 pt) Consider the global over-the-counter derivatives market.

(a) Which amounts are larger, notional amounts outstanding or gross market values ?

(b) What are the two most widely used underlying assets in terms of notional amounts outstanding in the past ten years ?

Exercise 4 (4 pts) The risk-free rate is 0%. Consider the following derivative contracts written on the same underlying asset. The underlying asset will have a price of S_T at the maturity date of the derivative contracts.

The first derivative contract is a European call option with an exercise price of $K = 50$€and maturity $T = 1$ year with a current price of $c = 5$€.

The second derivative contract is a European put with the same exercise price and maturity. Its current price is $p = 15$€.

(a) What is the current price of the underlying asset such that there is no arbitrage opportunity ?

The third derivative contract Z has a maturity of T = 1 year and will pay at maturity a cash flow equal to the absolute value of $2 \times (S_T - K)$€. Its current price is $Z_0 = 45$€.

(b) Draw the profit diagram of derivative Z as a function of S_T.

(c) Is there an arbitrage opportunity ? If yes, build an arbitrage strategy.

Exercise 5 (4 pts) Consider European call and put options on one share of a non-dividend paying stock, with the same maturity of 2 years and strike price of K. The risk-free interest rate is $r = 2.44\%$ per annum. Both, call and put options have the same price of $c = p = \$3$. The current stock price is \$100.

(a) What is the non-arbitrage strike ?

(b) Suppose an American call on the same stock, with same maturity and strike, is worth $C = \$3.5$. Does this price leads to an arbitrage opportunity ? Why ?

(c) If yes, propose an arbitrage strategy that involves the money market, one share of the stock, the American call and the European put.

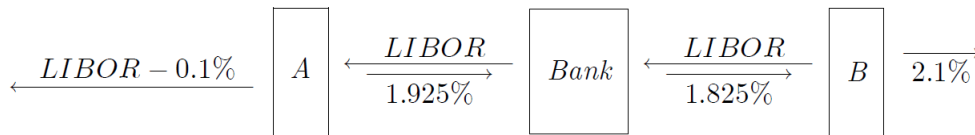   (c_1) Describe the arbitrage strategy at date $t = 0$.

     (c_1) What is the profit at date T in case of no early exercise of the American call?

     (c_3) What is the profit at date T in case of an (accidental) early exercise of the American call at date $t \in (0; T)$?

Exercise 6 (1 pt) The six-month and one-year zero rates are both 7% per annum (continuous compounding). For a bond that has a life of 18 months and pays a coupon of 6% per annum (with semiannual payments), the yield is 7.5% per annum (continuous compounding).

    What is the 18-month zero rate?

Exercise 7 (2 pts) We consider the following swap on a $20 million five-year loan, which is net a bank, acting as intermediary, 0.1% per annum and is equally attractive to both companies A and B. The total gain to all parties from the swap is 0.25% per annum.

$$\xleftarrow{LIBOR - 0.1\%} \boxed{A} \underset{\xrightarrow{1.925\%}}{\xleftarrow{LIBOR}} \boxed{Bank} \underset{\xrightarrow{1.825\%}}{\xleftarrow{LIBOR}} \boxed{B} \xrightarrow{2.1\%}$$

    What are the associated fixed short-term rate and floating rate per annum that the market offered (before the design of the swap) to companies A and B on a $20 million five-year loan?

Exercise 8 (1 pt) Consider foreign exchange swaps and currency swaps.

    (a) What is the difference between the two?

    (b) According to the Bank for International Settlements, which of the two is the most widely used?

Exercise 9 (1 pt) Call options on a stock are available with strike prices of $8, $13, and $19 and expiration dates in three months. Their prices are $3, $1.5, and $0.7, respectively.

    What is the profit of the corresponding butterfly spread if the price of the underlying asset is $9.5 at maturity?

Exercise 10 (4 pts) Suppose that the LIBOR/swap rate curve (risk free rate) is flat equal to 2%. A 2-year corporate bond provides a coupon of 3% per year payable semiannually and it has a yield of $2, 8\%$ (continuous compounding). Assume that defaults can take place at the end of the year (immediately before a coupon or principal payment) and that the recovery rate is 35%.

    (a) What is the market price of the bond?

    (b) What is the equivalent risk-free bond's value?

    (c) What is the bond risk premium?

    (d) What is the loss in case of default at each possible date of default?

    (e) Estimate the risk-neutral default probability $\bar{p}$ on the assumption that it is the same each year.