Game Theory (Microeconomics)

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Chap.1 Simultaneous games

Simultaneous games
Outline

1. Simultaneous games
2. Elimination of dominated strategies
3. Experimental evidence: Iterated strict dominance
4. Nash Equilibrium
5. More strategies
6. Multiple equilibria
7. Focal Point
8. Experimental evidence: Nash equilibrium
9. Mixed strategies
10. Empirical evidence: mixed strategies

Simultaneous game

A simultaneous game is defined by:

- A finite set of n players $N = \{1, 2, ..., n\}$
- Strategy sets $S_1, ..., S_n$
- Payoff functions $u_i : S_1 \times ... \times S_n \longmapsto \mathbb{R}$ for each $i \in N$.
- Notation:
  - A strategy profile $s = (s_1, ..., s_n)$ specifies a strategy for each player.
  - Denote $s_{-i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_n)$ the strategy of i's opponents

Beauty contest

- Players?
  - all students $N = \{1, ..., 79\}$
- Strategy sets?
  - $S_i = [0, 100]$ for all $i \in N$
- Payoff functions?
  - $u_i = 1$ if closest to half the average, 0 otherwise.

Grade game

- Without showing your neighbor what you are doing, write down on a form either the letter α or the letter β.
  - Think of this as a 'grade bid'.
  - We will randomly pair your form with one other form.
  - Neither you nor your pair will ever know with whom you were paired.
- Here is how grades may be assigned for this course:
  - if you put α and your pair puts β, then you will get grade A, and your pair grade C;
  - if both you and your pair put α, then you both will get grade B^-;
  - if you put β and your pair puts α, then you will get grade C, and your pair grade A;
  - if both you and your pair put β, then you will both get grade B^+.

Grade game

		Your pair	
		αlpha	β
You	αlpha	(B-, B-)	(A, C)
	β	(C, A)	(B+, B+)

- Vocabulary
  - The possible choices, α or β, are called '**strategies**'.
  - The grades - e.g., (A, C)-, are '**outcomes**'.
- Q.: What do you play?

Grade game

- Number of students having played α (resp. β):
- What strategy should a rational person choose in the Grade Game?
  - To answer this, we first need to know what that person cares about.
  - What 'payoff' does each outcome yield for this person?
- Game theory can not tell us what payoffs to assign to outcomes.
  - This depends on the preferences (and moral sentiments?) of the players, not just you but also your opponents.
- But game theory has a lot to say about how to play the game once payoffs are known.

Grade game
Possible payoffs: selfish players

- Assume every player is selfish, only caring about her own grade then (assuming she prefers A to B etc.) the payoffs associated to the outcome might be as follows:
  - $A = 3$ points; $B^+ = 1$ points; $B^- = 0$ points; and $C = -1$ points.
- So, the payoffs matrix writes as:

		Your pair	
		αlpha	β
You	αlpha	(0, 0)	(3, -1)
	β	(-1, 3)	(1, 1)

- Q.: What should you choose in this case?

Grade game
Possible payoffs: selfish players

		Your pair	
		alpha	β
You	alpha	(0, 0)	(3, -1)
	β	(-1, 3)	(1, 1)

- Q.: What should you choose in this case?
- A.: If your pair chooses α, then you choosing α yields a higher payoff than you choosing β.
  - If your pair chooses β, then again, you choosing α yields a higher payoff than you choosing β.
- So, you should always choose α because the payoff from α is strictly higher than that from β **regardless of others' choices**.

Definitions
Dominant and dominated strategies

Definition (Informal)

A strategy is **dominated** if, regardless of what any other players do, the strategy earns a player a smaller payoff than some other strategy.

Definition (Formal)

A strategy s_i is **(strictly) dominated** if there exits some $s_i' \in S_i$ such that for all $s_{-i} \in S_{-i}$ we have $u_i(s_i', s_{-i}) > u_i(s_i, s_{-i})$

Example

In the Grade game, β is a strictly dominated strategy

Lesson

As a "rational" player, you should never play a strictly dominated strategy.

Definitions
Dominant and dominated strategies

Definition (Informal)

A strategy is **dominant** if, regardless of what any other players do, the strategy earns a player a larger payoff than any other.

Definition (Formal)

A strategy s_i is **(strictly) dominant** if for every $s_i' \in S_i$ and all $s_{-i} \in S_{-i}$ we have $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.

Example

In the Grade game, α is a dominant strategy.

Lesson

As a "rational" player, you should always play a dominant strategy (if you have one to play).

Grade game
Possible payoffs: selfish players

		Your pair	
		alpha	β
You	alpha	(0, 0)	(3, -1)
	β	(-1, 3)	(1, 1)

- Unfortunately, the reasoning is the same for your pair:
  - given these payoffs, she will also choose α.
- You will end up both getting B^- even though there is a possible outcome (B^+, B^+) that is better for both of you.
  - To use some economics jargon: the outcome (B^-, B^-) is Pareto inefficient.

Grade game
Possible payoffs: selfish players

		Your pair	
		αlpha	β
You	αlpha	(0, 0)	(3, -1)
	β	(-1, 3)	(1, 1)

- Games like this one are called *Prisoners' Dilemmas*.
  - The *jointly* preferred outcome (B^+, B^+) arises when each chooses its *individually* worse strategy (i.e., β).

Lesson
Rational play by rational players can lead to bad outcomes.

Grade game
Possible payoffs: indignant altruistic players

- Suppose that each person cares not only about her own grade but also about the grade of the person with whom she is paired.
  - For example, each player likes getting an A but she feels guilty that this is at the expense of her pair getting a C.
    - The guilt lowers her payoff from 3 to -1.
    - Conversely, if she gets a C because her pair gets an A, indignation reduces the payoff from -1 to -3.

		Your pair	
		αlpha	β
You	αlpha	(0, 0)	(-1, -3)
	β	(-3, -1)	(1, 1)

- Q.: What should you choose in this case?

Grade game
Possible payoffs: indignant altruistic players

		Your pair	
		αlpha	β
You	αlpha	(0, 0)	(-1, -3)
	β	(-3, -1)	(1, 1)

- Q.: What should you choose in this case?
- A.: If your pair chooses α, then you choosing α yields a higher payoff than you choosing β.
  - If your pair chooses β, however, then you choosing β yields a higher payoff than you choosing α.
  - In this case, no strategy is dominated.
  - The best choice depends on what you think your pair is likely to do.
  - Later in the course, we will examine games like this called 'co-ordination games'.

Grade game
Possible payoffs: indignant altruistic players

Lesson
To figure out what actions you should choose in a game, a good first step is to figure out what are your payoffs (what do you care about) and what are other players' payoffs.

Grade game
Possible payoffs: selfish player vs indignant altruistic player

- Suppose you are a selfish player playing with an indignant altruistic player.

		Your pair	
		αlpha	β
You	αlpha	(0, 0)	(3, -3)
	β	(-1, -1)	(1, 1)

- Q.: What should you choose in this case?
- A.: Your strategy α strictly dominates your strategy β.

Grade game
Possible payoffs: indignant altruistic player vs selfish player

- Suppose you are an indignant altruistic player playing with a selfish player.

		Your pair	
		αlpha	β
You	αlpha	(0, 0)	(-1, -1)
	β	(-3, 3)	(1, 1)

- Q.: What should you choose in this case?
- A.: Neither of your strategies dominates the other.
  - But, your pair's strategy α strictly dominates her strategy β.
  - Therefore, if you know she is rational then you know she'll play α.
  - In which case, you should play α.

Grade game
Possible payoffs: indignant altruistic player vs selfish player

Lesson

If you do not have a dominated strategy, put yourself in your opponents' shoes to try to predict what they will do.

Example

In their shoes, you would not choose a dominated strategy.

Grade game
Conclusion

- What do real people do in Prisoners' Dilemmas?
  - Only about % of the class chose β in the grade game.
  - In larger experiments with 'normal people', about 30% chose (the analogue of) β.
  - Does this mean that Dauphine students are smarter than normal folk?
  - Not necessarily. It could just be that Dauphine students are selfish.

Prisoner's dilemma

- Conductor of orchestra under Stalin era.
- « Your friend Tchaikovsky has already confessed! »
- Choose between: to stay silent/to denounce.
- In all case:
  - to be denounced increases the sentence; and
  - to denounce decreases it.

Prisoner's dilemma

- 4 possible outcomes (conductor's years in jail):
  1. He denounces and he is not denounced: 1 year;
  2. He stays silent and he is not denounced: 3 years;
  3. He denounces and he is denounced: 10 years;
  4. He denounces but he is denounced: 25 years.

Prisoner's dilemma

- Strategy sets $S_1 = S_2 = \{\text{Denounce, Stay silent}\}$
- Payoffs, for $i = 1, 2$:
  - $u_i(Denounce, Denounce) = -10$;
  - $u_i(Stay\ silent, Stay\ silent) = -3$;
  - $u_1(Stay\ silent, Denounce) = u_2(Denounce, Stay\ silent) = -25$; and
  - $u_1(Denounce, Stay\ silent) = u_2(Stay\ silent, Denounce) = -1$.

Prisoner's dilemma

Represent the game in table:

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

Prisoner's dilemma

- The same holds for Tchaikovsky namesake.
- Individually rational strategy :
  - To denounce
- Conductor's best reply:
  - To denounce

Prisoner's dilemma

- Later, when they meet in the Gulag, they compare stories and realize that they have been had.
- If only they had the opportunity to meet and talk things over before they were interrogated, they could have agreed that neither would give in.
- However, once separated, each one get a better deal by double-crossing the other.
- Problem: As in the Grade game, the *jointly* preferred outcome arises when each chooses its *individually* worse strategy.

Prisoner's dilemma
Examples of prisoner's dilemma: Nuclear race

- Each superpower prefers the outcome where others, are disarmed while he is keeping his arsenal "just in case".
- To be disarmed while others keep their arsenal is inconceivable.
- Hence everyone prefers to keep his arsenal.
- One solution consists in:
  - committing to start on the road of nuclear disarmament...
  - ... then secretly breaking the pact.
- We shall study how to solve such avenues.

Prisoner's dilemma
Another example of prisoner's dilemma: Apps for Ipads

- The Web is a non-commercial entity that enables information spread and commerce as nothing that has come before.
- His success relies on two salient characteristics.
- **Universality**.
  - The web enables information to be accessed on any device, no matter who built it, what software it runs or who created the content.
  - If it is converted to HTML, we all can see it (and even save or print it).
- **Connectivity.**
  - Once a page is on the Web, it is theoretically connected to every other page.
  - It becomes part of the whole system.
  - Furthermore, linking allows us to vote for what we think is important. Links, after all, form the basis of how search engines like Google help us find what we're looking for.

Prisoner's dilemma
Another example of prisoner's dilemma: Apps for Ipads

- Today, universality and connectivity of the Web are threatened by *closed Internet applications* or "apps" that are designed to be proprietary, like those on devices such as *iPads* or *iPhones* and, to a lesser extent, on web sites like *Facebook*.
- This situation is like a prisoner's dilemma.
- There is a clear benefit to universality and connectivity.
- However, individual corporations stand to benefit if they can rig the game towards proprietary solutions (i.e. screw their buddy).
- If that happens, it will hurt consumers and threatens free enterprise and innovation.

Equilibrium and efficiency

- Prisoner's dilemma represents the classic conflict between:
  - individual incentives of players
  - joint payoff maximization
- In the equilibrium of a game, the total payoff is typically not maximized.
- In this sense, equilibria are typically "inefficient for players"
- Examples: pricing by firms, international negotiations, arms races

Climate Change: Stern Report (2006)

- Estimates from Stern 2006 report:
  - 4 degrees increase, the damage would be around 3% of GDP
  - 8 degrees increase, damage estimated between 11 to 20 %
- Estimates of costs: 1 to 2 % of GDP to limit the rise to $2-3$ degrees.

Climate Change: Stern Report (2006)

To go to the representation of the game need to make some assumptions:

- if both countries pay 2 % of GDP, no damage on climate
- if only one does, damage is 1.5 % of GDP
- if none pay, damage is 3 %

Climate Negotiations

		EU: Cooperate	EU: Not coop
US	Coop	-2, -2	-3.5, -1.5
US	Not coop	-1.5, -3.5	-3, -3

- Is there any strictly dominated strategy?
- Yes! “Cooperate” is strictly dominated by “Not cooperate”.
  - Here, “Not cooperate” is a strictly dominant strategy.

Climate Negotiations: More Dramatic Effects

In 2013, Stern declared to *The Guardian*: “I got it wrong on climate change – it’s far, far worse”

- if both countries pay 2 % of GDP, no damage on climate
- if only one does, damage is 6 % of GDP
- if none pay, damage is 12 %

Climate Negotiations: More Dramatic Effects

		EU: Coop	EU: Not coop
US	Coop	-2,-2	-8,-6
US	Not coop	-6,-8	-12,-12

- Is there any strictly dominated strategy?
- Yes! “Not cooperate” is strictly dominated by “Cooperate”
  - Here, “Cooperate” is a strictly dominant strategy.

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Iterated strict dominance

- It may happen that there is no dominant strategy but stil there are dominated strategies.
- Consider the following game:

	L	M	R
U	$(2,2)$	$(1,1)$	$(4,0)$
D	$(1,2)$	$(4,1)$	$(3,5)$

- Is there any dominant strategy?
  - No.
- Is there any strictly dominated strategy?
  - Yes: M.

Iterated strict dominance

Iterated strict dominance:

1. Column M dominated by column L: eliminate M
2. Once M eliminated, row D dominated by row U: eliminate D
3. Once M and D eliminated, column R dominated

- Iterated strict dominance leads to outcome (U,L)

Iterated strict dominance

- Iterated strict dominance applied to the beauty contest.
- What is the unique equilibrium?
- Prediction correct? Even if repeated?

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Nagel (AER, 1995): Testing the Beauty Contest

- Groups of 15 -18 subjects each
- The same group played for four periods
- After each round the response cards were collected
- All chosen numbers, the mean, and half the mean were announced
- The prize to the winner of each round was 20 DM (about $13)
- After four rounds, each player received the sum of his gains of each period

Nagel (AER, 1995): Testing the Beauty Contest
First-Period Choices

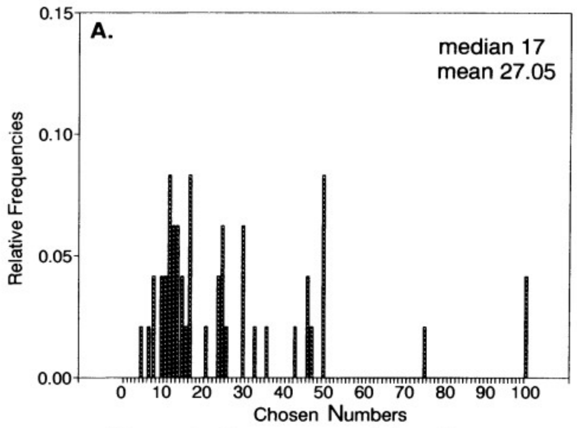
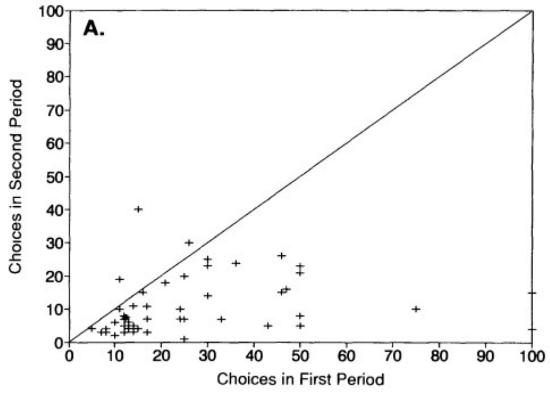


FIGURE 1. CHOICES IN THE FIRST PERIOD

- 6 % of the subjects chose numbers greater than 50
- and 8 % chose 50.

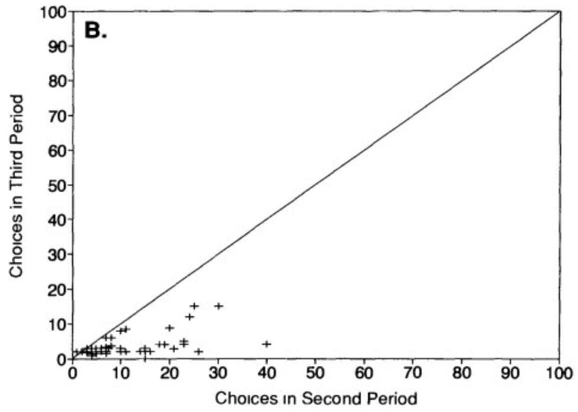
Nagel (AER, 1995): Testing the Beauty Contest
Choices from periods 1 to 2



A) TRANSITION FROM FIRST TO SECOND PERIOD

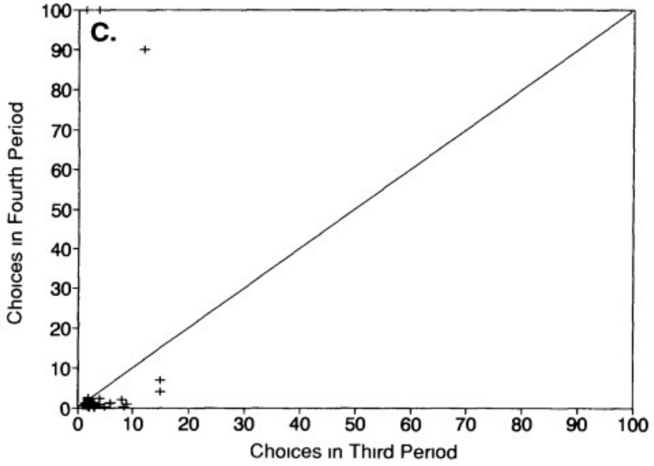
- A plot under the bisecting line indicates that the subject chose a lower number in period 2 than in period 1

Nagel (AER, 1995): Testing the Beauty Contest
Choices from periods 2 to 3



B) TRANSITION FROM SECOND TO THIRD PERIOD

Nagel (AER, 1995): Testing the Beauty Contest
Choices from periods 3 to 4



C) Transition from Third to Fourth Period

Nagel (AER, 1995): Testing the Beauty Contest
Conclusion

- The process is driven by iterative, naive best replies rather than by an elimination of dominated strategies.
- The process of iteration is finite and not infinite.
- There is a moving target, which approaches zero.
- Over time the chosen numbers approach the *equilibrium* or converge to it.
- (Many) people don't play *equilibrium* because they are confused.
- (Many) people don't play *equilibrium* because doing so (here, choosing 0) doesn't win;
  - rather they are cleverly anticipating the behavior of others, with noise.

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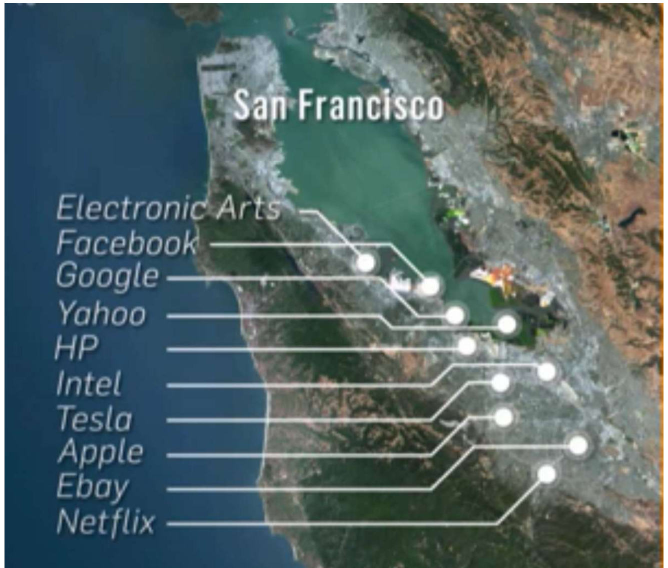
Nash equilibrium
Location game

- You and a competitor are to set up an ice cream parlour on the beach
- Once built, the location of the parlour is fixed for the season
- People are evenly distributed over the one kilometer long beach, and buy from the nearest vendor
- Ice creams are sold at fixed price
- You decide simultaneously on your location

Nash equilibrium
Location game

- Do you have a dominant strategy?
- Where do you go?

Nash equilibrium
Location game



Nash equilibrium
Location game

- To solve this game you need some belief about what the other player will do
- What you do depends on what you think he will do
- What you do depends on what you think he thinks you will do
- What you do depends on what you think he thinks you think he will do
- ...
- Need equilibrium concept to solve these iterations

Nash equilibrium
Definition

Definition (Informal)

A **Nash equilibrium** is an outcome where given what the other is doing, neither wants to change his own move.

Said differently, a Nash equilibrium is a strategy profile where :

- there is no unilateral profitable deviation; or

- each player's action is the best response to that of the other.

Nash equilibrium
Definition

Definition (Formal)

A strategy profile $(s_1^*, s_2^*, .., s_n^*)$ is a **Nash equilibrium** if for every i and every $s_i^{'} \in S_i$ we have $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i^{\prime}, s_{-i}^*)$.

- Think of two players. Denote the Nash equilibrium $\{s_1^*, s_2^*\}$. Nash equilibrium means:
  - If player 1 plays s_1^*, best player 2 can do is play s_2^*
  - If player 2 plays s_2^*, best player 1 can do is play s_1^*

Nash equilibrium
Best responses

- The *best response* to other player's strategy is the strategy for you that maximizes your payoff given what the others play
- The best response to a strategy s_{-i} by the opponents is the set of strategies that maximize your payoffs given that the others plays s_{-i} (maximizes $u_i(s_i^{\prime}, s_{-i})$)
- Nash equilibrium as we defined it is a fixed point of best responses

Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

BR1(P2 plays « Denounce »)=

Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

BR1(P2 plays « Denounce »)={Denounce};

Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

BR1(P2 plays « Denounce »)={Denounce};
BR1(P2 plays « Stays Silent »)=

Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

BR1(P2 plays « Denounce »)={Denounce};
BR1(P2 plays « Stays Silent »)={Denounce};

Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

BR1(P2 plays « Denounce »)={Denounce};
BR1(P2 plays « Stays Silent »)={Denounce};

Similarly:
BR2(P1 plays « Denounce »)={Denounce};
BR2(P1 plays « Stays Silent »)={Denounce}.

Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

The unique Nash equilibrium is:

Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10) N	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

The unique Nash equilibrium is: {Denounce, Denounce}.

Nash equilibrium
Back to the beach location game

- Is (0,1) a Nash equilibrium (i.e., both position themselves at the extremes of the beach)?
- Is (1/4,3/4) a Nash equilibrium?
- ...

Nash equilibrium
Fashion pricing

- You are working for Armani
- Main competitor is Ralph Lauren, with shop next door
- It is the end of the season, so unsold clothes are worthless
- Should you have sale or keep prices at normal high level?
- RL has similar dilemma..
- If only one shop has sale, that shop attracts some of the other shop's customers and possibly some new customers
- You and RL make independent and simultaneous decisions

Nash equilibrium
Fashion pricing

		RL: Sale	RL: No sale
Armani	Sale	40 , 40	50 , 30
	No sale	30 , 70	60 , 60

Nash equilibrium
Fashion pricing

		RL: Sale	RL: No sale
Armani	Sale	40* , 40*	50 , 30
	No sale	30 , 70*	60* , 60

- $BR^{Armani}(Sale) = \{Sale\}$
- $BR^{RL}(Sale) = \{Sale\}$
- There is a unique Nash equilibrium: $\{Sale, Sale\}$

Nash equilibrium
Dominant strategy

- Fashion pricing game is also solvable by iterated deletion of dominated strategy.

Property

If a game is solvable by iterated deletion of dominated strategies, then the solution is a Nash equilibrium.

Property

If all players have a dominant strategy, then the only Nash Equilibrium is one where all players play their dominant strategy.

Nash equilibrium
Interpretation

- Some justifications of the concept:
  - Introspection: correct conjectures about opponent's play
  - Self enforcing agreement: if players communicate and agree initially they will not deviate
  - Result of learning: situation that arises repeatedly

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More strategies

- Up till now only games with 2 players and 2 choices
- We examine now:
  - Games with more choices (but finite number)
  - Games with continuous strategy space

More strategies: 3x3

		COLUMN		
		Left	Middle	Right
ROW	Up	0, 1	9, 0	2, 3
	Straight	5, 9	7, 3	1, 7
	Down	7, 5	10, 10	3, 5

- Iterated strict dominance:

1. “Down” is a strictly dominant strategy: eliminate “Up” and “Straight”
2. Once “Up” and “Straight” are eliminated, “Middle” is a dominant strategy: eliminate “Left” and “Right”.

- Iterated strict dominance leads to outcome (Down,Middle)

More strategies: 4x3

		COLUMN		
		West	Center	East
ROW	North	2, 3	8, 2	7, 4
	Up	3, 0	4, 5	6, 4
	Down	10, 4	6, 1	3, 9
	South	4, 5	2, 3	5, 2

- No strategy can be eliminated (as long as we restrict to pure dominance).
- We shall see later on how to solve this game (use of mixed strategies).

More strategies: infinite number
Competition in an oligopoly

- Two firms i and j compete in quantity they produce (called Cournot competition).
- We consider here a situation where they make their choice simultaneously
- Strategy of player i is quantity q_i
- Given the choice of quantities produced (q_i, q_j), there is a resulting price that emerges in the market: what we call a demand function
- In this case we consider a very simple demand function: price on the market is given by $P = 1 - q_i - q_j$

More strategies: infinite number
Competition in an oligopoly: Objective of firms

- Each unit of good is of course costly to produce
- Here we assume that each unit costs c to produce so that the total cost of production for firm i is given by $C(q_i) = cq_i$
- If player i knows what player j does, choice is easy, it just maximizes profits, i.e. price$\times$quantity - cost:
  - In other words, firm i, if firm j produces q_j, chooses q_i to maximize

$$Pq_i - C(q_i) = (1 - q_i - q_j)q_i - cq_i$$

More strategies: infinite number
Competition in an oligopoly: Nash equilibrium as solution

- In practice you don't know for sure what the other one will do
- Depends on belief of what the others will do
- No obvious choice: i.e no dominant strategy
- So we look for the Nash Equilibrium

More strategies: infinite number
Competition in an oligopoly: Best responses

- To determine the Nash equilibrium, consider firm i. It takes the quantity of firm j as given and maximizes her own profits by choosing optimally q_i.
- Problem facing player i, given that opponent produces q_j is to maximize

$$\Pi(q_i) = q_i[1 - (q_i + q_j) - c] = -q_i^2 + q_i(1 - q_j - c)$$

- Reminder: to find a maximum, equalize the derivative to zero

$$\begin{aligned} \Pi'(q_i) &= 0 \\ -2q_i + (1 - q_j - c) &= 0 \end{aligned}$$

- So best response is

$$BR_i(q_j) = \frac{1-c}{2} - \frac{q_j}{2}$$

More strategies: infinite number
Competition in an oligopoly: Nash equilibrium

- A Nash equilibrium is a pair (q_i, q_j) such that q_i is a best response to q_j while q_j is itself a best response to q_i.
  - $q_i = BR(q_j) = \frac{1-c}{2} - \frac{q_j}{2}$ and $q_j = BR(q_i) = \frac{1-c}{2} - \frac{q_i}{2}$
  - Replace and get:
  $$q_i = \frac{1-c}{2} - \frac{1}{2}\left[\frac{1-c}{2} - \frac{q_i}{2}\right]$$
  - check yourself that the unique solution is
  $$q_i = q_j = (1-c)/3$$

Solution

The unique Nash equilibrium is for each firm to choose quantity $q = \frac{(1-c)}{3}$.

Simultaneous games
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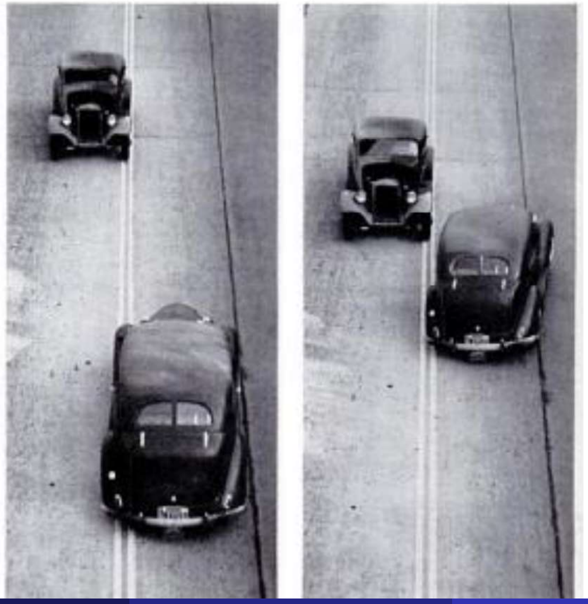
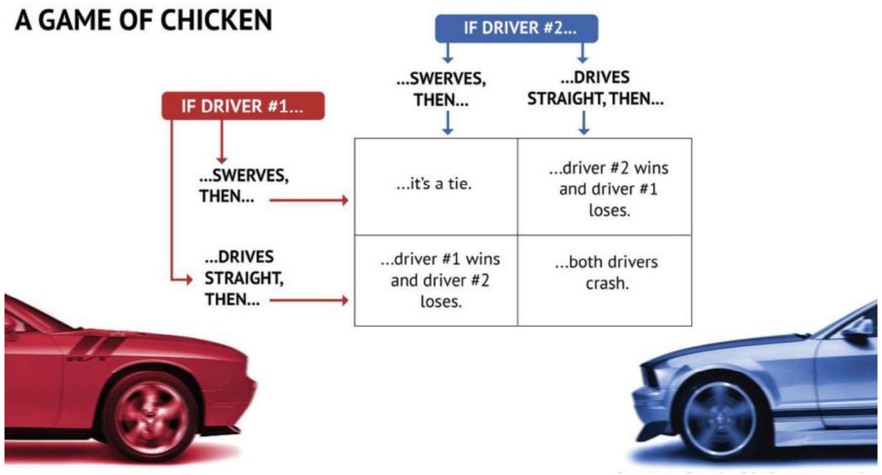
Multiple equilibria



Multiple equilibria



Multiple equilibria

There may be multiple Nash equilibria.

Example, the game of chicken (aka hawk-dove) .

		Player 2	
		Straight	Swerve
Player 1	Straight	(Crash, Crash)	(Win, Lose)
	Swerve	(Lose, Win)	(Tie, Tie)

Multiple equilibria

There may be multiple Nash equilibria.

Example, the game of chicken (aka hawk-dove) .

		Player 2	
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BR1(P2 plays « Straight »)=

Multiple equilibria

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Multiple equilibria

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Example, the game of chicken (aka hawk-dove) .

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	Swerve	(Lose, Win)	(Tie, Tie)

BR1(P2 plays « Straight »)={Swerve};
BR1(P2 plays « Swerve »)=

Multiple equilibria

There may be multiple Nash equilibria.

Example, the game of chicken (aka hawk-dove) .

		Player 2	
		Straight	Swerve
Player 1	Straight	(Crash, Crash)	(Win, Lose)
	Swerve	(Lose, Win)	(Tie, Tie)

BR1(P2 plays « Straight »)={Swerve};
BR1(P2 plays « Swerve »)={Straight};

Multiple equilibria

There may be multiple Nash equilibria.

Example, the game of chicken (aka hawk-dove) .

		Player 2	
		Straight	Swerve
Player 1	Straight	(Crash, Crash)	(Win, Lose)
	Swerve	(Lose, Win)	(Tie, Tie)

BR1(P2 plays « Straight »)={Swerve};
BR1(P2 plays « Swerve »)={Straight};
Nash equilibria:

Multiple equilibria

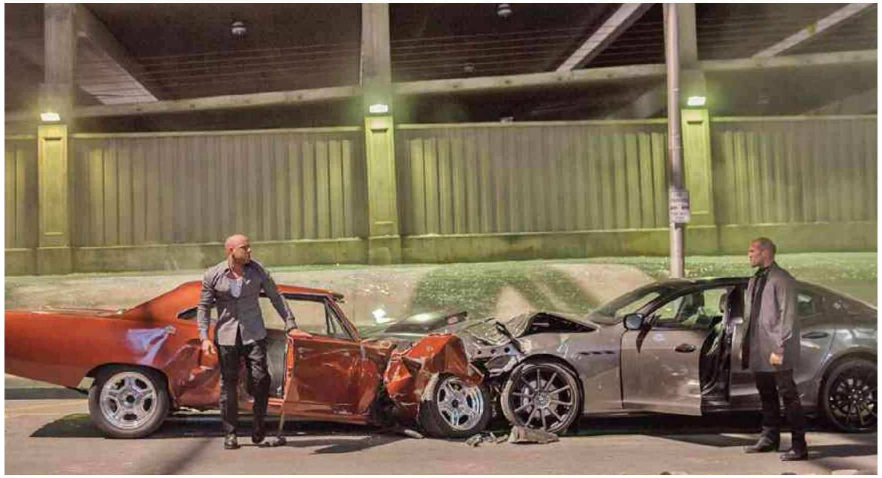
There may be multiple Nash equilibria.

Example, the game of chicken (aka hawk-dove) .

		Player 2	
		Straight	Swerve
Player 1	Straight	(Crash, Crash)	(Win, Lose) (N)
	Swerve	(Lose, Win) (N)	(Tie, Tie)

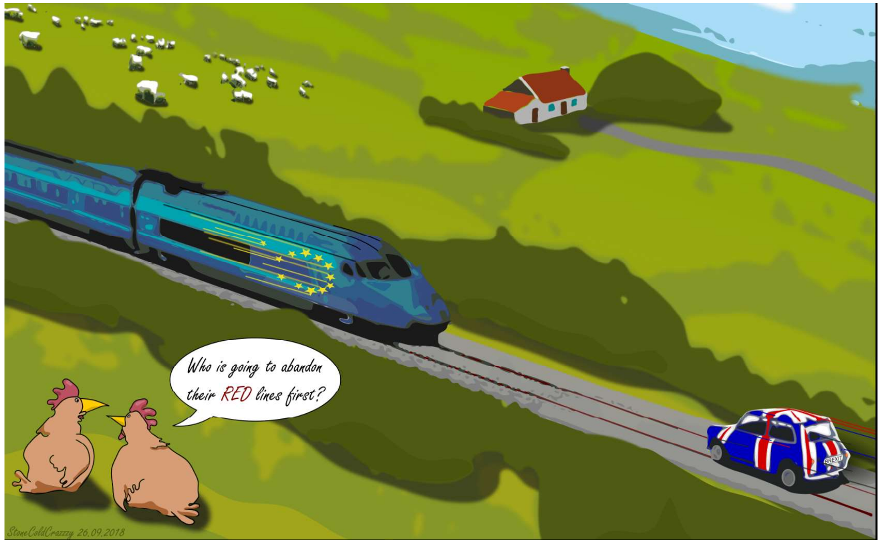
BR1(P2 plays « Straight »)={Swerve};
BR1(P2 plays « Swerve »)={Straight};
Nash equilibria: {(Straight, Swerve); (Swerve, Straight)}

Multiple equilibria



Multiple equilibria

Multiple equilibria



Multiple equilibria
Prisoner's dilemma: playing with your cousin

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-6,-25)
	Stays Silent	(-25,-6)	(-3,-3)

- What is the set of Nash equilibrium?

Simultaneous games
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7. **Focal Point**
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Focal Point

- How to select one equilibrium from multiple equilibria?
- Usually, the selection proceeds from social norms.
- On which side of the road to drive?
  - Dominant strategy: on the side used by other drivers.
- Which side to choose? No side is better than other.
  - UK, Australia, Japan: left-side.

Focal Point

- Choosing a date.
- Choosing a place to meet next week in Paris.
- The weather can modify the rdv location.
- Sunspot can make people moving from one equilibrium to another.
- Extrinsic fluctuations can cause financial crises.

Focal Point
Bank run from depositors

Bank run is a move from one equilibrium to another.

Self-fulfilling prophecy.

Northern Rock bank run on September 2007.
People queuing outside a branch in London to withdraw their savings due to fallout from the subprime crisis.

Focal Point
Bank run from depositors

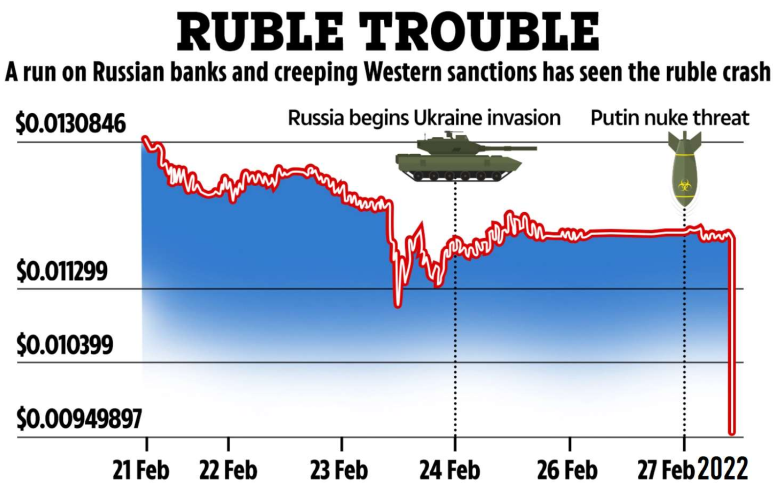
On March 21, 2013, people queue at an ATM outside a closed Laiki Bank branch in capital Nicosia, Cyprus.

"There are rumours that Laiki Bank (the Greek name for the Popular Bank) will never open again. I want to take out as much as I can," said a depositor.
"It's all about cash now. Only a gambler will take cheques in this situation," said a depositor.

Focal Point
Bank run from depositors

Long lines at Russia's ATMs as citizens rush to withdraw cash amid escalating EU sanctions on 27 Feb 2022

Focal Point
Bank run from depositors



Focal Point
Bank run from depositors

Des policiers gardent l'entrée d'une agence de la banque russe Sberbank devant laquelle des clients font la queue pour retirer leurs avoirs, le 25 février 2022 à Prague (Michal Cizek / AFP)

Focal Point
Bank run from depositors

Mar 2023

People wait outside the Silicon Valley Bank headquarters in Santa Clara, California, to withdraw funds after the federal government intervened upon the collapse of the bank. Photograph: Brittany Hosea

Focal Point
Bank run from bondholder

Bank run from bondholders

On March 2008, a bank run began on the securities and banking firm *Bear Stearns*. The non deposit-taking bank had financed huge long-term investments by selling short-maturity bonds, making it vulnerable to panic on the part of its bondholders.

Credit officers of rival firms began to say that *Bear Stearns* would not be able to make good on its obligations. Within two days, Bear Stearns's capital base of $17 billion had dwindled to $2 billion in cash. By the next morning, the *Fed* decided to lend *Bear Stearns* money (the first time since the Great Depression that it had lent to a nonbank).

Stocks sank, and that day *JPMorgan Chase* began to buy *Bear Stearns* as part of a government-sponsored bailout.

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Experimental evidence: Nash equilibrium
Ensminger (Oxford University Press, 2004): Public good game

- Ensminger (Oxford University Press, 2004): Testing Nash equilibrium in *Public good game*
- N players are grouped and each given an amount X (10 for example).
- Each player simultaneously decides how much to give, between 0 and X, to the *public good.*
- All contributions are made privately in an envelope so that no one but the experimenter knows the amount of each contribution.
- The total amount collected is then doubled by the experimenter and this amount is redistributed equally among everyone.
- For example: $N = 4$. If you have 10, you give 4 and the others give 20 in total. Total is 24, and each gets 0.5 of that. So you will get $10 - 4 + 0.5 * 24 = 18$.

Experimental evidence: Nash equilibrium
Ensminger (Oxford University Press, 2004): Public good game

- What is a public good?
- A **public good** is a good that is both *non-excludable* and *non-rivalrous*
  - *non-excludable*: non-paying consumers cannot be prevented from accessing it
    - E.g., fish stocks, forest, fresh air, national defense, street lighting ...
  - *non-rivalrous*: one person's consumption of the good does not affect another
    - E.g., cinemas, parks, satellite television, fresh air, national defense
- Need for public provision because these goods will tend to be privately under provided

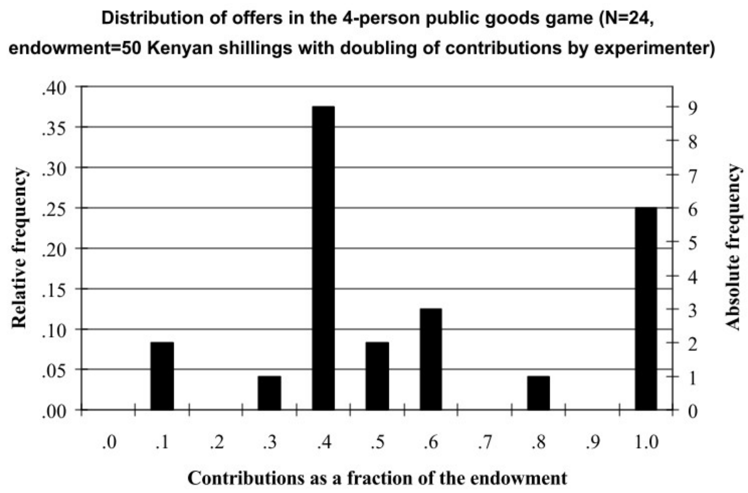
Experimental evidence: Nash equilibrium
Ensminger (Oxford University Press, 2004): Public good game

	Excludable	Non-excludable
Rivalrous	**Private goods** food, clothing, cars, personal electronics	**Common goods (Common-pool resources)** fish stocks, timber, coal
Non-rivalrous	**Club goods** cinemas, private parks, satellite television	**Public goods** free-to-air television, air, national defense

Experimental evidence: Nash equilibrium
Ensminger (Oxford University Press, 2004): Public good game

- Ensminger (2004): experiment in small society in Kenya
- Players grouped by 4 (anonymously) and given 50 shillings
- Can choose to keep the amount or contribute part or the whole of it to a public good
- Amount contributed doubled by experimenter and divided among the 4 players: so got back 50 percent of the total

Experimental evidence: Nash equilibrium
Ensminger (Oxford University Press, 2004): Public good game



- Results: on average contributions were 60 percent of endowments

Experimental evidence: Nash equilibrium
Ensminger (Oxford University Press, 2004): Public good game

- Players give more than in the NE (where contributions should be zero)
- Example player who gives 20 out of his 50 in a group where other three give 75 total, gets a payoff of:

$$50 - 20 + 0.5 * 95 = 77.5$$

- If the same player had given 0, he would get

$$50 + 0.5 * 75 = 87.5$$

- What explains this?

Experimental evidence: Nash equilibrium
Ensminger (Oxford University Press, 2004): Public good game

- How do we interpret these deviations?
  1. Players are not rational and cannot compute what is best for them
  2. Players do not adopt a pure selfish stance:
     - ⋆ altruism
     - ⋆ aversion for inequality
  3. Social norms

Experimental evidence: Nash equilibrium
Ensminger (Oxford University Press, 2004): Public good game

- **Altruism**: care not only about your own payoff but also payoff of others
- Can still apply tools of game theory but with different payoffs
- Specifically, suppose two players, you contribute X and other contributes Y
- Own payoff of player i is $P_i = 100 - X + 0.8 * (Y + X)$
- Other's payoff $P_{-i} = 100 - Y + 0.8 * (Y + X)$
- What player i is really maximizing if he is altruistic is $P_i + \alpha P_{-i}$
- Can solve for Nash equilibrium

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Mixed strategies

Inland Revenue

		Audit	Not audit
Tax Payer	Declare all income	3 , 1	3 , 2
	Lie on Income	0 , 4	5 , 0

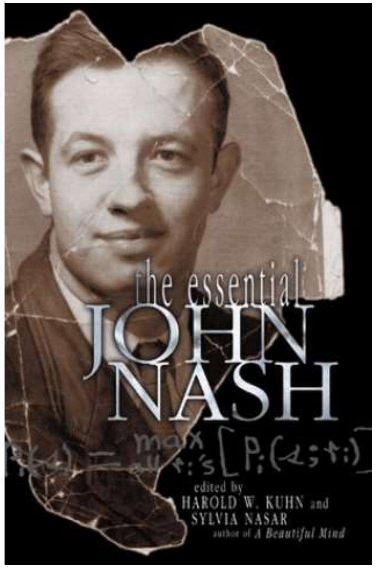
Figure: Tax game

- Does this game have any (pure) strategy equilibrium?

Mixed strategies

<u>Theorem (Nash, 1950):</u>

Every finite game has a strategy equilibrium.



Mixed strategies

- In previous classes, all the games we saw had a Nash equilibrium in what is called “pure strategy": i.e where all players play one action for sure
- In this game, if you know what the other player is going to choose, the strategy that makes you better off makes him worse off
- No equilibrium in pure strategies
- Other example: penalty kicks (most sports in fact)
- Intuitively the only outcome is an outcome where the other player does not know for sure what you are going to play: players randomize

Mixed strategies

- In the tax game no Nash Equilibrium where the player plays an action for sure: what is called pure strategy
- Exists other types of strategies, where the players randomize over actions: called *mixed strategy*

Definition

A **mixed strategy** is a strategy where the player randomizes over the set of actions. It is a probability distribution that assigns to each action (or pure strategy) a likelihood of being selected.

- A strategy is defined by the probability you place on each action
- It is as if you were giving these probabilities to a machine that picked accordingly and told you what strategy you should play
- Example a strategy in tax game for tax authority could be: “audit” with probability 0.4 and “not audit” with probability 0.6

Mixed strategies
How are payoffs calculated?

- Payoff for a player is a weighted average of payoff of each action where the weight is the probability: called expected payoff
- Suppose for instance that the tax payer plays a mixed strategy: “declare" with probability 0.2 and “lie” with 0.8.
- Then the payoff of the tax authority if it plays “audit” is:

$$0.2 \times 1 + 0.8 \times 4 = 3.4$$

Mixed strategies
Nash equilibrium

- Definition of Nash Equilibrium remains the same: combination of strategies such that if other players play their Nash equilibrium strategies, you also want to play your Nash equilibrium strategy
- Remember strategy is defined for a mixed strategy by a combination of probabilities
- So the probabilities are not any probabilities: they are defined at the equilibrium

Mixed strategies

- To determine an equilibrium in mixed strategies, we will always use the following essential property:

Property (Indifference)

In equilibrium, the players are indifferent (i.e get the same payoff) from all the strategies they play with positive probability.

- For example, if the tax payer in equilibrium plays “declare” with probability 0.2 and “lie” with 0.8, then his payoff if he played “declare” for sure and his payoff if he played “lie” for sure should be equal.

Mixed strategies

Example

What is the Nash equilibrium of the Tax payer game?

	L	R
U	$(3,1)$	$(3,2)$
D	$(0,4)$	$(5,0)$

Solution

Nash equilibrium is such that:

Player 1 plays U with probability $4/5$ and D with probability $1/5$

Player 2 plays L with probability $2/5$ and R with probability $3/5$

Mixed strategies

- To check that this is an equilibrium, we need to show that no player can do better by switching strategy if the other plays his Nash equilibrium strategy.
- Fix player 2 at his Nash equilibrium strategy: plays L with probability $2/5$ and R with probability $3/5$.
- We need to check that Player 1 is ready to play U with probability $4/5$ and D with probability $1/5$, i.e that he is indifferent between U and D:
  - Payoff from U: $\frac{2}{5} \times 3 + \frac{3}{5} \times 3 = 3$
  - Payoff from D: $\frac{2}{5} \times 0 + \frac{3}{5} \times 5 = 3$
- Fix player 1 at his Nash equilibrium strategy: plays U with probability $4/5$ and D with probability $1/5$. Is player 2 indifferent between L and R:
  - Payoff from L: $\frac{4}{5} \times 1 + \frac{1}{5} \times 4 = \frac{8}{5}$
  - Payoff from R: $\frac{4}{5} \times 2 + \frac{1}{5} \times 0 = \frac{8}{5}$

Mixed strategies
Finding it

- If the strategy is not given to you but you want to find it, just assume it is of the type:
  - Player 1 plays U with probability p and D with $1-p$
  - Player 2 plays L with probability q and R with $1-q$
- Player 1 has to be indifferent between U and D, so:

$$3 \times q + 3 \times (1-q) = 0 \times q + 5 \times (1-q) \Rightarrow q = \frac{2}{5}$$

- Player 2 has to be indifferent between L and R, so:

$$1 \times p + 4 \times (1-p) = 2 \times p + 0 \times (1-p) \Rightarrow p = \frac{4}{5}$$

Mixed strategies

- Idea of people randomizing before making decisions can appear unnatural.
- Mixed strategies are commonly used:
  - by poker players;
  - by traffic wardens (aka meter maids);
  - by tax controllers;
  - for rebates and special offers;
  - ...
- There are a number of interpretations given to justify this: could be population averages
- Players are indifferent, but the probabilities with which they randomize are very well defined: they leave the other players indifferent

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Chiappori et al. (2002): testing mixed strategies

- Mixed strategies can appear unnatural
- Need empirical evidence
- Chiappori, Levitt and Groseclose (AER, 2002) using empirical evidence from the French and Italian first-leagues containing 459 penalty kicks over a period of 3 years (1997-2000).

Chiappori et al. (2002): testing mixed strategies

- Well defined environment
- Number of players: 2
- Strategy sets can be well summarized by (Left, Middle, Right)
- No ambiguity on preferences of players
- Players play simultaneously
  - The maximum speed the ball can reach exceeds 125 mph. At this speed, the ball enters the goal about two-tenths of a second after having been kicked.
- The structure of this game is such that there is no pure-strategy equilibrium.

Chiappori et al. (2002): testing mixed strategies

- Seems clear that strikers and goalies randomize.
- But do probabilities played correspond to the theory?
  - Does the indifference property holds?
  - Do the probabilities with which each player randomize leave the other player indifferent?
- If the indifference property holds, the kicker's scoring probability should be the same whether he kicks L, C or R, and the goalkeeper's probability of averting a goal should the same whether he dives L, C or R.
  - If the players were not indifferent, then it would pay them to adjust their probabilities towards more frequent selection of the strategy with the higher scoring probability (in the case of the kicker) or the strategy with the higher probability of averting a goal (in the case of the goalkeeper).

Chiappori et al. (2002): testing mixed strategies

- Important elements of the theory:
  - Kicking at the center when the keeper stays is very damaging for the kicker (the scoring probability is zero)
  - A right-footed kicker (about 85 percent of the population) will find it easier to kick to his left (his "natural side") than his right; and vice versa for a left-footed kicker.
    - ⋆ For simplicity, for shots involving left-footed kickers, the direction will be reversed so that shooting left correspond to the "natural side" for all kickers.
  - Probability of scoring in the middle is lower than on the sides if goalie does not go the correct way

Chiappori et al. (2002): testing mixed strategies

- Indeed, we have:

Table 1—Observed Scoring Probabilities, by Foot and Side

Kicker	Goalie: Correct side	Goalie: Middle or wrong side
Natural side ("left")	63.6 percent	94.4 percent
Opposite side ("right")	43.7 percent	89.3 percent

- The scoring probability when the goalie is mistaken varies between 89 percent and 95 percent (depending on the kicking foot and the side of the kick), whereas it ranges between 43 percent and 64 percent when the goalkeeper makes the correct choice.
- Also, the scoring probability is always higher on the kicker's natural side.

Chiappori et al. (2002): testing mixed strategies

Prediction

The indifference property leads to several testable propositions:

(i) *The right-footed kicker selects L (his natural side) more often than R;*

(ii) *The goalkeeper selects L more often than R;*

(iii) *The goalkeeper selects L more often than the right-footed kicker;*

(iv) *The kicker selects C more often than the goalkeeper.*

Chiappori et al. (2002): testing mixed strategies

- It is straightforward to show that departures from these propositions lead to violations of the indifference property:
  - In the case of (i), if the right-footed kicker selects L and R with equal probability, the goalkeeper would not be indifferent between L and R, because he would avert a goal more often by selecting R (diving to the kicker's weaker side).
  - In the case of (ii), if the goalkeeper selects L and R with equal probability, the right-footed kicker would not be indifferent between L and R, because he would score more often by selecting L (kicking on his stronger side).
  - Selecting C is highly damaging for the kicker if the goalkeeper also selects C. For the kicker to be indifferent between C and either L or R, in accordance with (iv), the goalkeeper must only select C very rarely.

Chiappori et al. (2002): testing mixed strategies

TABLE 3—OBSERVED MATRIX OF SHOTS TAKEN

	Kicker			
Goalie	Left	Middle	Right	Total
Left	117	48	95	260
Middle	4	3	4	11
Right	85	28	75	188
Total	206	79	174	459

- Predictions (i) & (ii): the kicker and the goalie are both more likely to go L than R.
  - This prediction is confirmed: in the data, 260 jumps are made to the (kicker's) left, and only 188 to the right.
  - The same pattern holds for the kicker, although in a less spectacular way (206 against 174).

Chiappori et al. (2002): testing mixed strategies

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Goalie	Left	Middle	Right	Total
Left	117	48	95	260
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Right	85	28	75	188
Total	206	79	174	459

- Prediction (iii): The goalkeeper selects L more often than the right-footed kicker.
  - The result emerges very clearly in the data: goalies play "left" 260 times (56.6 percent of kicks), compared to 206 (44.9 percent) instances for kickers.

Chiappori et al. (2002): testing mixed strategies

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	Kicker			
Goalie	Left	Middle	Right	Total
Left	117	48	95	260
Middle	4	3	4	11
Right	85	28	75	188
Total	206	79	174	459

- Prediction (iv): The kicker selects C more often than the goalkeeper.
  - The result emerges very clearly in the data: kickers play “center” 79 times in the sample, compared to only 11 times for goalies.