87-F-5

Strategic Cost-Reduction Investment* and Economic Welfare

by

Masahiro Okuno-Fujiwara
Faculty of Economics,
The University of Tokyo

and

Kotaro Suzumura
Department of Economics,
The University of Pennsylvania
The Institute of Economic Research,
Hitotsubashi University

May 6, 1987

PREPARED UNDER
THE PROJECT ON APPLIED ECONOMICS
RESEARCH INSTITUTE FOR THE JAPANESE ECONOMY#

---

* Thanks are due to the editor and the anonymous referees of this journal for their detailed comments and suggestions. We are grateful to Professors David Cass, Lionel Mckenzie, Andrew Postlewaite and William Thomson for their incisive comments, and to Ms. Ayako Sugiyama of Hitotsubashi University for her help in numerical computation. Needless to say, the responsibility for any remaining defect rests solely with us.

Abstract

We consider an industry where related products are produced by Oligopolistically competitive firms having symmetric opportunity to use strategic cost-reduction investment. Both in the case of Cournot-Nash quantity competition and Bertrand-Nash price competition, we derive a necessary and sufficient condition for the strategic commitment to be socially excessive or insufficient at the margin. It is shown that the commitment is socially insufficient in the normal case of Bertarand-Nash competition, whereas the commitment is likely to be socially excessive in the normal case of Cournot-Nash competition and the likelihood increases as the number of firms in the industry increases.

1. Introduction

Is there any intrinsic reason to expect that the oligopolistic competition will generate an excessive or insufficient commitment to cost-reduction investment in the aggregate? This question is a matter of great interest and subtlely, around which many contributions appeared recently.[1] The central issue has been that the strategic considerations may motivate oligopolistic competitors to overcommit themselves to cost-reduction investment in order to deter rival's entry and/or aggressiveness.

Most of the preceding contributions are commonly concerned with the two-stage game of oligopolistic competition. In the usual set up, the firms make strategic commitment to cost-reduction investment in the first-stage fully anticipating the equilibrium to be established in the second-stage, in which either quantity or price competition takes place.[2] But deep cleavage develops in the literature from that point on.

(a) Some are concerned with the commitment of the incumbent firm in the face of a potential entrant (Bulow, Geanakoplos and Klemperer [3], Dixit [4], Fudenberg and Tirole [8], Spulber [18], among others), whereas others are concerned with the competition among symmetrically situated incumbent firms using cost-reduction investment as the first-stage strategic variable (Brander and Spencer [2], in particular).

(b) To bring in a verdict of excessiveness/insufficiency of investment, some use as a benchmark the investment level voluntarily chosen in the absence of strategic considerations (Brander and Spencer [2, Sec. 2], Bulow, Geanakoplos and Klemperer [3], Fudenberg and Tirole [8], among others), whereas others invoke the social welfare attainable in the absence of strategic considerations as the benchmark (Brander and Spencer [2, Sec. 3]).

In this paper, we are concerned with an industry where related products are produced by n oligopolistically competitive firms, where $2 \leqq n < +\infty$, which are endowed with the symmetric opportunity to use strategic commitment to cost-reduction investment. We examine whether these investments are socially excessive or insufficient from the viewpoint of social welfare. In the symmetric treatment of competing firms as well as in the use of social welfare benchmark, we follow Brander and Spencer [2], but with three essential differences. First, Brander and Spencer [2] obtained the result that excessive cost-reduction investment may occur in the two-stage model of duopoly, so that the problem of social excessiveness/insufficiency of commitment is not related to the number of firms in the industry. In contrast, we are interested in knowing whether the socially excessive/insufficient investment is more or less likely as there are more oligopolistic competitors involved.[3] Second, Brander and Spencer [2] considered only Cournot-Nash quantity competition in the second-stage game, whereas we are examining both Cournot-Nash quantity competition and Bertrand-Nash price competition.[4] This seems to be of importance, since the form of competition in the second-stage game may exert serious influence on the social excessiveness/insufficiency thesis. Indeed, if firms compete through prices, it is shown that there exists underinvestment, rather than overinvestment, more often than not. Third, to provide clearer economic sense of our social welfare-theoretic excessiveness/insufficiency thesis, we invoke the concept of strategic substitutes and strategic complements introduced by Bulow, Geanakoplos and Klemperer [3], which classifies products in accordance with whether more "aggressive" play by one firm in a market lowers or raises competing firm's marginal profitabilities in the market. This seems worthwhile in view of the well-substantiated assertion that "the choice of strategic assumption (strategic substitutes or complements) is the crucial determinant of the results of many oligopoly models (Bulow, Geanakoplos and Klemperer [3, p. 490])".[5]

Both in the case of Cournot-Nash quantity competition and Bertrand-Nash price competition in the second-stage game, we will provide a necessary and sufficient condition for the commitment to be socially excessive/insufficient at the margin. As an implication of our analysis, we can assert that the cost-reduction investment is always socially insufficient at the margin if products are strategic complements and ordinary substitutes irrespective of the form of second-stage competition. Therefore, the socially excessive investment can surface only when strategic substitutes prevail. Note that products are more likely to be strategic substitutes (resp. strategic complements) in the case of quantity (resp. price) competition.

An important special case in which the socially excessive investment does obtain is the case in which products are strategic substitutes and homogeneous, and Cournot-Nash competition prevails in the second-stage game. This special case is of importance especially for the following reason. In the postwar period, the Japanese government tried to intervene the industrial organization of some industries in the name of preventing "excessive competition in capacity investment". The industries in which the so-called "excessive competition" allegedly prevailed were characterized by the following features:[6] (1) The product of the industry is homogeneous; (2) The production in this industry necessitates a prior commitment to fixed productive capacity; (3) The industrial organization is oligopolistic, and the strategic consideration does play an essential role. Our excessive commitment thesis suggests that the "excessive competition" in terms of social welfare can indeed occur in an industry characterized by the above features despite the fact "a widespread belief that increasing competition will increase welfare (Stiglitz [19, p. 184])" strongly dominates.

The plan of the paper is as follows. Our model of the two-stage game of oligopolistic competition is expounded in Section 2. In Section 3, we introduce Bertrand market surplus function and Cournot market surplus function as our welfare criteria, and establish, in each case, a necessary and sufficient condition for the strategic commitment to be socially excessive/insufficient at the margin. Section 4 is devoted to the Cournot-Nash competition and establishes our socially excessive commitment thesis. Section 5 concludes the main text with a couple of clarifying observations. Three appendices are added at the end of the paper. Appendix A shows that our model satisfies the so-called diagonal dominance property, whereas Appendix B derives the stability condition of our model. In Appendix C, we compare the level of strategic cost-reduction investment directly with the socially optimal level in order to illustrate a <u>global</u>, rather than a <u>marginal</u>, version of the excessive commitment thesis.

2. The Model

2.1. Consider an industry in which n firms $(2 \leqq n < +\infty)$ are producing related products. It is assumed that firms are engaging in the two-stage game of oligopolistic competition. In the first-stage, each firm makes strategic commitment to cost-reduction investment fully anticipating the equilibrium to be established in the second-stage, where firms compete with each other in the product market. The strategies in the second-stage are either quantities or prices. The equilibrium concept we work with is that of Nash throughout. Therefore, the equilibrium of the second-stage game is either Cournot-Nash equilibrium (in the case of quantity competition) or Bertrand-Nash equilibrium (in the case of price competition), and that of the whole game is subgame perfect equilibrium.

2.2. Assume that the utility function of the representative consumer takes the form

$$U(\mathbf{x},y) = u(\mathbf{x}) + y \tag{1}$$

where $\mathbf{x} = (x_1,x_2,\ldots,x_n) \in \mathbb{R}^n_+$ and $y \in \mathbb{R}_+$ denote, respectively, the consumption vector of the products of this industry and the consumption of the competitively produced numeraire good. Let the budget constraint be given by

$$\sum_{j=1}^{n} p_j x_j + y = M, \tag{2}$$

where p_j is the price of product j and M is income. Let $(\mathbf{x}^*,y^*)$ be defined as the maximizer of (1) subject to (2). Assuming twice continuous differentiability of $u(\mathbf{x})$ and interior optimum, we may easily verify that $\mathbf{x}^*$ is independent of M, so that we may write $(\mathbf{x}^*, y^*) = (\mathbf{x}(\mathbf{p}), y(\mathbf{p},M))$, where $\mathbf{p} = (p_1,p_2,\ldots,p_n) \in \mathbb{R}^n_+$

Let the indirect utility function be defined by

$$\begin{aligned} V(\mathbf{p},M): &= U(\mathbf{x}(\mathbf{p}),\ y(\mathbf{p},M)) \\ &= u(\mathbf{x}(\mathbf{p})) + y(\mathbf{p},M) \\ &= u(\mathbf{x}(\mathbf{p})) - \sum_{j=1}^{n} p_j x^j(\mathbf{p}) + M, \end{aligned}$$

where $x^j(\mathbf{p})$ is the j-th component of $\mathbf{x}(\mathbf{p})$. To simplify notation, define $v_i(\mathbf{p}): = u(\mathbf{x}(\mathbf{p})) - \Sigma^n_{j=1} p_j x^j(\mathbf{p})$. Then we have

$$v_i(\mathbf{p}) := \frac{\partial}{\partial p_i} v(\mathbf{p}) = - x^i(\mathbf{p}) \quad (i = 1,2,\ldots,n), \tag{3}$$

which is nothing other than the Roy's identity in our model. To derive (3), we need the first-order condition

$$u_i(\mathbf{x}(\mathbf{p})) := \frac{\partial}{\partial x_i} u(\mathbf{x}(\mathbf{p})) = p_i \quad (i = 1,2,\ldots,n). \tag{4}$$

Throughout this paper, we assume that $u(\mathbf{x})$, hence $v(\mathbf{p})$, is <u>symmetric</u> and that the products are <u>ordinary substitutes</u>, viz.,

$$u_{ij}(\mathbf{x}) := \frac{\partial^2}{\partial x_i \partial x_j} u(\mathbf{x}) < 0 \quad (i,j = 1,2,\ldots,n). \tag{5}$$

It follows that

$$v_{ii}(\mathbf{x}(\mathbf{p})) + (n-1)\, v_{ij}(\mathbf{x}(\mathbf{p})) > 0 \qquad (i \neq j,\ i,j = 1,2,\ldots,n) \tag{6}$$

holds, where $v_{ij}(\mathbf{x}(\mathbf{p})) := (\partial^2/\partial x_i \partial x_j)\, v(\mathbf{x}(\mathbf{p}))$ $(i,j = 1,2,\ldots,n)$. The property (6), to be called the <u>diagonal dominance</u>, will be proved in Appendix A. We will also assume that

$$\begin{cases} x^i_i(\mathbf{p}) := \dfrac{\partial}{\partial p_i} x^i(\mathbf{p}) < 0 \\ \\ x^i_j(\mathbf{p}) := \dfrac{\partial}{\partial p_j} x^i(\mathbf{p}) > 0 \quad (i \neq j) \end{cases} \tag{7}$$

for all $i,j = 1,2,\ldots,n$, which is another way of defining ordinary substitutes. It will be shown in Appendix A that (5) must be strengthened into

$$u_{ii}(\mathbf{x}) < u_{ij}(\mathbf{x}) < 0 \quad (i \neq j;\ i,j = 1,2,\ldots,n) \tag{5*}$$

in order that (7) holds.

2.3. To formalize our two-stage game structure in a compact form, let s_i denote the strategic variable of firm i in the second-stage game. In the case of quantity competition s_i stands for q_i, the quantity produced by firm i, whereas s_i stands for p_i, the price of product produced by firm i, in the case of price competition. The strategic variable of firm i in the first-stage game is denoted by K_i, to be called cost-reduction investment of firm i.

Let the second-stage cost function of firm i be given by $C(q_i;K_i)$, assumed to be twice continuously differentiable. We will write $C_q(q_i;K_i) = (\partial/\partial q_i)\, C(q_i;K_i)$ and $C_{qq}(q_i;K_i) = (\partial^2/\partial q_i^2)\, C(q_i;K_i)$. Likewise, other first-order and second-order partial derivatives will be denoted as $C_K(q_i;K_i)$, $C_{qK}(q_i;K_i)$ and $C_{KK}(q_i;K_i)$. Throughout this paper, we assume that $C_q(q_i;K_i) \geqq 0$, $C_K(q_i;K_i) < 0$, $C_{qq}(q_i;K_i) \geqq 0$, $C_{KK}(q_i;K_i) > 0$ and $C_{qK}(q_i;K_i) < 0$ hold for all $(q_i;K_i) \in \mathbb{R}^2_{++}$. Thus, marginal cost is always non-negative and non-decreasing in output. Furthermore, the investment is cost-reducing at a decreasing rate, and larger investment always decreases marginal cost.

The second-stage payoff of firm i may be written as

$$\pi^i(\mathbf{s};K_i) = \begin{cases} s_i \dfrac{\partial}{\partial s_i} w(\mathbf{s}) - C(s_i;K_i) & \text{if Cournot-Nash} \\[2ex] s_i \left\{-\dfrac{\partial}{\partial s_i} w(\mathbf{s})\right\} - C\left(-\dfrac{\partial}{\partial s_i} w(\mathbf{s});\ K_i\right) & \text{if Bertrand-Nash,} \end{cases}$$

where $w(\mathbf{s})$ is defined by

$$w(\mathbf{s}) = \begin{cases} u(\mathbf{q}) & \text{if Cournot-Nash} \\ v(\mathbf{p}) & \text{if Bertrand-Nash.} \end{cases}$$

Let $\mathbf{s}^*(\mathbf{K})$ denote the second-stage Nash equilibrium when the investment profile $\mathbf{K} = (K_1, K_2, \ldots, K_n)$ is parametrically given. Then the first-stage payoff of firm i is given by

$$\Pi^i(\mathbf{K}) = \pi^i(\mathbf{s}^*(\mathbf{K});\ K_i) - K_i,$$

assuming that the cost-reduction investment K_i is measured by the fixed expenditure for equipment installation. Let $\mathbf{K}^*$ denote the first-stage Nash equilibrium, so that $(\mathbf{K}^*;\ \mathbf{s}^*(\mathbf{K}^*))$ is the subgame perfect equilibrium of the two-stage game. Our concern is whether $\mathbf{K}^*$ represents a socially excessive/insufficient commitment from the point of view of social welfare.

Throughout this paper, we assume that the second-stage Nash equilibrium is symmetric, viz., $s_i^*(\mathbf{K}) = s_j^*(\mathbf{K})$ holds for all $i, j = 1, 2, \ldots, n$ if the investment profile $\mathbf{K} = (K_1, K_2, \ldots, K_n)$ is symmetric, so that we obtain $K_i = K_j$ $(i, j = 1, 2, \ldots, n)$. The first-stage Nash equilibrium is also assumed to be symmetric, viz., $K_i^* = K_j^*$ for all $i, j = 1, 2, \ldots, n$.

2.4. Assuming interior second-stage Nash equilibrium, $\mathbf{s}^*(\mathbf{K})$ is characterized by

$$\frac{\partial}{\partial s_i} \pi^i(\mathbf{s}^*(\mathbf{K});\ K_i) = 0 \qquad (i = 1, 2, \ldots, n). \tag{8}$$

Differentiating (8) with respect to K_i and K_h $(h \neq i)$, we obtain

$$\sum_{j=1}^{n} \frac{\partial^2}{\partial s_i \partial s_j} \pi^i(\mathbf{s}^*(\mathbf{K});\ K_i) \cdot \frac{\partial}{\partial K_i} s_j^*(\mathbf{K}) + \frac{\partial^2}{\partial s_i \partial K_i} \pi^i(\mathbf{s}^*(\mathbf{K});\ K_i) = 0 \qquad (9)$$

$$(i = 1,2,\ldots,n).$$

and

$$\sum_{j=1}^{n} \frac{\partial^2}{\partial s_i \partial s_j} \pi^i(\mathbf{s}^*(\mathbf{K});\ K_i) \cdot \frac{\partial}{\partial K_h} s_j^*(\mathbf{K}) = 0 \qquad (10)$$

$$(i = 1,2,\ldots,n;\ h \neq i).$$

In view of our assumption of symmetry, we can define the following three variables independent of indices, i and j:

$$\alpha(\mathbf{K}):\ = \frac{\partial^2}{\partial s_i^2} \pi^i(\mathbf{s}^*(\mathbf{K});\ K_i)$$

$$\beta(\mathbf{K}):\ = \frac{\partial^2}{\partial s_i \partial s_j} \pi^i(\mathbf{s}^*(\mathbf{K});\ K_i) \quad (i \neq j)$$

and

$$\gamma(\mathbf{K}):\ = \frac{\partial^2}{\partial s_i \partial K_i} \pi^i(\mathbf{s}^*(\mathbf{K});\ K_i).$$

By virtue of the second order condition, $\alpha(\mathbf{K}) < 0$ holds. Throughout the rest of this paper, the sign of $\beta(\mathbf{K})$ plays an important role. We say, following Bulow, Geanakoplos and Klemperer [3], that products are <u>strategic complements</u> (resp. <u>strategic substitutes</u>) if $\beta(\mathbf{K}) > 0$ (resp. < 0).[7] Finally, the sign of $\gamma(\mathbf{K})$ hinges on the type of competition in the second-stage. In the case of quantity competition, we obtain

$$\gamma(\mathbf{K}) = -\, C_{qK}(q_i(\mathbf{K});\ K_i) > 0, \qquad (11)$$

whereas in the case of price competition, we obtain

$$\gamma(\mathbf{K}) = -\frac{\partial}{\partial p_i} x^i(\mathbf{p}^*(\mathbf{K})) \cdot C_{qK}(x^i(\mathbf{p}^*(\mathbf{K})); K_i), \tag{12}$$

which is negative in view of (7).

Our assumption of symmetry also allows us to define

$$\theta(\mathbf{K}) := \frac{\partial}{\partial K_i} s_j^*(\mathbf{K}) \quad (i \neq j)$$

and

$$\omega(\mathbf{K}) := \frac{\partial}{\partial K_i} s_i^*(\mathbf{K})$$

without specifying the indices of firms.

Using this notation, we may rewrite (9) and (10) as

$$\alpha(\mathbf{K})\omega(\mathbf{K}) + (n-1)\,\beta(\mathbf{K})\theta(\mathbf{K}) + \gamma(\mathbf{K}) = 0$$

and

$$\alpha(\mathbf{K})\theta(\mathbf{K}) + (n-2)\,\beta(\mathbf{K})\theta(\mathbf{K}) + \beta(\mathbf{K})\omega(\mathbf{K}) = 0,$$

which can be solved for $\theta(\mathbf{K})$ and $\omega(\mathbf{K})$ to obtain

$$\theta(\mathbf{K}) = \frac{\beta(\mathbf{K})\gamma(\mathbf{K})}{\Delta(\mathbf{K})}$$

and

$$\omega(\mathbf{K}) = -\frac{\{\alpha(\mathbf{K}) + (n-2)\beta(\mathbf{K})\}\gamma(\mathbf{K})}{\Delta(\mathbf{K})},$$

where $\Delta(\mathbf{K}) := \{\alpha(\mathbf{K}) - \beta(\mathbf{K})\}\{\alpha(\mathbf{K}) + (n-1)\beta(\mathbf{K})\}$. $\Delta(\mathbf{K})$ is positive by virtue of the stability condition for the second stage Nash equilibrium derived in Appendix B.

The sign of (13) and (14), which is crucial for the following analysis, depends on the type of competition as well as on the strategic relatedness of the products. Consider, first, the case of quantity competition. By virtue of the stability condition and the sign of $\alpha(\mathbf{K})$, $\beta(\mathbf{K})$ and $\gamma(\mathbf{K})$ found above, $\theta(\mathbf{K}) > 0$ (resp. < 0) if and only if strategic complements (resp. strategic substitutes) prevail, whereas $\omega(\mathbf{K}) > 0$ holds no matter what. Next consider the case of price competition. Invoking the stability condition and making use of (12) instead of (11), we conclude that $\theta(\mathbf{K}) < 0$ (resp. > 0) if and only if strategic complements (resp. strategic substitutes) prevail, whereas $\omega(\mathbf{K}) < 0$ holds no matter what. For the sake of easy reference, these results are summarized in Table 1(a) and Table 1(b), respectively.

Finally, assuming interior first-stage Nash equilibrium, the equilibrium investment profile $\mathbf{K}^*$ is characterized by

$$\frac{\partial}{\partial K_i} \Pi^i(\mathbf{K}^*) = 0 \qquad (i = 1,2,\ldots,n),$$

which completes the description of our model.

3. Social Welfare and Strategic Cost-Reduction Investment

3.1. If there were omnipotent government, the "first-best" policy in relation to this oligopolistic industry would be to control all of the number of firms in the industry, the level of cost-reduction investment of each and every firm, and the level of output or price of each and every firm, so that the net market surplus is maximized. Such a government never exists in reality, and our evaluation of the social excessiveness/insufficiency of strategic cost-reduction investment should be based on a "second-best" performance criterion that has at least some empirical relevance. Our contention is that the oligopolistic quantity/price competition in the second-stage is something which lies beyond the controlling power of the government, and we ask whether the strategic cost-reduction investment in the first-stage is socially excessive/insufficient from the viewpoint of such a "second-best" government.

3.2. In the case of price competition in the second-stage, such a "second-best" welfare criterion is provided by what we call <u>Bertrand market surplus function</u>:

$$W^B(\mathbf{K}) := u(\mathbf{x}(\mathbf{p}^*(\mathbf{K}))) - \sum_{j=1}^{n} p_j^*(\mathbf{K})\, x^j(\mathbf{p}^*(\mathbf{K})) + \sum_{j=1}^{n} \Pi^j(\mathbf{K})$$

$$= v(\mathbf{p}^*(\mathbf{K})) + \sum_{j=1}^{n} \Pi^j(\mathbf{K}), \qquad (15)$$

viz., the sum of consumer's surplus and profits.

We are interested in the sign of $(\partial/\partial K_i)W^B(\mathbf{K}^*)$. If it is positive (resp. negative), a marginal increase (resp. decrease) of K_i from the equilibrium level K_i^* increases the value of Bertrand market surplus function, so that the equilibrium level of investment is socially insufficient (resp. excessive) at the margin.

Differentiating (15) with respect to K_i and evaluating at $\mathbf{K}^*$, we obtain

$$\frac{\partial}{\partial K_i} W^B(\mathbf{K}^*) = D_i^B(\mathbf{K}^*) + S_i^B(\mathbf{K}^*), \qquad (16)$$

where

$$D_i^B(\mathbf{K}^*) := \sum_{j=1}^{n} \{p_j^*(\mathbf{K}^*) - C_q^j(\mathbf{K}^*)\} \sum_{h=1}^{n} \frac{\partial}{\partial p_h} x^j(\mathbf{p}^*(\mathbf{K}^*)) \cdot \frac{\partial}{\partial K_i} p_h^*(\mathbf{K}^*) \qquad (17)$$

and

$$S_i^B(\mathbf{K}^*) := - C_K(x^i(\mathbf{p}^*(\mathbf{K}^*)), K_i^*) - 1$$

$$= - \{p_i^*(\mathbf{K}^*) - C_q^i(\mathbf{K}^*)\} \sum_{j \neq i} \frac{\partial}{\partial p_j} x^i(\mathbf{p}^*(\mathbf{K}^*)) \cdot \frac{\partial}{\partial K_i} p_j^*(\mathbf{K}^*), \quad (18)$$

and $C_q^j(\mathbf{K}^*) := C_q(x^j(\mathbf{p}^*(\mathbf{K}^*)); K_j^*)$ for short. Note that $D_i^B(\mathbf{K}^*)$, the distortion effect, is the familiar sum of marginal distortions that an increase of K_i creates in each market, whereas $S_i^B(\mathbf{K}^*)$, the strategic effect, is the term which would be zero in the absence of strategic considerations. As we shall show below, distortion effect creates incentive to reduce investment regardless of the type of product and of competition. It is the strategic effect which sometimes creates incentive to invest in excessive magnitude.

Using symmetry and (3), we can reduce (17) into

$$D_i^B(\mathbf{K}^*) = - \{p_i^*(\mathbf{K}^*) - C_q^i(\mathbf{K}^*)\}\{v_{ii}(\mathbf{p}^*(\mathbf{K}^*)) + (n-1)v_{ij}(\mathbf{p}^*(\mathbf{K}^*))\} \Gamma(\mathbf{K}^*), \quad (19)$$

where $i \neq j$ and

$$\Gamma(\mathbf{K}^*) := \omega(\mathbf{K}^*) + (n-1)\theta(\mathbf{K}^*)$$

$$= \frac{\gamma(\mathbf{K}^*)\{\beta(\mathbf{K}^*) - \alpha(\mathbf{K}^*)\}}{\Delta(\mathbf{K}^*)} \quad (20)$$

by virtue of (13) and (14). Invoking (6), (11) and the stability conditions, we may conclude that $D_i^B(\mathbf{K}^*) > 0$, viz., the distortion effect always generates socially insufficient cost-reduction investment at equilibrium.

On the other hand, (18) can be similarly reduced into

$$S_i^B(\mathbf{K}^*) = \{p_i^*(\mathbf{K}^*) - C_q^i(\mathbf{K}^*)\} \ (n-1)\theta(\mathbf{K}^*)v_{ij}(\mathbf{p}^*(\mathbf{K}^*)), \quad (21)$$

where $i \neq j$. By virtue of Table 1(a), (3) and (7), we may conclude that $S_i^B(\mathbf{K}^*) > 0$ (resp. < 0) if products are strategic complements (resp. strategic substitutes). These results are summarized in Table 2(a).

In the present case of Bertrand-Nash price competition, it would normally be the case that products are strategic complements.[8] Both $D_i^B(\mathbf{K}^*)$ and $S_i^B(\mathbf{K}^*)$ are then positive, and we obtain:

Proposition 1: Suppose that Bertrand-Nash price competition prevails in the second-stage and that products are strategic complements. Then the equilibrium cost-reduction investment is socially insufficient at the margin from the viewpoint of social welfare.

Thus the socially excessive cost-reduction investment at the margin, which Brander and Spencer [2] pointed out in the context of Cournot-Nash duopoly, does not normally surface in the context of Bertrand-Nash oligopoly.

What if products are strategic substitutes? Substituting (19) - (21) into (16) and noting Table 1(a):

Proposition 2: Suppose that Bertrand-Nash price competition prevails in the second-stage and that products are strategic substitutes. Then the equilibrium cost-reduction investment is socially insufficient, optimal, or socially excessive at the margin if and only if

$$v_{ii}(\mathbf{p}^*(\mathbf{K}^*)) \lesseqgtr \frac{\alpha(\mathbf{K}^*)}{\beta(\mathbf{K}^*)} \{v_{ii}(\mathbf{p}^*(\mathbf{K}^*)) + (n-1)\, v_{ij}(\mathbf{p}^*(\mathbf{K}^*))\}.$$

Since the case covered by Proposition 2 does not seem to be typical, we refrain from exploring its implications any further.

3.3. Suppose instead that the quantity competition prevails in the second-stage. A "second-best" welfare criterion in this case is defined by

$$W^C(\mathbf{K}) := u(\mathbf{q}^*(\mathbf{K})) - \sum_{j=1}^{n} \{C(q_j^*(\mathbf{K}); K_j) + K_j\}, \tag{22}$$

which we call Cournot market surplus function.

Differentiating (22) with respect to K_i and evaluating at $\mathbf{K}^*$, we obtain

$$\frac{\partial}{\partial K_i} W^C(\mathbf{K}^*) = D_i^C(\mathbf{K}^*) + S_i^C(\mathbf{K}^*), \tag{23}$$

where

$$D_i^C(\mathbf{K}^*) := \sum_{j=1}^{n} \{u_j(\mathbf{q}^*(\mathbf{K}^*)) - C_q^j(\mathbf{K}^*)\} \frac{\partial}{\partial K_i} q_j^*(\mathbf{K}^*) \tag{24}$$

and

$$S_i^C(\mathbf{K}^*) = - q_i^*(\mathbf{K}^*) \sum_{j \neq i} u_{ij}(\mathbf{q}^*(\mathbf{K}^*)) \frac{\partial}{\partial K_i} q_j^*(\mathbf{K}^*) \tag{25}$$

and $C_q^j(\mathbf{K}^*) := C_q(q_j^*(\mathbf{K}^*); K_j^*)$.

Under our assumption of symmetry, (24) is reduced into

$$D_i^C(\mathbf{K}^*) = - u_{ii}(\mathbf{q}^*(\mathbf{K}^*)) \, q_i^*(\mathbf{K}^*) \, \Gamma(\mathbf{K}^*), \tag{26}$$

which is positive in view of (5), (11), (20) and the stability condition. In this case as well, the distortion effect unambiguously generates socially insufficient cost-reduction investment at equilibrium.

On the other hand, (24) can be reduced into

$$S_i^C(\mathbf{K}^*) = -(n-1)\theta(\mathbf{K}^*)\, q_i^*(\mathbf{K}^*) u_{ij}(\mathbf{q}^*(\mathbf{K}^*)) \tag{27}$$

under symmetry, where $i \neq j$. By virtue of Table 1(b) and (5), we conclude that $S_i^C(\mathbf{K}^*) > 0$ (resp. < 0) if products are strategic complements (resp. strategic substitutes). These results are summarized in Table 2(b).

If quantity competition prevails in the second stage, the normal case would be the one where products are strategic substitutes.[9] However, it is the case of strategic complements that the sign of $(\partial/\partial K_i)\, W^C(\mathbf{K}^*)$ is unambiguous, and we obtain the following:

Proposition 3: Suppose that Cournot-Nash quantity competition prevails in the second-stage game and that products are strategic complements. Then the equilibrium cost-reduction investment is always socially insufficient at the margin from the viewpoint of social welfare.

In the normal case of strategic substitutes, we may substitute (26) and (27) into (23) to obtain

$$\frac{\partial}{\partial K_i} W^C(\mathbf{K}^*) = -\, q_i(\mathbf{K}^*) u_{ii}(\mathbf{q}^*(\mathbf{K}^*))\theta(\mathbf{K}^*)\{1 + (n-1)\frac{u_{ij}(\mathbf{q}^*(\mathbf{K}^*))}{u_{ii}(\mathbf{q}^*(\mathbf{K}^*))} - \frac{\alpha(\mathbf{K}^*)}{\beta(\mathbf{K}^*)}\},$$

where (13) and (14) are utilized. Invoking Table 1(b) and (5), we obtain:

Proposition 4: Suppose that Cournot-Nash quantity competition prevails in the second-stage game and that products are strategic substitutes. Then the equilibrium cost-reduction investment is socially insufficient, socially optimal, or socially excessive at the margin if and only if

$$1 + (n-1)\frac{u_{ij}(\mathbf{q}^*(\mathbf{K}^*))}{u_{ii}(\mathbf{q}^*(\mathbf{K}^*))} \lesseqgtr \frac{\alpha(\mathbf{K}^*)}{\beta(\mathbf{K}^*)} . \tag{28}$$

We are particularly interested in the possibility of socially excessive investment at the margin. In the next section, we further investigate the condition (28) in order to clarify the likelihood of such a phenomenon to surface. We investigate (28) by analyzing several special yet important cases. Interpretation of these results must await Section 5.

4. Socially Excessive Investment Thesis

4.1. Consider first the case where the inverse demand function $p_i = u_i(\mathbf{q})$ $(i = 1,2,...,n)$ is linear. We can readily verify that $\alpha(\mathbf{K}) = 2u_{ii}(\mathbf{q}^*(\mathbf{K})) - C_{qq}(q_i^*(\mathbf{K}); K_i)$ and $\beta(\mathbf{K}) = u_{ij}(\mathbf{q}^*(\mathbf{K}))$ hold in this case. Then the necessary and sufficient condition for socially excessive investment boils down to

$$-\frac{C_{qq}(q_i^*(\mathbf{K}^*); K_i^*)}{u_{ij}} - 1 < \frac{1}{\rho}\{(n-1)\rho^2 - 2\},$$

where $u_{ij} := u_{ij}(\mathbf{q}^*(\mathbf{K}^*)) < 0$ and $\rho := u_{ij}(\mathbf{q}^*(\mathbf{K}^*))/u_{ii}(\mathbf{q}^*(\mathbf{K}^*)) > 0$ are constant by the assumption of linearity. Note that $0 < \rho < 1$ holds by virtue of (5*). Note also that $\phi(\rho) := \{(n-1)\rho^2 - 2\}/\rho$ is an increasing function of ρ and that $\phi(\rho) \to n - 3$ as $\rho \to 1$. Thus:

Proposition 5: Suppose that Cournot-Nash quantity competition prevails in the second-stage game, products are strategic substitutes, and the inverse demand functions are linear. Then the cost reduction investment is more likely to be socially excessive at the margin if (a) the larger is the number of firms n, (b) the smaller is the slope of the marginal cost function at equilibrium,

$C_{qq}(q_i^*(\mathbf{K}^*);K_i^*)$, and (c) the larger is ρ, hence products are more substitutable.

4.2. Consider second the case of Cournot-Nash competition with strategic substitutes, where output is physically homogeneous. Let the inverse demand function be $p = f(Q)$, where $Q: = \Sigma_{j=1}^n q_j$, with $f'(Q) < 0$ for all $Q > 0$ such that $f(Q) > 0$. Assume further that the cost function $C(q_i;K_i)$ is separable, so that $C(q_i;K_i) = c(K_i)q_i$ with $c(K_i) > 0$ and $c'(K_i) < 0$ for all $K_i > 0$. Let $\delta(Q): = Qf''(Q)/f'(Q)$, the elasticity of the slope of f. In what follows, we assume that there exists a finite constant δ_0 such that $\delta(Q) \geqq \delta_0$ for all $Q > 0$. [10]

In this case, the gross benefit function $u(\mathbf{q})$ is written as

$$u(\mathbf{q}) = \int_0^{\Sigma_{j=1}^n q_j} f(x)dx,$$

which implies that $u_i(\mathbf{K}^*) = f(\Sigma_{j=1}^n q_j^*(\mathbf{K}^*))$, $u_{ii}(\mathbf{K}^*) = u_{ij}(\mathbf{K}^*) = f'(\Sigma_{j=1}^n q_j^*(\mathbf{K}^*))$ and $u_{iii}(\mathbf{K}^*) = u_{iij}(\mathbf{K}^*) = f''(\Sigma_{j=1}^n q_j^*(\mathbf{K}^*))$. It then follows from Proposition 4 that $(\partial/\partial K_i) W^C(\mathbf{K}^*) < 0$ holds if and only if

$$\omega(\mathbf{K}^*) + 2(n-1)\,\theta(\mathbf{K}^*) < 0. \tag{29}$$

Products being strategic substitutes by assumption, we have $\theta(\mathbf{K}^*) < 0$. Hence (29) holds if and only if

$$\frac{\omega(\mathbf{K}^*)}{\theta(\mathbf{K}^*)} + 2(n-1) = -\frac{\alpha(\mathbf{K}^*)}{\beta(\mathbf{K}^*)} + n > 0, \tag{30}$$

where we used (13) and (14).

In this homogeneous output model with strategic substitutes, we may verify that

$$\alpha(\mathbf{K}^*) = 2f'(Q^*(\mathbf{K}^*)) + q_i^*(\mathbf{K}^*)f''(Q^*(\mathbf{K}^*)) < 0 \tag{31}$$

and

$$\beta(\mathbf{K}^*) = f'(Q^*(\mathbf{K}^*)) + q_i^*(\mathbf{K}^*)f''(Q^*(\mathbf{K}^*)) < 0, \tag{32}$$

where $Q^*(\mathbf{K}^*) := \Sigma_{j=1}^n q_j^*(\mathbf{K}^*)$. It should also be noted that, in view of (32),

$$n + \delta(Q^*(\mathbf{K}^*)) = \frac{n\beta(\mathbf{K}^*)}{f'(Q^*(\mathbf{K}^*))} > 0. \tag{33}$$

It follows from (30) - (32) that $(\partial/\partial K_i) W^C(\mathbf{K}^*) < 0$ if and only if

$$(n-1)\{n + \delta(Q^*(\mathbf{K}^*))\} > n. \tag{34}$$

Since $\delta(Q^*(\mathbf{K}^*)) \geqq \delta_0$ by assumption, a sufficient condition for (34) is that n satisfies an inequality

$$n^2 - (2 - \delta_0)n - \delta_0 > 0. \tag{35}$$

Let $N(\delta_0)$ be defined as the larger root of the quadratic equation $n^2 - (2 - \delta_0)n - \delta_0 = 0$. Obviously, $N(\delta_0) > 0$, and the critical inequality (35) is satisfied by any n which exceeds $N(\delta_0)$.

We have thus obtained the following:

Proposition 6: Suppose that the products are homogeneous and strategic substitutes, and the inverse demand function $p = f(Q)$ is such that

$\delta(Q) := Qf''(Q)/f'(Q)$ is uniformly bounded from below by $\delta_0 > -\infty$. Suppose also that the cost function is separable, so that $C(q_i; K_i) = c(K_i)q_i$. Then there exists a critical number $N(\delta_0) > 0$ such that the cost-reduction investment is socially excessive at the margin if the number of firms n exceeds $N(\delta_0)$.

Thus again we obtained the result that investment becomes more likely to be socially excessive as the number of firms becomes larger. Needless to say, the relevance of Proposition 6 hinges squarely on the size of the critical number $N(\delta_0)$. Indeed, if $N(\delta_0)$ is astronomical, the excessive investment result becomes almost irrelevant. Let us examine two parameterizable special cases of our model with a view to gauging the numerical order of $N(\delta_0)$.

Example 4.1.

Suppose that the inverse demand function is linear, viz., $f(Q) = a - bQ$ for some $a > 0$ and $b > 0$. Then the products are always strategic substitutes as may easily be seen, and it is clearly the case that $\delta(Q) = 0$ for all $Q > 0$. Therefore $\delta_0 = 0$ will serve as the uniform lower bound, and $N(\delta_0)$, being the larger root of $n^2 - 2n = 0$, is given by 2. ‖

Example 4.2.

Suppose that the inverse demand function is constantly elastic, viz., $f(Q) = Q^{-\varepsilon}$ for some $\varepsilon > 0$. Then the products are strategic substitutes if and only if $n > \varepsilon + 1$, which follows from the condition that $\beta(\mathbf{K}) = \varepsilon\{(\varepsilon + 1)/n - 1\} \{Q^*(\mathbf{K})\}^{-\varepsilon-1} < 0$. By computation we obtain $\delta(Q) = -\varepsilon - 1$ for all $Q > 0$, so that $\delta_0 = -\varepsilon - 1$ will serve as the uniform lower bound, and that $N(\delta_0)$ will be the larger root of the quadratic equation $n^2 - (\varepsilon + 3)n + (\varepsilon + 1) = 0$. Clearly, $N(\delta_0)$ exceeds 2, and increases continuously as ε becomes large. However, even for a large enough ε, say $\varepsilon = 5$ (which corresponds to the price elasticity of demand of 0.2), we have $N(\delta_0) = 4 + \sqrt{10}$, which does not seem to be absurdly large at all. ||

We have thus shown that the homogeneous output Cournot model with strategic substitutes contains fairly large cases where strategic cost-reduction investment is socially excessive at the margin if the number of firms exceeds a number determined solely by the demand condition, and this critical number of firms may be reasonably small.

How about the case where $n = 2$ (duopoly) in the homogenous output model? Still assuming that $C(q_i;K_i) = c(K_i)q_i$, we can obtain

$$\frac{\partial}{\partial K_i} W^C(\mathbf{K}^*) \lesseqgtr 0 \ \ \underline{\text{if and only if}} \ \ 1 \gtreqless \frac{1}{1 + q_i^*(\mathbf{K}^*)\dfrac{f''(Q^*(\mathbf{K}^*))}{f'(Q^*(\mathbf{K}^*))}}$$

which boils down to $(\partial/\partial K_i)W^C(\mathbf{K}^*) \lesseqgtr 0$ if and only if $f''(Q^*(\mathbf{K}^*)) \lesseqgtr 0$ in view of $f'(Q^*(\mathbf{K}^*)) < 0$. Thus:

Proposition 7: Suppose that $n = 2$, the products are strategic substitutes and homogeneous, and the cost function is separable, viz., $C(q_i;K_i) = c(K_i)q_i$. Then the strategic cost-reduction investment is socially excessive, socially optimal, or socially insufficient at the margin if and only if the inverse demand function is concave, linear or convex.

5. Clarifying Remarks

5.1. Two clarifications seem to be in order. The first is about the contrast between Proposition 1 on the "normal" case in Bertrand-Nash competition and Proposition 4 on the "normal" case in Cournot-Nash competition.

According to the former, price competition in the second-stage normally generates the strategic effect to work in the same direction as the distortion effect. Consequently, socially insufficient investment unambiguously occurs. On the other hand, quantity competition in the second-stage normally generates the strategic effect to work in the opposite direction of the distortion effect. Furthermore, our examination of some special, yet not at all pathological cases in Section 4 suggests that there are reasons to expect that socially excessive investment is more likely to occur if there are more firms engaging in this type of competition. Why this contrast?

The second is about the intuition behind the socially excessive investment in the case of Cournot-Nash quantity competition. What is the intuitive reason behind the strategic effect to become dominant over the distortion effect as the number of firms becomes large?

5.2. We start with the first. Suppose the second stage is competed via prices. In Figure 1(a), $p_1 = r_1^0(p_2)$ (resp. $p_2 = r_2^0(p_1)$) is the reaction curve of firm 1 (resp. firm 2) in the second-stage when the first-stage investment profile is given by $\mathbf{K}^0$. Suppose firm 1 invested more than K_1^0 in the first-stage. Then the reaction curve of firm 1 would be $p_1 = r_1^1(p_2)$ instead of $p_1 = r_1^0(p_2)$. The second stage Bertrand-Nash equilibrium would be E^1 instead of E^0, resulting in a lower (i.e. <u>more aggressive</u>) price charged by firm 2. This induced aggression by firm 2 would affect the profitability of firm 1 by

$$\{p_1^*(\mathbf{K}^0) - C_q(x_1(\mathbf{p}^*(\mathbf{K}^0)); K_1^0)\} \cdot \frac{\partial}{\partial p_2} x_1(\mathbf{p}^*(\mathbf{K}^0)) \cdot \frac{\partial}{\partial K_1} p_2^* (\mathbf{K}^0) < 0.$$

This <u>negative</u> impact of its own investment <u>via</u> the induced aggression by the opponent taken into consideration, firm 1 will be strategically motivated to invest <u>less</u> than socially desirable.

Next consider the case where quantity competition prevails in the second stage. In Figure 1(b), $q_1 = r_1^0(q_2)$ (resp. $q_2 = r_2^0(q_1)$) is the reaction curve of firm 1 (resp. firm 2) in the second-stage when the first-stage investment profile is specified by $\mathbf{K}^0$. If firm 1 invested more in the first-stage game, the Cournot-Nash equilibrium would be E^1 rather than E^0, resulting in a lower (i.e. less aggressive) quantity produced by firm 2. This induced concession by firm 2 would affect the profitability of firm 1 by

$$q_1^*(\mathbf{K}^0)\ u_{12}(\mathbf{q}^*(\mathbf{K}^0)) \cdot \frac{\partial}{\partial K_1} q_2^*(\mathbf{K}^0) > 0$$

at the margin. Taking this positive impact of its own investment via the induced concession by the opponent into consideration, firm 1 will be strategically motivated to invest more than socially desirable.

Thus it is precisely the property that products are strategic complements or substitutes that determines the sign of the strategic effect.

5.3. We now turn to the second issue; why does the negative strategic effect tend to dominate the positive distortion effect as the number of firms becomes large? We shall concentrate on the case of homogenous output, while extending the following analysis for more general case is straightforward. Given an investment profile $\mathbf{K}^0$ in the first-stage game, define the reaction curve of firm i by

$$r_i^0(Q_{-i}) = \underset{q_i > 0}{\arg\max}\ \{q_i f(q_i + Q_{-i}) - C(q_i^0; K_i)\},$$

where Q_{-i} denotes the aggregate output of firms other than i, which helps us define the cumulative reaction curve $R_i^0(Q)$ by[11]

$$q_i = R_i^0(Q) \ \underline{\text{if and only if}}\ q_i = r_i^0(Q - q_i).$$

The second-stage Cournot-Nash equilibrium can be defined by $(R_1^0(Q^*), R_2^0(Q^*),\dots,R_n^0(Q^*))$, where $Q^*: = Q^*(\mathbf{K}^0)$ is defined by

$$Q^* = \sum_{i=1}^{n} R_i^0(Q^*).$$

Note also that the distortion effect and the strategic effect are given in this case by

$$D_i^C(\mathbf{K}^0) = - f'(Q^*)q_i^*(\mathbf{K}^0) \frac{\partial}{\partial K_i} Q^*(\mathbf{K}^0) \tag{36}$$

and

$$S_i^C(\mathbf{K}^0) = - f'(Q^*)q_i^*(\mathbf{K}^0) (n-1) \frac{\partial}{\partial K_i} q_j^*(\mathbf{K}^0), \tag{37}$$

respectively.

Thus, with other things equal, the distortion effect increase at the rate of n-1. The question is how the ratio of $\frac{\partial}{\partial K_i} Q^*(\underset{\sim}{K}^0)$ and $\frac{\partial}{\partial K_i} q_j^*(\underset{\sim}{K}^0)$ change as the number of firms increases. We shall analyze this question in the following.

Take any $j \neq i$. Firm j's cumulative reaction curve $R_j^0(Q)$ is drawn in Figure 2. Other firms' cumulative reaction curves are vertically added to $R_j^0(Q)$ to obtain $\Sigma_{k=1}^n R_k^0(Q)$, which cuts the 45° line to define the equilibrium E^0. Suppose that firm i increases its investment in the first-stage from K_i^0 to K_i^1. The aggregate cumulative reaction curve will then shift up to $\Sigma_{k=1}^n R_k^1(Q)$, where $R_k^1(Q) = R_k^0(Q)$ for all $k \neq i$. At new equilibrium E^1, the aggregate output and the output of firm j will become $Q^*(\mathbf{K}^1)$ and $q_j^*(\mathbf{K}^1)$, respectively, where $K_k^1 = K_k^0$ for all $k \neq i$. If $\Delta K_i = K_i^1 - K_i^0$ is small enough, we obtain

$$Q^*(\mathbf{K}^1) - Q^*(\mathbf{K}^0) = \frac{\partial}{\partial K_i} Q^*(\mathbf{K}^0) \cdot \Delta K_i$$

and

$$q_j^*(\mathbf{K}^1) - q_j^*(\mathbf{K}^0) = \frac{\partial}{\partial K_i} q_j^*(\mathbf{K}^0) \cdot \Delta K_i ,$$

so that

$$\frac{\partial}{\partial K_i} q_j^*(\mathbf{K}^0) \Big/ \frac{\partial}{\partial K_i} Q^*(\mathbf{K}^0) = \text{slope of } R_j^0(Q), \tag{38}$$

which is independent of n. Putting (36) - (38) together, it is straightforward that the strategic effect becomes dominant over the distortion effect as n increases. It should also be clear from the argument that, even if the slope of $R_j^0(Q)$ is not constant, as long as the slope does not become flat too quickly as Q increases, the strategic effect starts to dominate the distortion effect as the equilibrium Q increases as a result of the increase in n.

What underlies our socially excessive investment thesis is, therefore, the fact that normally the distortion effect does not increase as much as (or diminishes more quickly than) the strategic effect does. This property is evident in the case of homogenous product, linear demand and cost functions, which is implicit in Figure 2. In this case, the cumulative reaction curve of each firm is independent of the number of firms n, whereas the aggregate cumulative reaction curve goes up as n increases.

5.4. In conclusion, our main results in this paper may be briefly summarized as follows. If Bertrand-Nash price competition prevails in the second-stage game, the strategic cost-reduction investment is socially insufficient at the margin in the normal case. If, on the other hand, Cournot-Nash quantity competition prevails in the second-stage game, the strategic cost-reduction investment is often socially excessive at the margin in the normal case, and the likelihood increases as the number of firms in the industry becomes large.

Appendix A: Diagonal Dominance

Differentiating (4) with respect to p_i and p_j $(j \neq i)$, we obtain

$$\sum_{k=1}^{n} u_{ik} x_i^k = 1 \tag{A.1}$$

and

$$\sum_{k=1}^{n} u_{ik} x_j^k = 0 \quad (i \neq j), \tag{A.2}$$

where $u_{ik} = u_{ik}(\mathbf{x}(\mathbf{p}))$ and $x_j^k = x_j^k(\mathbf{p})$ for simplicity. Invoking symmetry, (A.1) and (A.2) can be reduced into

$$(n-1)\, u_{ik} x_i^k + u_{ii} x_i^i = 1 \quad (i \neq k) \tag{A.3}$$

$$\{(n-2)\, u_{ik} + u_{ii}\}\, x_i^k + u_{ik} x_i^i = 0 \quad (i \neq k)\ , \tag{A.4}$$

respectively. Solving (A.3) and (A.4) for x_i^k $(i \neq k)$ and x_i^i, we obtain

$$x_i^k = \frac{u_{ik}}{(u_{ik} - u_{ii})\,\{(n-1)\, u_{ik} + u_{ii}\}} \quad (i \neq k) \tag{A.5}$$

and

$$x_i^i = - \frac{(n-2)\, u_{ik} + u_{ii}}{(u_{ik} - u_{ii})\,\{(n-1)\, u_{ik} + u_{ii}\}}\ . \tag{A.6}$$

By virtue of (3), (A.5) and (A.6), we obtain

$$v_{ii} + (n-1) v_{ik} = - \frac{1}{(n-1) u_{ik} + u_{ii}}\ ,$$

which is positive if (5) is assumed.

Note in passing that $x_i^i < 0$ and $x_k^i > 0$ $(i \neq k)$ hold if (5*) is assumed.

Appendix B: Stability of the Second-Stage Nash Equilibrium

Consider a natural myopic adjustment process for the second-stage Nash equilibrium, where each firm increases its strategic variable if positive marginal profitability prevails, viz.,

$$\dot{s}_i = \sigma \pi_i^i(\mathbf{s}), \tag{B.1}$$

where $\dot{s}_i$ is the time-derivative of s_i, and $\sigma > 0$ denotes the speed of adjustment.[12,13] Let $\mathbf{s}^*$ be the second-stage Nash equilibrium, and let (B.1) be linearly approximated by

$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \\ \vdots \\ \dot{s}_n \end{bmatrix} = \sigma \begin{bmatrix} \alpha & \beta & \cdots & \beta \\ \beta & \alpha & \cdots & \beta \\ \vdots & & \ddots & \vdots \\ \beta & \beta & \cdots & \alpha \end{bmatrix} \begin{bmatrix} s_1 - s_1^* \\ s_2 - s_2^* \\ \vdots \\ s_n - s_n^* \end{bmatrix} \tag{B.2}$$

around $\mathbf{s}^*$. For local stability of $\mathbf{s}^*$, the coefficient matrix Ω of (B.2) must have eigenvalues with negative real parts, which will be the case if Ω is negative definite. Therefore we require

$$(-1)^k \det \Omega_k > 0 \quad (k = 1,2,\ldots,n), \tag{B.3}$$

where Ω_k is the $k \times k$ principal sub-matrix of Ω. Simple computation entails that (B.3) is equivalent to

$$(-1)^k (\alpha - \beta)^{k-1} \{\alpha + (k-1)\beta\} > 0 \quad (k = 1,2,\ldots,n),$$

which holds true if

$$\alpha - \beta < 0 \tag{B.4}$$

$$\alpha + (k-1)\beta < 0 \quad (k = 1,2,\ldots,n).$$

As a matter of fact, (B.4) can be simplified into

$$\alpha - \beta < 0,\ \alpha + (n-1)\beta < 0. \tag{B.5}$$

Indeed, if $\beta < 0$, then $\alpha + (k-1)\beta < 0$ for all $k = 1,2,\ldots,n$ under (B.5). If $\beta > 0$ instead, then $(k-1)\beta < (n-1)\beta$ for all $k < n$, so that $\alpha + (k-1)\beta < \alpha + (n-1)\beta < 0$.

In the main text, we are referring to (B.5) as the stability condition for short.[14]

If products are homogeneous, we can obtain $\alpha = 2f'(Q) + qf''(Q) - C_{qq}(q)$ and $\beta = f'(Q) + qf''(Q)$, so that $\alpha < 0$, $\alpha - \beta < 0$ and $\alpha + (n-1)\beta < 0$ can be rewritten, respectively, as follows:

$$2f'(Q) + qf''(Q) - C_{qq}(q) < 0, \tag{B.6}$$

$$f'(Q) < C_{qq}(q), \tag{B.7}$$

and

$$f'(Q) - C_{qq}(q) + n\{f'(Q) + qf''(Q)\} < 0. \tag{B.8}$$

Suppose that (B.7) and

$$f'(Q) + qf''(Q) < 0 \tag{B.9}$$

hold. Then (B.6) - (B.8) are satisfied, hence the process (B.1) is locally stable in the case where products are homogeneous. Note that (B.7) and (B.9) are nothing other than Hahn's [9] stability condition for homogeneous output Cournot oligopoly.[15] Therefore our stability condition (B.5) may be regarded as a generalization of well-known Hahn condition to the case of symmetric product differentiation.

Appendix C: Global Excessive Investment Thesis

In the main text, we are concerned with the socially excessive/insufficient investment at the margin in the sense that a marginal decrease/increase of cost-reduction investment from the strategically chosen level increases market surplus. Since our welfare criteria, viz., market surples functions, may not be concave, there is a possibility that the marginally welfare-improving adjustment is in fact a "wrong" adjustment from the viewpoint of global optimization. In view of this possibility, which cannot be excluded offhand in the second-best exercise, it would be of some interest to compare directly the socially second-best level of cost-reduction investment with that of the subgame perfect equilibrium. Although such a direct comparison may be performed only when our model is greatly simplified, it is still worthwhile to see that our excessive commitment thesis holds even globally for at least such parametrizable economies. With this purpose in mind, let us consider a homogeneous output Cournot competition with a separable cost function $C(q_i;K_i) = c(K_i)q_i$. Let K^* be the Nash equilibrium investment level, and let K^{**} be the socially second-best investment level which is defined by

$$K^{**} = \underset{K > 0}{\arg\max} \left[\int_0^{\Sigma_{j=1}^n q_j^*(\mathbf{K})} f(x)dx - \sum_{j=1}^n \{c(K)q_j^*(\mathbf{K}) + K\}\right],$$

where $\mathbf{K} = (K, K, \ldots, K)$.

It can be shown that K^* and K^{**} are characterized by

$$- c'(K^*) q^*(\mathbf{K}^*) = \frac{n + 1 + \delta(Q^*(\mathbf{K}^*))}{2n + (2 - \frac{1}{n})\delta(Q^*(\mathbf{K}^*))} \qquad (C.1)$$

and

$$- c'(K^{**}) q^*(\mathbf{K}^{**}) = \frac{n + 1 + \delta(Q^*(\mathbf{K}^{**}))}{n + 2 + \delta(Q^*(\mathbf{K}^{**}))} \qquad (C.2)$$

respectively, where $Q^*(\mathbf{K}) := nq^*(\mathbf{K})$.

Example C(1)

Suppose that the inverse demand function is linear, viz., $f(Q) = a - bQ$, where $a > 0$ and $b > 0$, and that the marginal cost function is given by $c(K) = K^{-\eta}$, where $\eta > 0$. It then follows that (C.1) and (C.2) boil down to

$$- c'(K^*) \frac{a - c(K^*)}{b} = \frac{(n + 1)^2}{2n}$$

and

$$- c'(K^{**}) \frac{a - c(K^{**})}{b} = \frac{(n + 1)^2}{n + 2},$$

respectively. Furthermore, the function $\zeta(K)$ defined by

$$\zeta(K) := - c'(K) \frac{a - c(K)}{b}$$

becomes a single-peaked function and K^* as well as K^{**} satisfies $\zeta'(K^*) < 0$ and $\zeta'(K^{**}) < 0$ owing to the second-order conditions for profit and/or welfare maximization.

Noting that

$$\frac{(n+1)^2}{n+2} \gtreqless \frac{(n+1)^2}{2n} \quad \text{if and only if} \quad n \gtreqless 2,$$

we can now conclude that $K^* > K^{**}$ for $n \geqq 3$. Therefore, the Nash equilibrium investment level is socially excessive not only at the margin but also globally if there exist at least three firms in the industry. ||

Example C(2)

Suppose that the inverse demand function is of constant elasticity family, i.e., $f(Q) = Q^{-\varepsilon}$, where $\varepsilon > 0$. Suppose also that $c(K) = K^{-\eta}$, where $\eta > 0$. Then (C.1) and (C.2) reduce into

$$\frac{\eta}{n}\left(1 - \frac{\varepsilon}{n}\right)^{\frac{1}{\varepsilon}} (K^*)^{\frac{\eta}{\varepsilon} - \eta - 1} = \frac{n - \varepsilon}{\frac{2n-1}{n}(n-\varepsilon-1) + 1} \tag{C.3}$$

and

$$\frac{\eta}{n}\left(1 - \frac{\varepsilon}{n}\right)^{\frac{1}{\varepsilon}} (K^{**})^{\frac{\eta}{\varepsilon} - \eta - 1} = \frac{n - \varepsilon}{n-\varepsilon-1}, \tag{C.4}$$

respectively. By virtue of the second-order condition, the function $\xi(K)$ defined by

$$\xi(K) := \frac{\eta}{n}\left(1 - \frac{\varepsilon}{n}\right)^{\frac{1}{\varepsilon}} K^{\frac{\eta}{\varepsilon} - \eta - 1}$$

is monotonically decreasing, and $n > \varepsilon + 1$ holds.

Comparing the RHS of (C.3) with that of (C.4), we may verify that there exists a critical number, say $n^*(\varepsilon)$, such that $K^* > K^{**}$ holds true for $n > n^*(\varepsilon)$. Here again, therefore, the Nash equilibrium investment level is socially excessive in the global sense if there are sufficient number of firms in the industry.

Needless to say, the relevance of this result hinges on the size of the critical number $n^*(\varepsilon)$. Table 3 provides a series of critical numbers $n^*(\varepsilon)$ for the specified values of ε. As can be seen from this table, the social excessiveness of Nash equilibrium commitment occurs even when the number of firms is reasonably small. ‖

Footnotes

1. See, among many others, Brander and Spencer [2], Bulow, Geanakoplos and Klemperer [3], Dixit [4], Eaton and Grossman [6], Fudenberg and Tirole [8], Spence [17], Spulber [18] and Ware [23].

2. See, however, Ware [23] who proposed a three-stage model of capacity commitment as a modification of Dixit's [4] model of two-stage competition between incumbent and potential entrant.

3. Recent papers by Mankiw and Winston [11] and Suzumura and Kiyono [20] explored some other senses in which competition can be socially excessive by treating the number of firms endogenously. (See, also, Weizsäcker [24] and [25].) In contrast, the number of firms is an exogenous parameter in our present model.

4. However, unlike the model of Singh and Vives [15] where each firm can choose either quantity or price as its strategic variable <u>independently of each other</u>, we treat the case of <u>uniform</u> quantity competition and <u>uniform</u> price competition.

5. As a matter of fact, Brander and Spencer [2, p. 227] assume that products are strategic substitutes (without using this terminology which was coined later), leaving the readers with an impression that this is the only situation of interest. See, also, Eaton and Grossman [6].

6. Several concrete industries with these three features include iron and steel, petroleum refining, petrochemicals, certain other chemicals, cement, paper and pulp, and sugar refining. See Komiya [10] and Suzumura and Okuno-Fujiwara [21] for this and other features of the Japanese industrial policy.

7. As a matter of fact, Bulow, Geanakoplos and Klemperer [3] defined products to be strategic complements (resp. strategic substitutes) if an aggressive change of a strategy variable generates an upward (resp. a downward) change of rival's marginal profitability. Therefore, in the case of price competition, the relevant partial derivatives would be

$$\frac{\partial^2}{\partial(\frac{1}{p_i})\,\partial(\frac{1}{p_j})}\hat{\pi}^i(\frac{1}{p_1}, \frac{1}{p_2}, \ldots, \frac{1}{p_n}) \quad (i \neq j),$$

where $\hat{\pi}^i(1/p_1, 1/p_2, \ldots, 1/p_n) := \pi^i(p_1, p_2, \ldots, p_n)$. However, since

$$\frac{\partial^2}{\partial p_i \partial p_j}\pi^i(p_1, p_2, \ldots, p_n) = (-\frac{1}{p_i^2}) \cdot (-\frac{1}{p_j^2}) \cdot \frac{\partial^2}{\partial(\frac{1}{p_i})\,\partial(\frac{1}{p_j})}\hat{\pi}^i(\frac{1}{p_1}, \frac{1}{p_2}, \ldots, \frac{1}{p_n}),$$

we may safely use our symmetric definition throughout.

8. Consider the case of duopoly $(n = 2)$ and let the reaction function of firm i, $r_i(s_j)$ $(i \neq j;\ i,j = 1,2)$, be defined by $\pi^i_i(r_i(s_j),s_j) = 0$ in an obvious notation. It follows that

$$r'_i(s_j) = -\pi^i_{ij}(r_i(s_j),s_j)/\pi^i_{ii}(r_i(s_j),s_j).$$

Since $\pi^i_{ii}(r_i(s_j),s_j) < 0$ by the second-order condition, the slope $r'_i(s_j)$ is determined by the sign of $\pi^i_{ij}(r_i(s_j),s_j)$. Therefore, the usual shape of the reaction curves, as is depicted in, e.g., Dixit [4] and our Figure 1, corresponds to the case where products are strategic complements (resp. strategic substitutes) in the case of price competition (resp. quantity competition).

9. See footnote 8.

10. Suppose that $f''(Q) \leqq 0$, so that the inverse demand function is <u>concave</u>. It then follows that $\delta(Q) \geqq 0$ holds, which means that $\delta_0 = 0$ will serve as the uniform lower bound we are asking for. Based on this observation, one referee suggested that the uniform lower boundedness of $\delta(Q)$ is an assumption which is not very restrictive.

11. On the concept and uses of the cumulative reaction curve, see Novshek [12], Roberts and Sonnenschein [13], and Yakowitz and Szidarovszky [26].

12. For the sake of notational simplicity, we omit **K** throughout this Appendix. Since **K** is fixed here, this should not cause any confusion.

13. See Dixit [5] for this adjustment process in the special case of duopoly. See, also, Al-Nowaihi and Levine [1], Hahn [9] and Seade [14].

14. The sufficiency of (B.6) for the local stability of (B.1) can alternatively be obtained by invoking Theorem 2(ii) of Al-Nowaihi and Levine [1].

15. Note also that (B.10) is nothing other than Novshek's [12] condition for the existence of a Cournot equilibrium.

References

[1] Al-Nowaihi, A., and Levine, P.L., "The Stability of the Cournot Oligopoly Model: A Reassessment." Journal of Economic Theory, Vol. 35 (1985), pp. 307-321.

[2] Brander, J.A., and Spencer, B.J., "Strategic Commitment with R & D: The Symmetric Case." Bell Journal of Economics, Vol. 14 (1983), pp. 225-235.

[3] Bulow, J.I., Geanakoplos, J.D., and Klemperer, P.D., "Multimarket Oligopoly: Strategic Substitutes and Complements." Journal of Political Economy, Vo. 93 (1985), pp. 488-511.

[4] Dixit, A., "The Role of Investment in Entry Deterrence." Economic Journal, Vol. 90 (1980), pp. 95-106.

[5] Dixit, A., "Comparative Statics for Oligopoly." International Economic Review, Vol. 27 (1986), pp. 107-122.

[6] Eaton, J., and Grossman, G.M., "Optimal Trade and Industrial Policy under Oligopoly." Quarterly Journal of Economics, Vol. 100 (1986), pp. 383-406.

[7] Friedman, J.W., Oligopoly Theory, Cambridge: Cambridge University Press, 1983.

[8] Fudenberg, D., and Tirole, J., "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look." American Economic Review: Papers and Proceedings, Vol. 74 (1984), pp. 361-366.

[9] Hahn, F.H., "The Stability of the Cournot Oligopoly Solution." Review of Economic Studies, Vol. 29 (1962), pp. 328-331.

[10] Komiya, R., "Planning in Japan" in M. Bornstein, ed., Economic Planning: East and West, New York: Ballinger, 1975, pp. 189-227.

[11] Mankiw, N.G., and Winston, M.D., "Free Entry and Social Inefficiency." Rand Journal of Economics, Vol. 17 (1986), pp. 48-58.

[12] Novshek, W., "On the Existence of Cournot Equilibrium." Review of Economic Studies, Vol. 52 (1985), pp. 85-98.

[13] Roberts, J., and Sonnenschein, H., "On the Existence of Cournot Equilibrium without Concave Profit Functions." Journal of Economic Theory, Vol. 13 (1976), pp. 112-117.

[14] Seade, J.K., "The Stability of Cournot Revisited." Journal of Economic Theory, Vol. 15 (1980), pp. 15-27.

[15] Singh, N., and Vives, X., "Price and Quantity Competition in a Differentiated Duopoly." Rand Journal of Economics, Vol. 15 (1984), pp. 546-555.

[16] Spence, A.M., "Entry, Investment, and Oligopolistic Pricing." Bell Journal of Economics, Vol. 8 (1977), pp. 534-544.

[17] Spence, A.M., "Cost Reduction, Competition, and Industry Performance." Econometrica, Vol. 52 (1984), pp. 101-121.

[18] Spulber, D.F., "Capacity, Output, and Sequential Entry." American Economic Review, Vol. 71 (1981), pp. 503-514.

[19] Stiglitz, J.E., "Potential Competition May Reduce Welfare." American Economic Review: Papers and Proceedings, Vol. 71 (1981), pp. 184-189.

[20] Suzumura, K., and Kiyono, K., "Entry Barriers and Economic Welfare." Review of Economic Studies, Vol. 54 (1987), pp. 157-167.

[21] Suzumura, K., and Okuno-Fujiwara, M., "Industrial Policy in Japan: Overview and Evaluation." Paper presented at the Conference on "Trade Friction and Economic Policy," The Graduate School of Business Administration, New York University, April 10-11, 1986.

[22] Vives, X., "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation." Journal of Economic Theory, Vol. 36 (1985), pp. 166-175.

[23] Ware, R., "Sunk Costs and Strategic Commitment: A Proposed Three-Stage Equilibrium." Economic Journal, Vol. 94 (1984), pp. 370-378.

[24] Weizsäcker, C.C. von, "A Welfare Analysis of Barriers to Entry." Bell Journal of Economics, Vol. 11 (1980), pp. 399-420.

[25] Weizsäcker, C.C. von, Barriers to Entry: A Theoretical Treatment, Berlin: Springer-Verlag, 1980.

[26] Yakowitz, S. and Szidarovszky, "A New Proof of the Existence and Uniqueness of the Cournot Equilibrium." International Economic Review, Vol. 18 (1977), pp. 787-789.

Table 1: Effects of Investment on Equilibrium Output

(a) Bertrand-Nash Price Competition

	$\theta(\mathbf{K})$	$\omega(\mathbf{K})$
Strategic substitutes	+	–
Strategic complements	–	–

(b) Cournot-Nash Quantity Competition

	$\theta(\mathbf{K})$	$\omega(\mathbf{K})$
Strategic substitutes	–	+
Strategic complements	+	+

Table 2: Distortion Effect and Strategic Effect

(a) Bertrand-Nash Price Competition

	Distortion effect	Strategic effect
Strategic substitutes	+	-
Strategic complements	+	+

(b) Cournot-Nash Quantity Competition

	Distortion effect	Strategic effect
Strategic substitutes	+	-
Strategic complements	+	+

Table 3: Critical Number of Firms*

ε	$n^*(\varepsilon)$
0.2	2.76619
0.4	2.92066
0.6	3.08062
0.8	3.24536
1.0	3.41421
1.2	3.58661
1.4	3.76205
1.6	3.94012
1.8	4.12046
2.0	4.30278
2.2	4.48680
2.4	4.67231
2.6	4.85913
2.8	5.04709
3.0	5.23607
3.2	5.42594
3.4	5.61661
3.6	5.80799
3.8	6.00000
4.0	6.19258
4.2	6.38568
4.4	6.57924
4.6	6.77321
4.8	6.96757
5.0	7.16228

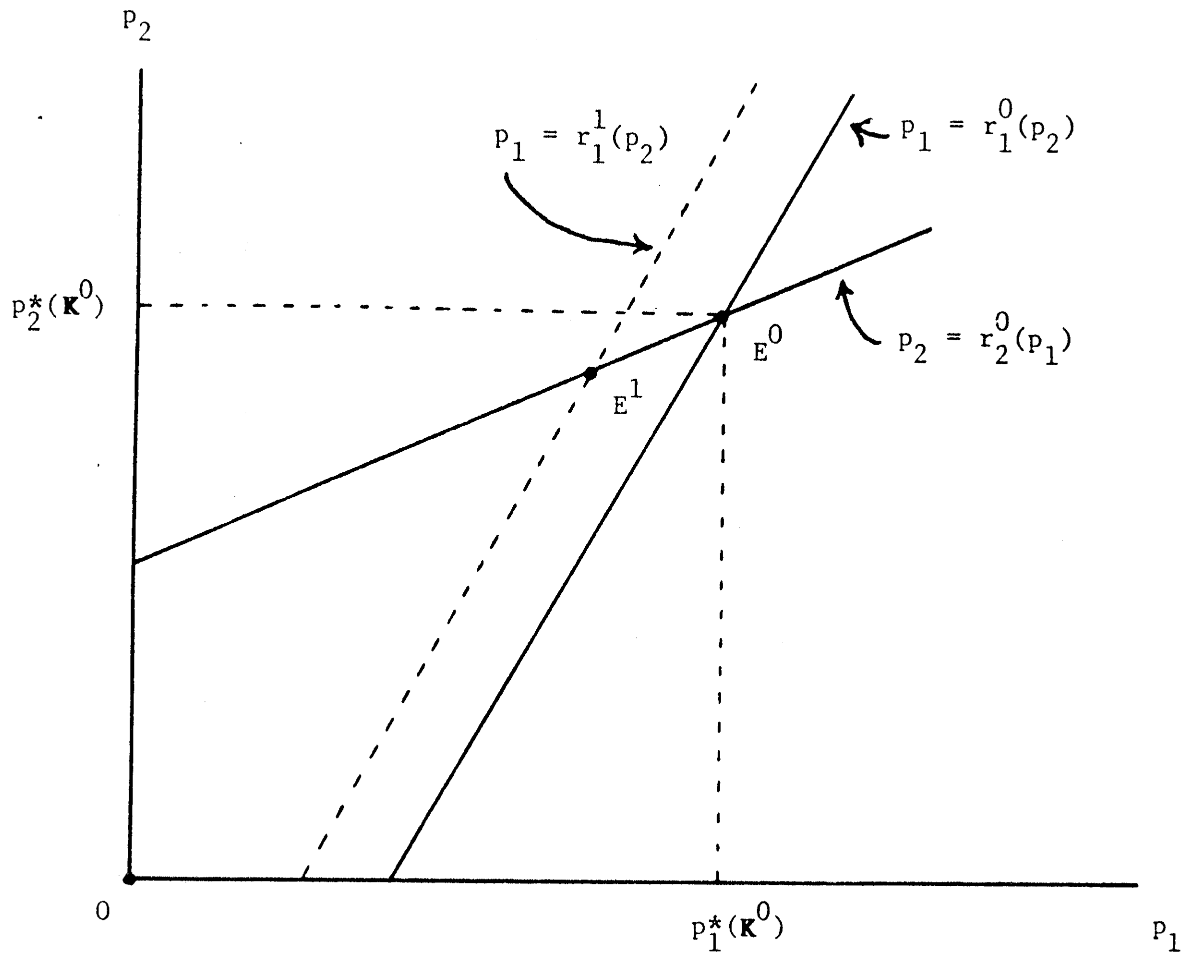
* The critical number $n^*(\varepsilon)$ is calculated by solving the equation

$$\frac{n-\varepsilon}{n-\varepsilon+1} = \frac{n-\varepsilon}{\frac{2n-1}{n}(n-\varepsilon-1) + 1}$$

for n.

Figure 1: Effect of Investment on Reaction Curves

(a) Bertrand-Nash Price Competition



(b) Cournot-Nash Quantity Competition

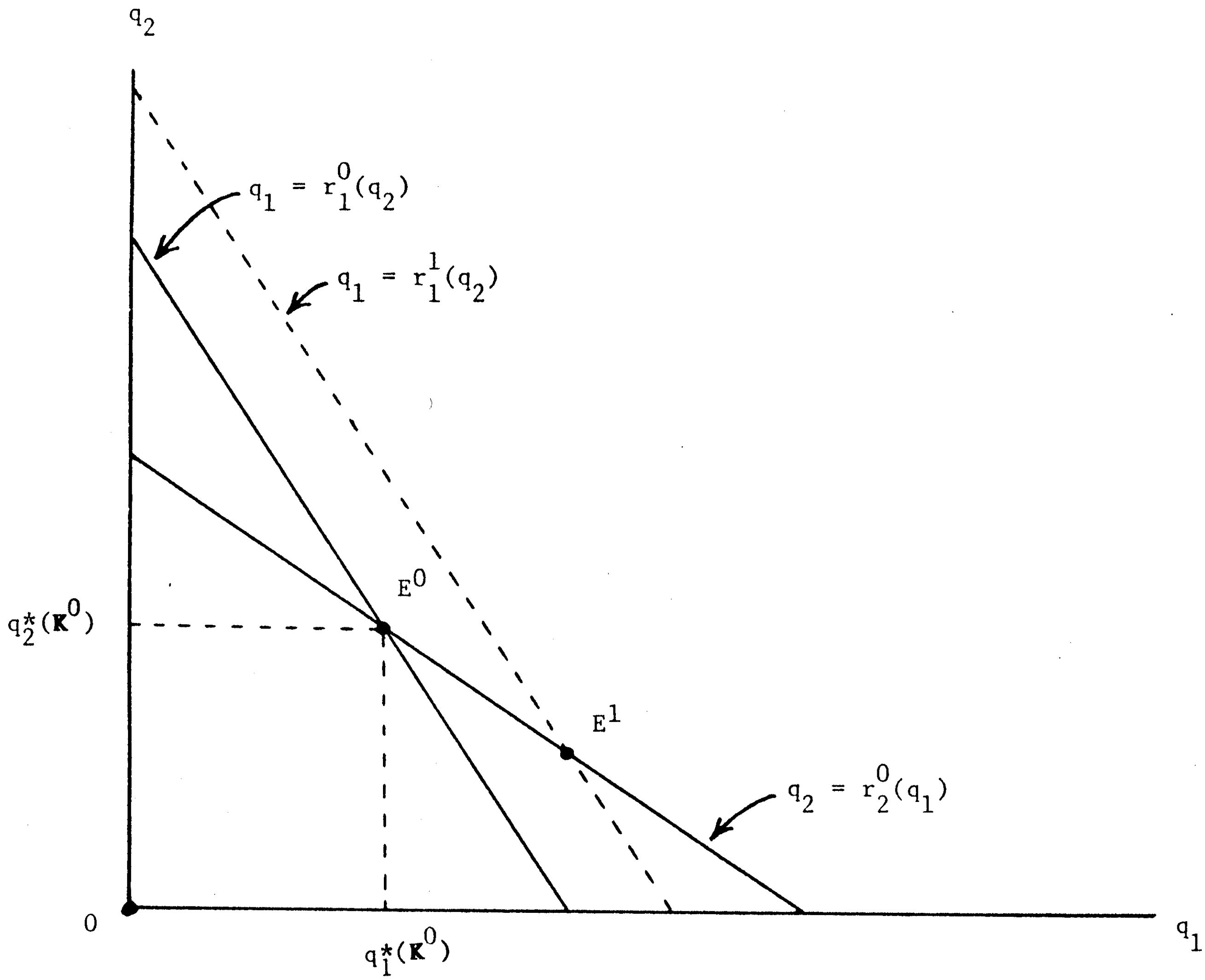


Figure 2: Cumulative Reaction Curve