Game Theory with Application in Economics and Finance

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Duration: 90 mn. No document, no calculator allowed. Answers can be formulated in French or English.

Problem. Running Out of Bank Runs (15 pts)

In 2023, three U.S. banks experienced runs. In one of them, the Silicon Valley Bank, depositors and investors withdrew around USD 42 billion in just one day, marking the second largest collapse in the history of the U.S. banking system. It is considered as the 'first social media bank run'.

There are n depositors. Each makes a unit deposit in the same bank, meaning that total deposits equal the number of depositors n. Denoting $r \in (0,1)$ to be the bank's reserve ratio, the amount $r \times n$ is kept by the bank as its total reserves. The rest of the money is invested and cannot be recouped by the bank before maturity.

All of a sudden, a rumor spreads on social media claiming that many depositors are considering withdrawing their money. All depositors make a simultaneous choice between action W, withdrawing their deposit before maturity, and action L, leaving their deposit at the bank.

We denote n_W to be the number of depositors selecting W, hence $n - n_W$ represents the number of depositors playing L.

- If the total amount of money to be withdrawn by all depositors choosing W does not exceed the total reserves, $n_W \leq r \times n$, we observe business as usual.
  - In such case the depositors playing W get their full deposit 1, and those playing L wait until maturity to receive their deposit with some positive investment return $i > 0$, i.e. $1 + i$ in total.
- If the amount of intended withdrawals exceeds the total reserves, $n_W > r \times n$, the bank experiences a run.
  - In such case those playing W split the available amount of total reserves evenly, each receiving $\frac{r \times n}{n_W}$.
  - Those playing L lose their deposit (we normalize their payoff to zero).

For the purposes of the proofs let us define $\bar{n}_W(r)$ as the maximum number of withdrawals that can be accommodated without triggering a bank run. Consequently, $\bar{n}_W(r) + 1$ is the minimum number of withdrawals that start a run. From these definitions it follows that $\bar{n}_W(r) + 1 > r \times n \geq \bar{n}_W(r)$.

Part A. Two depositors (6 pts)

Assume $n = 2$.

A.1) Assume $r < \frac{1}{2}$.

- ***A.1.a) (1 pt)*** *What is the value of $\bar{n}_W(r)$? Write player 1's corresponding payoff $g_1(x, y)$ associated to any pair of actions $(x, y) \in \{L, W\}^2$.*
- ***A.1.b) (1 pt)*** *What is the set of pure-strategy Nash equilibrium?*
- ***A.1.c) (1 pt)*** *What is the set of Pareto efficient outcomes?*
- ***A.1.d) (1 pt)*** *Draw the corresponding payoff matrix and report your previous answers using arrows and the symbols (N) and (P) to indicate the outcomes that are Nash equilibrium and/or Pareto efficient.*

A.2) (2 pts) *Assume $r \geq \frac{1}{2}$. Same questions as before.*

Part B. More than two depositors (5 pts)

Assume n depositors, with $n \geq 3$.

B.1) (1 pt) *Let $s_{-i} \in \{W, L\}^{n-1}$ denote the profile of actions chosen by other depositors than player i, and $n_W(s_{-i})$ denote the corresponding number of depositors (other than player i) selecting W. What is player i's best response correspondence when $n_W(s_{-i}) < \bar{n}_W(r)$? (Hint: it suffices to compare $g_i(s_{-i}, W)$ to $g_i(s_{-i}, L)$.)*

B.2) (1 pt) *Same question when $n_W(s_{-i}) = \bar{n}_W(r)$.*

B.3) (1 pt) *Same question when $n_W(s_{-i}) > \bar{n}_W(r)$.*

B.4) (1 pt) *What is the maximal number of pure-strategy Nash equilibrium?*

B.5) (1 pt) *Give a condition on the rate r guaranteeing that a bank run cannot be an equilibrium outcome.*

Part C. Incorporating deposit insurance (4 pts)

Over a hundred countries have an explicit deposit insurance scheme. In our framework, the scheme will consist of the government's guarantee of a minimum payout ratio, $I \in [0, 1]$. This is the portion of the deposit that depositors receive in the case of a bank run – from the bank and the government combined. If a bank run occurs, the government simply tops up the amount a depositor receives from the bank to the I level. Formally, if a bank run occurs a depositor who selected L (resp. W) receives I (resp. $\max\{I, \frac{rn}{n_W}\}$).

Assume again $n = 2$ and $r < \frac{1}{2}$.

C.1) (2 pts) *Draw the corresponding payoff matrix and report the symbols (N) and (P) to indicate the outcomes that are Nash equilibrium and/or Pareto efficient.*

The collapse of Silvergate Capital, Silicon Valley Bank, and Signature Bank within the same week of March 2023 initiated a huge loss of confidence in regional, midsize, and small US banks. In such a context, withdrawing by transferring one's savings from one bank to another becomes risky (the alternative option to withdraw a huge amount of cash is risky as well). Let $\gamma > 0$ denote the cost associated to action W.

The US Treasury, Federal Reserve, and Federal Deposit Insurance Corporation (FDIC) announced a systemic risk exception to guarantee uninsured deposits, irrespective of a significant portion of these deposits exceeding the official limit of \$250,000 and thus being usually ineligible for FDIC insurance. Assume now the government's guarantee is full: $I = 1$.

C.2) (2 pts) *Draw the corresponding payoff matrix and report the symbols (N) and (P) to indicate the outcomes that are Nash equilibrium and/or Pareto efficient.*

Exercise. Infinitely repeated Prisoner's dilemma (5 pts)

Consider the following indefinitely repeated Prisoner's Dilemma, with $\alpha \geq 0$ modifying the incentives for deviation from cooperation as follows. Both players have the same discount factor $\delta \in (0, 1)$.

$1\backslash 2$	t_2	c_2
t_1	$1, 1$	$4+\alpha, 0$
c_1	$0, 4+\alpha$	$3, 3$

1) (3 pts) *Characterize the smallest threshold $\bar{\delta}$ beyond which mutual cooperation is sustainable at the equilibrium of infinitely repeated game using a grim trigger strategy.*

2) (2 pts) *How does this threshold vary with α? Interpret.*