Arbitrage and Pricing – Exam

Université Paris Dauphine-PSL - Master 1 I.E.F. (272)

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April 2024. Duration : 1h30. No document allowed. Calculator allowed.
Answers can be formulated in English or French.

Part A : Valuable strike of an American option (4 pts)

Consider a one-period binomial model for the stock-price movement over the following year. The current stock price is S_0. The "up" and "down" moves are u and d, with $d < e^r < u$, where $r \geq 0$ denote the (continuously compounded) risk-free interest rate. Consider American call options on this stock with the expiration date at the end of the period/year and strike $K \geq dS_0$.

A.1) (3 pts) *What is the maximal strike price $\bar{K}$ for which there is early exercise ?*

A.2) (1 pt) *Compute such a value (rounded to the nearest euro) for the numerical application : $S_0 = 100$ euros, $r = 2\%$, $u = 1.08$, $d = 0.96$.*

Part B : Forward tree (9 pts)

The risk-free (continuously compounded) interest rate is $r > 0$. Consider one share of a non-dividend-paying stock that is currently worth S_0, with volatility (which corresponds to the annualized standard deviation of the realized returns on the stock) equal to σ. The volatility parameter to a time period of length t is then $\sigma\sqrt{t}$.

B.1) (1 pt) *What is the forward price for delivery of one share of the stock at time T, denoted as $F_{0,T}(S)$?*

The values S_u^1 and S_d^1 of the stock, in the up and down states-of-the-world respectively, in the forward tree are modeled so that the return of the forward contract is, in a sense, centered between the returns for the up and down states. We set $S_u^1 = F_{0,T}(S)e^{\sigma\sqrt{T}}$ and $S_d^1 = F_{0,T}(S)e^{-\sigma\sqrt{T}}$.

B.2) (1 pt) *What is the value of the corresponding up and down moves u and d that depict the possible evolution of the underlying asset ?*

B.3) (1 pt) *What assumption needs to be made on σ so that the no-arbitrage condition with respect to u, d, and r holds for the binomial asset pricing model ?*

B.4) (1 pt) *What are the values of the ratios S_u^1/S_d^1 and $\frac{u}{d}$?*

B.5) (1 pt) *What is the expression for the risk-neutral probability p in the forward tree ? Using the ratio $\frac{u}{d}$ from the previous answer, give an expression of this probability of the form $p = \frac{1}{x+1}$, where x denotes a parameter you must find.*

B.6) (1 pt) *What is the limit of p as $T \to 0$?*

Consider a non-dividend-paying stock with a current price of 70 per share. Its volatility is given to be 0.25. The continuously-compounded, risk-free interest rate equals 4%.

B.7) (3 pts) *What is the price of a one-year, at-the-money European call option on this stock consistent with the above stock-price model, denoted as C_0 ?*

Part C : Put–call parity and Greeks (5 pts)

Let C_t and P_t denote the values of a european call and put at time t on the same non-dividend paying stock S_t, with same strike price K and maturity T.

C.1) (1 pt) *What is the put-call parity equation at time t ?*

C.2) (1 pt) *Use this equation to derive the relationship between the deltas of the two options. (Hint : the explicit values of the corresponding deltas are not required, just use the notation $\Delta(C)$ and $\Delta(P)$ to mention them.)*

C.3) (1 pt) *Same question for the gammas.*

C.4) (1 pt) *Same question for the vegas.*

C.5) (1 pt) *Same question for the thetas.*

Part D : Multiple Choice Questions (2 pts)

D.1) (1 pt) *What is the impact on the value of European call option prices if the company decides to increase the dividend yield paid to the shareholders ?*

(a) The call-option price will drop.
(b) The call-option price will increase.
(c) The call-option price will always remain constant.
(d) The impact on the price of the call cannot be determined using the binomial option pricing model.
(e) There is not enough information provided.

D.2) (1 pt) *Which of the following American-type options will never be exercised early to get strictly higher profit ?*

(a) Put on a dividend-paying stock
(b) Call on a dividend-paying stock
(c) Call on a non-dividend-paying stock
(d) Put on a non-dividend-paying stock
(e) All of the above should be exercised early in a certain scenario.