Industrial Organization - Solution to the Final Exam

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22 pts = 20 pts + 2 bonus pts

Duration: 90 mn. No document, no calculator allowed.

Exercise A. Repeated Monopolistic Competition (9 pts)

A.1) (2 pts) For any given pair of competitor prices (p_j, p_k), firm i's optimal price p_i maximizes the profit:

$$p_i \times q_i(p_i) = p_i \times (100 - 3p_i + \sum_{j \neq i} p_j) = p_i \times \left(100 + \sum_{j \neq i} p_j\right) - 3p_i^2$$

From the F.O.C., the profit is maximized at p_i^* such that:

$$\frac{\partial}{\partial p_i}\left(p_i^* \times \left(100 + \sum_{j \neq i} p_j\right) - 3p_i^{*2}\right) = 0$$

That is:

$$100 + \sum_{j \neq i} p_j - 6p_i^* = 0$$

This profit function is strictly concave with a maximum in

$$p_i^* = \frac{100 + \sum_{j \neq i} p_j}{6} \tag{1}$$

A Nash equilibrium solves the system:

$$\begin{cases} p_1^* = \frac{100+p_2^*+p_3^*}{6} \\ p_2^* = \frac{100+p_1^*+p_3^*}{6} \\ p_3^* = \frac{100+p_1^*+p_2^*}{6} \end{cases}$$

Summing these three equalities we obtain:

$$p_1^* + p_2^* + p_3^* = \frac{300 + 2(p_1^* + p_2^* + p_3^*)}{6}$$

So $p_1^* + p_2^* + p_3^* = \frac{300}{4} = 75$. From (1), for any $i \in \{1, 2, 3\}$ we have:

$$7p_i^* = 100 + p_i^* + p_j^* + p_k^* = 175$$

Therefore, there is a unique solution, given by $p_1^* = p_2^* = p_3^* = \frac{175}{7} = 25$. At equilibrium, each firm produces $q_i = 100 - 3p_i^* + \sum_{j \neq i} p_j^* = 100 - 75 + 50 = 75$. The corresponding profit is $p_i^* \times q_i(p_i^*) = 75 \times 25 = 1875$.

A.2) (2 pts) The sum of the three firms' profits is a symmetric function of p_1, p_2, and p_3. Solving for the maximum can be done by replacing p_1, p_2, and p_3 with a symmetric price p. The problem is then to solve $\max_p 3p(100 - p)$. The solution is given by $p = 50$. The total profit is then $3 \times 50(100 - 50) = 7500$, and each firm's profit is 2500. Observe that this "cooperative" solution is not immune against unilateral profitable deviation (it is not a Nash equilibrium).

A.3) (2 pts) In the infinitely repeated game, consider the following i's trigger strategy:

At $t = 1$, $p_i = 50$ ("cooperative" price);

at $t > 1$, $p_i = 50$ if 50 is the only price that has been observed from all firms in the past; otherwise, charge $p_i = 25$.

A.4) (3 pts) Let us show that there are some values of δ_i such that the strategy profile where each firm adopts the previous trigger strategy supports a SPNE. Under such a strategy profile, by unilaterally deviating at stage t, firm i obtains at max the amount $\max_{p_i \geq 0} p_i(200 - 3p_i) = \frac{10000}{3}$, for $p_i = \frac{100}{3}$. Then for all subsequent stages, firm i obtains at max 1875. Hence, firm i's deviation is not profitable if:

$$2500 \sum_{k=t}^{\infty} \delta_i^k \geq \frac{10000}{3} \delta_i^t + 1875 \sum_{k=t+1}^{\infty} \delta_i^k$$

That is:

$$2500 \frac{\delta_i^t}{1 - \delta_i} \geq \frac{10000}{3} \delta_i^t + 1875 \frac{\delta_i^{t+1}}{1 - \delta_i}$$

which writes as:

$$2500 \geq \frac{10000}{3}(1 - \delta_i) + 1875\delta_i$$

so we get $\delta_i \geq \frac{10000 - 7500}{10000 - 5625} = \frac{4}{7}$. Therefore, when $\delta_i \geq \frac{4}{7}$ for all $i = 1, 2$, and 3, "cooperation" relying on such a trigger strategy is sustainable as an equilibrium of the infinitely repeated game.

Now, let us check that this equilibrium is perfect. From the previous paragraph, we already know that all three players adopt an equilibrium behavior in any subgame that belongs to the path of "cooperation". In addition, any subgame that does not belong to the path of "cooperation" triggers a punishment behavior that consists of charging the price of 25 for all firms. Such pricing corresponds to a Nash equilibrium of the stage game.

Exercise B. Socially Excessive R&D in Patent Race (13 pts)

B.1) (2 pts) A firm that successfully engages in R&D while its rival does not choose a price p^m, or quantity q^m, that maximizes its profit

$$\pi^m = \max_{q \in \mathbb{R}^+} p(q) \times q = (1 - q) \times q = -q^2 - q$$

That is, $q^m = p^m = \frac{1}{2}$ and $\pi^m = \frac{1}{4}$.

The resulting consumer surplus is worth

$$CS^m = \int_0^{q^m} (1-Q)dQ - p^m \times q^m = \left[Q - \frac{Q^2}{2}\right]_0^{\frac{1}{2}} - \left(\frac{1}{2}\right)^2 = \frac{1}{8}.$$

B.2) (2 pts) Under Bertrand competition, a firm that successfully engages in R&D as its rival replicate p.p.c. At equilibrium, it charges at marginal cost and makes zero profit.

The resulting price is $p^B = 0$, the profit is $\pi^B = 0$, and the aggregate output is $Q^B = 1$.

The resulting consumer surplus is worth

$$CS^B = \int_0^{Q^B} (1-Q)dQ - p^B \times Q^B = \left[Q - \frac{Q^2}{2}\right]_0^1 = \frac{1}{2}.$$

B.3) (2 pts) Under Cournot competition, a firm that successfully engages in R&D as its rival chooses a quantity q^C that maximizes its profit given its rival output $\bar{q}$:

$$q^C \in \arg\max_{q \in \mathbb{R}^+} (1 - (q^C + \bar{q}))q^C$$

The F.O.C. gives $q^C(\bar{q}) = \frac{1-\bar{q}}{2}$. Note that the S.O.C. is satisfied: $\frac{\partial^2 \pi_i}{\partial q_i^2} = -2 < 0$.

The Cournot equilibrium firm's output satisfies:

$$q^C(\bar{q}(q^C)) = q^C$$

From $\bar{q}(q^C) = \frac{1-q^C}{2}$, this yields to $q^C = \frac{1}{3}$.

The resulting aggregate output, and price are $Q^C = 2q^C = \frac{2}{3}$, $p^C = 1 - Q^C = \frac{1}{3}$, and profit $\pi^C = p^C \times q^C = \frac{1}{9}$.

The resulting consumer surplus is worth

$$CS^C = \int_0^{Q^C} (1-Q)dQ - p^C \times Q^C = \left[Q - \frac{Q^2}{2}\right]_0^{\frac{2}{3}} - \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}.$$

B.4) (1 pt) Under Bertrand competition, both firms conducting R&D is a Nash equilibrium if

$$\rho^2 \pi^B + \rho(1-\rho)\pi^m - f \geq 0$$

That is $f \leq \frac{\rho(1-\rho)}{4} \equiv f_2^B$.

B.5) (2 pts) Under Cournot competition, both firms conducting R&D is a Nash equilibrium if

$$\rho^2 \pi^C + \rho(1-\rho)\pi^m - f \geq 0$$

That is $f \leq \frac{\rho^2}{9} + \frac{\rho(1-\rho)}{4} = \rho\frac{9-5\rho}{27} \equiv f_2^C$. We have $f_2^C > f_2^B$, so this condition is less demanding than the one obtained under Bertrand competition.

In particular, when $f \in (f_2^B; f_2^C)$ the R&D cost is:

- sufficiently low for both firms conducting R&D at equilibrium under Cournot competition,
- but it is too high for them conducting R&D under Bertrand competition.

This comes from the expected profits resulting from a firm's decision to engage in R&D which – assuming zero profit when renouncing to R&D (irrespective of Bertrand or Cournot market) – are higher under Cournot competition.

B.6) (2 pts) From society's point of view, it is optimal to have one research division rather than two under Bertrand competition if:

$$\rho(\pi^m + CS^m) - f \geq \rho^2(\pi^B + CS^B) + 2\rho(1-\rho)(\pi^m + CS^m) - 2f$$

That is $f \geq \rho^2 CS^B + \rho(1-2\rho)(\pi^m + CS^m) = \frac{\rho^2}{2} + \frac{3}{8}\rho(1-2\rho) = \rho\frac{3-2\rho}{8} \equiv f_2^{B,pub}$.

Excessive R&D is observed when $f \in (f_2^{B,pub}; f_2^B)$ so that the R&D cost is:

- sufficiently high for having one research division rather than two being optimal; and
- sufficiently low for both firms conducting R&D being a Nash equilibrium.

Here, we have $f_2^B = \rho\frac{2-2\rho}{8} \leq \rho\frac{3-2\rho}{8} = f_2^{B,pub}$ for any level of probability ρ. So, there is no levels of R&D cost f and probability ρ that lead to an excessive R&D compared to what is socially optimal.

B.7) (2 pts) From society's point of view, it is optimal to have one research division rather than two under Cournot competition if:

$$\rho(\pi^m + CS^m) - f \geq \rho^2(\pi^C + CS^C) + 2\rho(1-\rho)(\pi^m + CS^m) - 2f$$

That is $f \geq \rho^2(\pi^C + CS^C) + \rho(1-2\rho)(\pi^m + CS^m) = \frac{3}{9}\rho^2 + \frac{3}{8}\rho(1-2\rho) = \rho\frac{9-10\rho}{24} \equiv f_2^{C,pub}$.

Excessive R&D is observed when $f \in (f_2^{C,pub}; f_2^C)$.

The condition $f_2^C = \rho\frac{9-5\rho}{27} > \rho\frac{9-10\rho}{24} = f_2^{C,pub}$ is equivalent to $\rho > \frac{9}{50} = 0.18$.

So, given the probability of success is not too small ($\rho > 0.18$), any level of R&D cost satisfying $f \in (f_2^{C,pub}; f_2^C)$ leads to an excessive R&D compared to what is socially optimal.