Arbitrage&Pricing
Paris Dauphine University - Master IEF (272)

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Additional Exercises from Exam

Exercise 1. There are two periods, $t \in \{0, 1\}$. There are two assets. One non-risky asset (money that can be borrowed or lend) that returns $r = 2\%$ with discrete compounding at time 1. And one risky asset which is a stock of price $S_0 = 20$ at time 0. At date 1, there is either an upward or a downward move. The price of the stock is then either $S_1^u = 24$ or $S_1^d = 19$.

Suppose the market price of an European put option on the stock with strike 22€ at time 0 is 2.25€.

a) What should be the non-arbitrage price of the put option at date 0?

b) Construct an arbitrage portfolio that uses one unit of the put option.

Exercise 2. Consider a stock whose price starts at $S_0 = 67$€ and evolves according to a two-steps binomial tree where each upward (resp. downward) move increases (resp. decreases) the value by 5%. The risk-free interest rate is 2% and is continuously compounded. At date $t = 0$, a financial institution issues two derivatives that each matures at time $t = 2$.

According to the underlying contracts, the buyer of the first (resp. second) derivative has the right to buy one unit of the stock at time $t = 2$ (resp. at any time $t \in \{0, 1, 2\}$), for a price $0.45\,(S_2 - 66) + 50$ (resp. $0.45\,(S_t - 66) + 50$).

a) Draw the binomial tree that depicts the evolution of the stock price through time t, with $t \in \{0, 1, 2\}$.

b) Draw the binomial tree that depicts the evolution of the first derivative no-arbitrage price, denoted as E_t, through time t, with $t \in \{0, 1, 2\}$.

c) Draw the binomial tree that depicts the evolution of the second derivative no-arbitrage price, denoted as A_t, through time t, with $t \in \{0, 1, 2\}$.

Exercise 3. Assume that a non-dividend paying stock has an expected return of μ and a volatility of σ with the log return of the stock price been normally distributed.

A financial institution has just announced that it will trade a derivative that pays off an euro amount equal to $\ln S_{T_1}$ at time T_2 where S_{T_1} denotes the values of the stock price at time

$T_1 > 0$, and time T_2 gives rise to an inherent payment delay, $T_2 - T_1 > 0$.

We denote by r the per-period and continuously compounded risk-free interest rate.

a) What is the price, f, of the derivative at time $t \in [0, T_2]$ according to a risk-neutral valuation?

b) Is the price, f, of the derivative continuous at time T_1? Prove your statement.

c) Verify that your price satisfies the Black-Scholes-Merton differential equation:

$$\Theta + rS\Delta + \frac{\sigma^2}{2} S^2 \Gamma = rf$$

d) Suppose the risk-free interest rate is 2.5%, and the volatility is 18%. The derivative has been originated thirty periods ago and has a remaining life of a fifty periods. The inherent payment delay is ten periods. Assume the stock price is currently 24€. Give the no-arbitrage current price of the derivative.