Industrial Organization - Exercises Chapter 1

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Exercise 1: Bertrand equilibrium with subadditive different costs (Dastidar, Economics Letters, 2011)

Consider a simultaneous move price choice game in a homogeneous product, asymmetric cost duopoly. The cost function for firm i, $i = 1, 2$, when producing quantity $q \in [0; 1]$ is $C_i(q) = \bar{c_i} + c_i q$ if $q > 0$ and zero otherwise, where $\bar{c_i} \in (0; \frac{(1-c_i)^2}{4})$ (resp. $c_i \in (0; 1)$) is the fixed (resp. variable) cost of production, so that the cost function is strictly subadditive (i.e., $C_i(x + y) < C_i(x) + C_i(y)$ for any two quantities x and y).

Firm i chooses which price p_i to quote in the interval $[0, 1]$. There is a demand function for the lowest posted price p, $D(p) = 1 - p$.

In price competition firms have to meet the demand that they face at the posted price. The firm which quotes the lowest price gets all the demand. Any firm which quotes a price higher than its rival gets no demand. If there is a tie at any price, the two firms share the demand equally.

1) *How does write $\pi_i(p_i, p_j)$ the profit going to firm i when it quotes price p_i and its competitor charges price p_j?*

2) *What price p_i^m would firm i quote if it was a monopoly? Which assumption does guarantee that firm i's monopoly profit is strictly positive?*

3) *What price $\tilde{p_i}$ is firm i's* "monopoly breakeven price" *(i.e., the price at which firm i's monopoly profit is zero and just below (resp. above) which it is negative (resp. positive))?*

4) *Can we have altogether $\tilde{p_1} > p_2^m$ and $\tilde{p_2} > p_1^m$? Why?*

5) *Suppose $\tilde{p_2} \geq p_1^m$. Give a pure strategy Bertrand equilibrium (p_1^*, p_2^*). Prove that there is no unilateral profitable deviation.*

6) *Conversely, suppose $\tilde{p_1} \geq p_2^m$. Give a pure strategy Bertrand equilibrium (p_1^*, p_2^*). (The proof is not required.)*

7) *Suppose $\tilde{p_1} \neq \tilde{p_2}$, $\tilde{p_1} < p_2^m$, and $\tilde{p_2} < p_1^m$. We want to show that there is no pure strategy Bertrand equilibrium (p_1^*, p_2^*). Find a unilateral profitable deviation in each of the following case. (Without loss of generality, when $p_1^* \neq p_2^*$ we will assume that $p_1^* < p_2^*$.)*

7.a) *$p_1^* = p_2^* < \tilde{p_i}$ for at least one firm i.*

7.b) *$p_1^* = p_2^* > p_i^m$ for at least one firm i.*

7.c) *$p_1^* = p_2^* \geq \max\{\tilde{p_1}, \tilde{p_2}\}$.*

7.d) *$\tilde{p_2} < p_1^* < p_2^*$.*

7.e) *$p_1^* < p_2^*$ and $p_1^* \leq \tilde{p_2}$.*

8) *Suppose again $\tilde{p_1} \neq \tilde{p_2}$, $\tilde{p_1} < p_2^m$, and $\tilde{p_2} < p_1^m$. Without loss of generality, assume $\tilde{p_1} < \tilde{p_2}$. We want to show that there is a Bertrand equilibrium (p_1^*, p_2^*) that relies on a firm 2's mixed strategy that consists in randomizing uniformly on the interval $[\tilde{p_2}; \tilde{p_2} + a]$, with $a > 0$ small enough.*

8.a) *Does $p_1^* \geq p_2 \tilde{+} a$? Explain.*

8.b) *Does $p_1^* < \tilde{p_2}$? Explain.*

8.c) *Does $p_1^* \in (\tilde{p_2}; \tilde{p_2} + a)$? Explain. (Hint: assume $p_1^* \in (\tilde{p_2}; \tilde{p_2} + a)$, compute firm 1's expected payoff and show that it has a unilateral profitable deviation when the parameter a is small enough.)*

8.d) *So what is firm 1's best response?*

Exercise 2: Is a horizontal merger privately and/or socially beneficial ? (Liu and Wang, Economics Letters, 2015)

Consider an industry with $m + k$ symmetric firms producing homogeneous goods at a constant marginal cost $\bar{c} \in (0, 1)$. Firm $i = 1, ..., m$ is an insider that may agree to merge and firm $i = m + 1, ..., m + k$ is an outsider. The inverse market demand is linear and is given by $P(Q) = 1 - Q$, where P is price and Q is industry output.

Conditional on the insiders merge or not, these firms compete in different modes. If all firms behave non-cooperatively, i.e., there is no merger, we assume that these firms compete like Cournot oligopolists. If the m insiders merge, we assume that the merger results in two changes. First, the insiders act like a single firm and choose their aggregate output to maximize their joint profits. Second, the merger obtains a strategic advantage of becoming the leader in the market (Stackelberg competition).

Part A. Cournot competition

A.1) *Give the symmetric equilibrium of the Cournot competition in which the equilibrium output of the ith firm,* $i = 1, ..., m+k$, *and the industry output are represented as* $q_i^c = q^c$ *and* $Q^c = (m + k)q^c$, *respectively.*

A.2) *What are the corresponding market price and ith firm profit* ($i = 1, ..., m + k$), *represented as* P^c *and* $\pi_i^c = \pi^c$, *respectively ?*

Part B. Stackelberg competition

We consider now the game with the following sequence of moves and assumption. In the first stage, the insiders merge and choose their aggregate output q^m to maximize their joint profits. In the second stage, each outsider simultaneously chooses its output to maximize its own profit. We assume the outsiders output choice is symmetric, so that $q_i = q^o$, for all $i = m + 1, ..., m + k$.

B.1) *Give the subgame perfect Nash equilibrium (SPNE) of this game in which the industry output is represented as* $Q^S = q^m + kq^o$.

B.2) *What are the associated industry output* Q^s, *profit of the merger* π^m, *and profit of each outsider* π^o *?*

Part C. Profits and consumer welfare effects of a leading merger

C.1) *Is becoming a leader and merging profitable to the insiders ? Why ?*

C.2) *Under what condition is the merger profitable to the outsiders ?*

C.3) *While a merger generally changes all firms' outputs, consumers care about the changes in industry output only. Under what condition does the industry output (and the resulting consumer welfare) increase ?*

C.4) *Conclude.*