

# Arbitrage&Pricing

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### Exercises Chapter 4

**Exercise 1** *What is the price of a European Call option on a non-dividend paying stock when the stock price is 26€, the strike price is 25€, the risk-free interest rate is 10% per annum, the volatility is 30% per annum, and the time to maturity is three months?*

**Exercise 2** *What is the price of a European Put option on a non-dividend paying stock when the stock price is 69€, the strike price is 70€, the risk-free interest rate is 5% per annum, the volatility is 35% per annum, and the time to maturity is six months?*

**Exercise 3** *Assume that a non-dividend paying stock has an expected return of  $\mu$  and a volatility of  $\sigma$  with the log return of the stock price been normally distributed.*

*Prove that a 95% confidence interval for  $S_T$  is given by  $(S_0e^{(\mu-\frac{1}{2}\sigma^2)T-1.96\sigma\sqrt{T}}; S_0e^{(\mu-\frac{1}{2}\sigma^2)T+1.96\sigma\sqrt{T}})$ .*

**Exercise 4** *Assume that a non-dividend paying stock has an expected return of  $\mu$  and a volatility of  $\sigma$  with the log return of the stock price been normally distributed.*

*A financial institution has just announced that it will trade a derivative that pays off an euro amount equal to  $\ln S_T$  at time  $T$  where  $S_T$  denotes the values of the stock price at time  $T$ .*

a) *What is the price,  $f$ , of the derivative at time  $t$  in term of the stock price,  $S$ , at time  $t$  according to a risk-neutral valuation? (We denote by  $r$  the risk-free interest rate.)*

b) *Verify that your price satisfies the Black-Scholes-Merton differential equation:*

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{\sigma^2}{2} S^2 \frac{\partial^2 f}{\partial S^2} = rf$$