Arbitrage&Pricing
Paris Dauphine University - Master IEF (272)

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Exercises Chapter 1

Exercise 1 *There are two periods, $t \in \{0, 1\}$.*

There are two assets. One non-risky asset (money that can be borrowed or lend) that returns $r = \frac{1}{4}$ at time 1.

And one risky asset which is a stock of price $S_0 = 4$ at time 0. At date 1, a coin is tossed. The price of the stock at time 1 is $S_1 = \begin{cases} 8 \ \text{if Head} \\ 2 \ \text{if Tail} \end{cases}$.

Assume our initial wealth is (money) $x = 1.2$€ and we want to buy at time 0, $\Delta_0 = \frac{1}{2}$ shares of the stock.

(a) How much do we have to borrow?

(b) What is our in debt, I_t, to the money market, at time $t \in \{0, 1\}$?

(c) What is our stock value, $\Delta_0 S_t$, at time $t \in \{0, 1\}$?

(d) What is our portfolio value, X_t, at time $t \in \{0, 1\}$?

(e) At time 1, what is the value of an European call option with underlying asset S and strike $K = 5$?

(f) Compare the result of (d) with the one obtained in (e).

(g) Under NAO, what is the value of our European call option at time 0?

Exercise 2 *Consider Exercise 1. Suppose the price of the option at time 0 is 1.21€. Construct an arbitrage portfolio that uses one unit of the option.*

Exercise 3 *Consider Exercise 1. Suppose the price of the option at time 0 is 1.19€. Construct an arbitrage portfolio that uses one unit of the option.*

Exercise 4 *Consider a financial market with a money account, a stock, and an European call option on the stock with strike price $K = 98$.*

Suppose at time $t = 0$, the stock price is $S_0 = 100$ and the price at time $t = 1$ is either

$S_1(H) = 112$ or $S_1(T) = 84$.

Suppose the interest rate on the money account is $r = 5\%$.

We want to obtain a no-arbitrage price for the call option.

The following table lists the payoff structure of the financial market.

	$t = 0$	$t = 1, T$	$t = 1, H$
Money	1	1.05	1.05
Stock	100	84	112
Option	?	$(84 - 98)^+ = 0$	$(112 - 98)^+ = 14$

Under NAO, what is the price of the option at time $t = 0$?

Exercise 5 Suppose the price of the option in Exercise 4 is 5€.

Construct the arbitrage that uses one unit of the option.

Exercise 6 Suppose the price of the option in Exercise 4 is 15€.

Construct the arbitrage that uses one unit of the option.