Arbitrage&Pricing

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LEDa

Chapter 5

Chapter 5: The Greek Letters
Outline

Chapter 5: The Greek Letters
Outline

Introduction

- This chapter covers what are commonly referred to as the “Greek letters”, or simply the “Greeks”.
- Each Greek letter measures a different dimension to the risk in an option position and the aim of a trader is to manage the Greeks so that all risks are acceptable.
  - It measures the sensitivity of the value of a portfolio to a small change in a given underlying parameter.
  - So that component risks may be treated in isolation, and the portfolio rebalanced accordingly to achieve a desired exposure.
- The analysis presented in this chapter is applicable to market makers in options on an exchange as well as to traders working in the over-the-counter market for financial institutions.

Chapter 5: The Greek Letters
Outline

Black-Scholes formula at time t

Theorem (Black-Scholes-Merton Formula for Call at time t)

The price of European call at time t, C_t, write as

$$C_t = S_t \mathcal{N}(d_1) - Ke^{-r(T-t)}\mathcal{N}(d_2)$$

where S_t is the spot price of a non-dividend-paying stock,

$$d_1 \equiv \frac{\ln(\frac{S_t}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{(T-t)}}$$

and

$$d_2 \equiv \frac{\ln(\frac{S_t}{K}) + (r - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{(T-t)}} = d_1 - \sigma\sqrt{(T-t)}.$$

Black-Scholes formula at time t

- Similarly, with the same d_1 and d_2 of the previous Theorem, we obtain the price of a Put with similar characteristics.

Corollary (Black-Scholes-Merton Formula for Put at time t)

The price of European put at time t, P_t,, write as

$$P_t = Ke^{-r(T-t)}\mathcal{N}(-d_2) - S_t\mathcal{N}(-d_1)$$

Chapter 5: The Greek Letters
Outline

Delta
Definition

Definition

Delta (Δ) measures the rate of change of the theoretical option value with respect to changes in the underlying asset's price. It is the first derivative of the value of the option with respect to the underlying entity's price.

- For instance, the delta of a European call option on a non-dividend-paying stock is

$$\Delta\left(C_t\right)=\frac{\partial C_t}{\partial S_t}$$

Delta
The expression of Delta

- The expression of Δ writes as

$$\begin{aligned}\Delta\left(C_t\right) &= \frac{\partial C_t}{\partial S_t}=\frac{\partial}{\partial S_t}\left(S_t\mathcal{N}(d_1)-Ke^{-r(T-t)}\mathcal{N}(d_2)\right)\\ &= \mathcal{N}(d_1)+S_t\mathcal{N}'(d_1)\frac{\partial d_1}{\partial S_t}-Ke^{-r(T-t)}\mathcal{N}'(d_2)\frac{\partial d_2}{\partial S_t}\end{aligned}$$

where $\mathcal{N}'(x)$ is the density function for a standardized normal distribution, that is,

$$\mathcal{N}'(x)\equiv\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}.$$

Delta
The expression of Delta

- From $d_2=d_1-\sigma\sqrt{(T-t)}$ we have

$$\mathcal{N}'(d_2)=\mathcal{N}'(d_1-\sigma\sqrt{(T-t)})=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(d_1-\sigma\sqrt{(T-t)}\right)^2}$$

with

$$\begin{aligned}\left(d_1-\sigma\sqrt{(T-t)}\right)^2 &= d_1^2-2d_1\sigma\sqrt{(T-t)}+\sigma^2\left(T-t\right)\\ &= d_1^2-2\left(\ln(\frac{S_t}{K})+(r+\frac{\sigma^2}{2})\left(T-t\right)\right)\\ &\quad +\sigma^2\left(T-t\right)\\ &= d_1^2-2\left(\ln(\frac{S_t}{K})+r\left(T-t\right)\right)\end{aligned}$$

Delta
The expression of Delta

- So that

$$\begin{aligned}\mathcal{N}'(d_2) &= \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}+\left(\ln(\frac{S_t}{K})+r(T-t)\right)}=\mathcal{N}'(d_1)e^{\ln(\frac{S_t}{K})+r(T-t)}\\ &= \mathcal{N}'(d_1)\frac{S_t}{Ke^{-r(T-t)}}.\end{aligned}$$

- Hence

$$S_t\mathcal{N}'(d_1)=Ke^{-r(T-t)}\mathcal{N}'(d_2) \tag{1}$$

  - We shall use again this result in the expression of Θ.

Delta
The expression of Delta

- Now, from $\frac{\partial d_1}{\partial S_t} = \frac{\partial d_2}{\partial S_t}$ and (1) we obtain

$$S_t \mathcal{N}'(d_1)\frac{\partial d_1}{\partial S_t} = Ke^{-r(T-t)}\mathcal{N}'(d_2)\frac{\partial d_2}{\partial S_t}$$

- Therefore

$$\Delta(C_t) = \mathcal{N}(d_1) + \left(S_t\mathcal{N}'(d_1)\frac{\partial d_1}{\partial S_t} - Ke^{-r(T-t)}\mathcal{N}'(d_2)\frac{\partial d_2}{\partial S_t}\right) = \mathcal{N}(d_1)$$

  - Similarly, the delta of a European put option on a non-dividend-paying stock is

$$\Delta(P_t) = \frac{\partial P_t}{\partial S_t} = \frac{\partial}{\partial S_t}\left(Ke^{-r(T-t)}\mathcal{N}(-d_2) - S_t\mathcal{N}(-d_1)\right) = -\mathcal{N}(-d_1)$$

Delta
Example

Example (A)

Consider a call option on a non-dividend-paying stock where the stock price is \$49, the strike price is \$50, the risk-free rate is 5%, the time to maturity is 20 weeks, and the volatility is 20%. In this case,

$$S_0 = 49;\ K = 50;\ r = 5\%;\ \sigma = 20\%;\ \text{and } T = 0.3846 \text{ (i.e., 20 weeks)}$$

So

$$d_1 = \frac{\ln(\frac{49}{50}) + (0.05 + \frac{0.2^2}{2})0.3846}{0.2\sqrt{0.3846}} = 0.0542$$

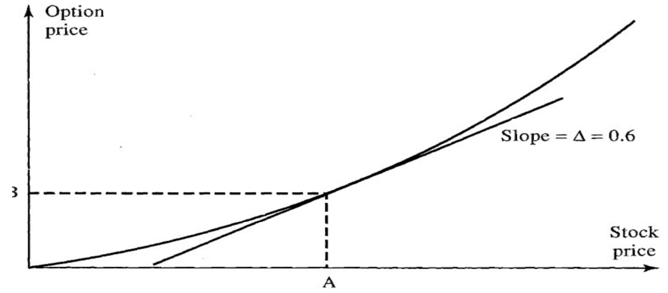
The option's delta is

$$\Delta(C_t) = \mathcal{N}(d_1) = \mathcal{N}(0.0542) = 0.522$$

When the stock price changes by ΔS, the option price changes by $0.522\Delta S$.

Delta
Interpretation

- Suppose that the delta of a call option on a stock is 0.6.
  - This means that when-the stock price changes by a small amount, the option price changes by about 60% of that amount.

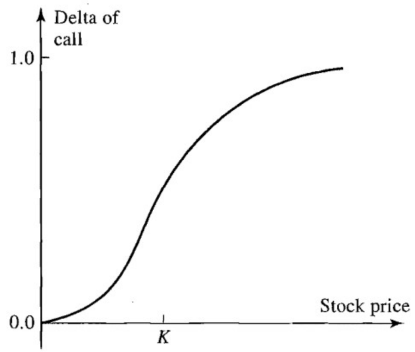


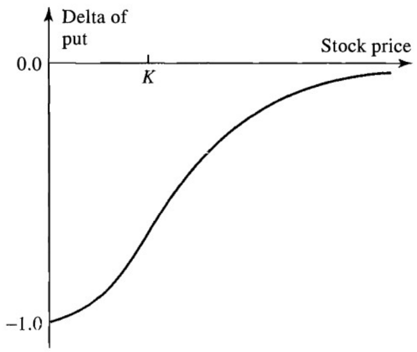
- As we have seen in previous chapters, the investor's position from having shorted an option could be hedged by buying Δ shares of the underlying asset.

Delta
Dynamic Delta Hedging

- The convexity of the curve in the previous figure has an intuitive interpretation:
  - Delta, which is the slope of the curve, is increasing with the stock price.
  - This is so because the higher the stock price at a given point in time, the more likely the European call option will be exercised at maturity.
    - At an extreme case, when the stock price is far away above the strike, the option will be exercised with probability one at maturity. So $\Delta = 1$.
    - At the opposite case, when the stock price is close to zero, the option will be exercised with probability zero at maturity. So $\Delta = 0$.
- The same reasoning hold with respect to a put, but in opposite direction.
  - When the stock price is far away above the strike, the option will be exercised with probability zero at maturity. So $\Delta = 0$.
  - When the stock price is close to zero, the option will be exercised with probability one at maturity. So $\Delta = -1$.

Delta
Dynamic Delta Hedging





Delta
Dynamic Delta Hedging

Definition
The **intrinsic value** is the amount of money the holder of the option would gain by exercising the option immediately.

- So a call with strike \$50 on a stock with price \$60 would have intrinsic value of \$10, whereas the corresponding put would have zero intrinsic value.

Definition
An option without any intrinsic value is said to be **out-of-the-money**.

- A call (resp. put) option is out-of-the-money when the strike price is above (resp. below) the current trading price of the underlying security.

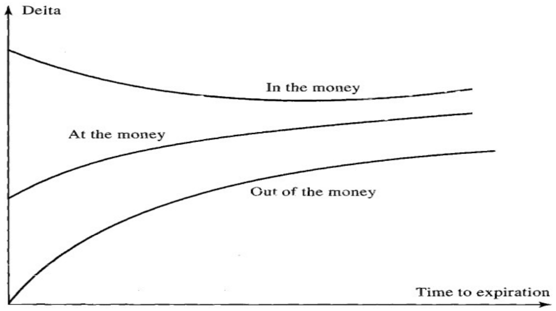
Delta
Dynamic Delta Hedging

Definition
An option that has a strike price that is equal to the current trading price of the underlying security is said to be **at-the-money**.

Definition
An option with intrinsic value is said to be **in-the-money**.

- A call (resp. put) option is in-the-money when the strike price is below (resp. above) the current trading price of the underlying security.

Delta
Dynamic Delta Hedging



Typical patterns for variation of delta with time to maturity for a call option.

- Moving from the right to the left on the horizontal axis has the interpretation to reduce the time to expiry.
  - At point zero, the option expires and the $\Delta = 1$ (resp. $\Delta = 0$) if the underlying security of the call option is in-the-money (resp. out-of-the-money).

Chapter 5: The Greek Letters
Outline

Theta
Definition

Definition

Theta (Θ) measures the rate of change of the theoretical option value with respect to the passage of time with all else remaining the same.

It is the first derivative of the value of the option with respect to the time.

- Theta is sometimes referred to as the **time decay**.
- The theta of a European call (resp. put) option on a non-dividend-paying stock is then $\Theta = \frac{\partial C_t}{\partial t}$ (resp. $\Theta = \frac{\partial P_t}{\partial t}$).
- With all else remaining the same: We assume S_t does not vary with t.

Theta
The expression of Theta

Question

What is the expression of $\frac{\partial C_t}{\partial t}$?

- From
$$C_t = S_t\mathcal{N}(d_1) - Ke^{-r(T-t)}\mathcal{N}(d_2)$$
we have
$$\begin{aligned}\Theta(C_t) &= \frac{\partial C_t}{\partial t} = \frac{\partial}{\partial t}\left(S_t\mathcal{N}(d_1) - Ke^{-r(T-t)}\mathcal{N}(d_2)\right) \\ &= S_t\mathcal{N}'(d_1)\frac{\partial d_1}{\partial t} - rKe^{-r(T-t)}\mathcal{N}(d_2) - Ke^{-r(T-t)}\mathcal{N}'(d_2)\frac{\partial d_2}{\partial t}\end{aligned}$$

Theta
The expression of Theta

- That is
$$\Theta(C_t) = S_t\mathcal{N}'(d_1)\frac{\partial d_1}{\partial t} - Ke^{-r(T-t)}\mathcal{N}'(d_2)\frac{\partial d_2}{\partial t} - rKe^{-r(T-t)}\mathcal{N}(d_2)$$
- According to (1), we have
$$S_t\mathcal{N}'(d_1) = Ke^{-r(T-t)}\mathcal{N}'(d_2)$$
- So, we obtain
$$\Theta(C_t) = S_t\mathcal{N}'(d_1)\left(\frac{\partial d_1}{\partial t} - \frac{\partial d_2}{\partial t}\right) - rKe^{-r(T-t)}\mathcal{N}(d_2).$$

Theta
The expression of Theta

- Now, we have

$$\frac{\partial d_1}{\partial t} - \frac{\partial d_2}{\partial t} = \frac{\partial}{\partial t}(d_1 - d_2) = \frac{\partial}{\partial t}\left(\frac{\sigma^2 (T-t)}{\sigma\sqrt{(T-t)}}\right) = \frac{\partial}{\partial t}\left(\sigma\sqrt{(T-t)}\right)$$
$$= \sigma\frac{\partial}{\partial t}\left(\sqrt{(T-t)}\right) = -\frac{\sigma}{2\sqrt{(T-t)}}$$

- Hence

$$\Theta(C_t) = -S_t \mathcal{N}'(d_1)\frac{\sigma}{2\sqrt{(T-t)}} - rKe^{-r(T-t)}\mathcal{N}(d_2).$$

Theta
The expression of Theta

- Similarly, the theta of a European put option on a non-dividend-paying stock is

$$\Theta(P_t) = -S_t \mathcal{N}'(d_1)\frac{\sigma}{2\sqrt{(T-t)}} + rKe^{-r(T-t)}\mathcal{N}(-d_2).$$

Theta
Example

Example (A')

Coming back to Example A, we have

$$\Theta(C_0) = -S_0 \mathcal{N}'(d_1)\frac{\sigma}{2\sqrt{T}} - rKe^{-rT}\mathcal{N}(d_2)$$

with

$$S_0 = 49;\ K = 50;\ r = 5\%;\ \sigma = 20\%;\ \text{and } T = 0.3846.$$

and

$$d_1 = 0.0542;\ \text{and } d_2 = \frac{\ln(\frac{49}{50}) + (0.05 - \frac{0.2^2}{2})0.3846}{0.2\sqrt{0.3846}} = -0.0698$$

Theta
Example

Example (A')

The option's theta is

$$\begin{aligned}\Theta(C_0) &= 49\mathcal{N}'(0.0542)\frac{0.2}{2\sqrt{0.3846}} - 0.05 \\ &\quad \times 49e^{-0.05\times 0.3846}\mathcal{N}(-0.0698) \\ &= -4.31\end{aligned}$$

So the theta is $-4.31/365 = -0.0118$ per calendar day, or $-4.31/252 = -0.0171$ per trading day.

Theta
Interpretation

$$\Theta\left(C_t\right) = -S_t\mathcal{N}'(d_1)\frac{\sigma}{2\sqrt{(T-t)}} - rKe^{-r(T-t)}\mathcal{N}(d_2)$$

- Three observations.
- (i) Theta is usually negative for an option.
  - This is because, as time passes with all else remaining the same, the option tends to become less valuable.

Theta
Interpretation

$$\Theta\left(C_t\right) = -S_t\mathcal{N}'(d_1)\frac{\sigma}{2\sqrt{(T-t)}} - rKe^{-r(T-t)}\mathcal{N}(d_2)$$

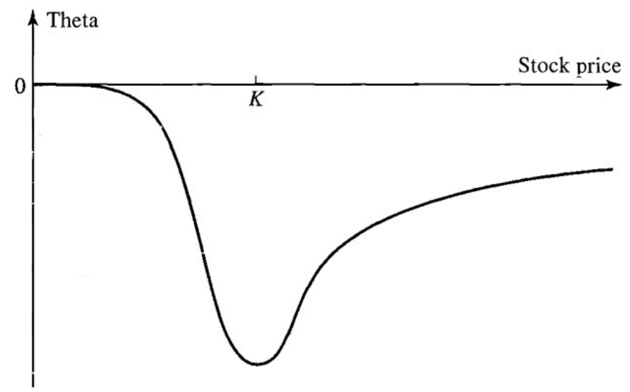
- (ii) When the stock price is very low, theta is close to zero.
  - Indeed, both terms go to zero as
  - for the 1st term, we have $\lim_{S_t\to 0} d_1 = \lim_{S_t\to 0}\left(\frac{\ln S_t}{\sigma\sqrt{(T-t)}} + \frac{-\ln K + (r+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{(T-t)}}\right) = -\infty$ and $\lim_{d_1\to-\infty}\mathcal{N}'(d_1) = \lim_{d_1\to-\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}} = 0$; and
  - for the 2nd term, we have $\lim_{d_2\to-\infty}\mathcal{N}(d_2) = 0$.

Theta
Interpretation

$$\Theta\left(C_t\right) = -S_t\mathcal{N}'(d_1)\frac{\sigma}{2\sqrt{(T-t)}} - rKe^{-r(T-t)}\mathcal{N}(d_2)$$

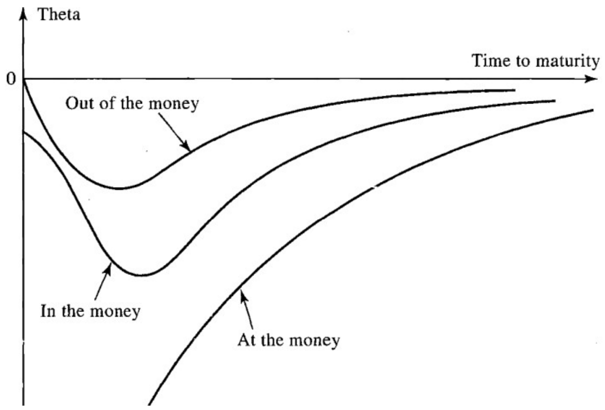
- (iii) As the stock price becomes very large, theta tends to $-rKe^{-rT}$
  - Indeed, for the 1st term, we have $\lim_{S_t\to+\infty} d_1 = \lim_{S_t\to+\infty} d_2 = +\infty$, $\lim_{d_1\to+\infty}\mathcal{N}'(d_1) = \lim_{d_1\to+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}} = 0$; and
  - for the 2nd term, we have $\lim_{d_2\to+\infty}\mathcal{N}(d_2) = 1$.

Theta
Interpretation



Variation of theta of a European call option with stock price.

Theta
Interpretation



Typical patterns for variation of theta of a European call option with time to maturity.

Chapter 5: The Greek Letters
Outline

Gamma
Definition

Definition

Gamma (Γ) measures the rate of change for delta with respect to the underlying asset's price. It is the first (resp. second) derivative of the delta (resp. value of the option) with respect to the underlying entity's price.

- The gamma of a European call on a non-dividend-paying stock is then

$$\begin{aligned}\Gamma(C_t) &= \frac{\partial^2 C_t}{\partial S_t^2} = \frac{\partial}{\partial S_t}(\Delta(C_t)) = \frac{\partial}{\partial S_t}(\mathcal{N}(d_1)) = \mathcal{N}'(d_1)\frac{\partial d_1}{\partial S_t} \\ &= \mathcal{N}'(d_1)\frac{\partial}{\partial S_t}\left(\frac{\ln(\frac{S_t}{K}) + (r+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{(T-t)}}\right) \\ &= \mathcal{N}'(d_1)\frac{1}{S_t\sigma\sqrt{(T-t)}}\end{aligned}$$

Gamma
Definition

- Similarly, the delta of a European put option on a non-dividend-paying stock is

$$\begin{aligned}\Gamma(P_t) &= \frac{\partial^2 P_t}{\partial S_t^2} = \frac{\partial}{\partial S_t}(\Delta(P_t)) = \frac{\partial}{\partial S_t}(\mathcal{N}(-d_1)) = \mathcal{N}'(-d_1)\frac{\partial}{\partial S_t}(-d_1) \\ &= \mathcal{N}'(-d_1)\frac{\partial d_1}{\partial S_t} = \frac{e^{-\frac{(-d_1)^2}{2}}}{\sqrt{2\pi}}\frac{\partial d_1}{\partial S_t} = \mathcal{N}'(d_1)\frac{\partial d_1}{\partial S_t} \\ &= \mathcal{N}'(d_1)\frac{1}{S_t\sigma\sqrt{(T-t)}} = \Gamma(C_t).\end{aligned}$$

Gamma
Example

Example (A")

Coming back to Example A', we have

$$\Gamma(C_0) = \mathcal{N}'(d_1)\frac{1}{S_0\sigma\sqrt{T}}$$

with

$$S_0 = 49;\ d_1 = 0.0542;\ \sigma = 20\%;\ \text{and}\ T = 0.3846.$$

The option's gamma is

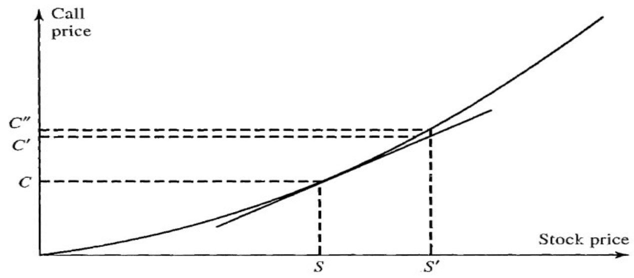
$$\Gamma(C_0) = \frac{1}{\sqrt{2\pi}}e^{-\frac{0.0542^2}{2}}\frac{1}{49 \times 0.2 \times \sqrt{0.3846}} = 0.065$$

When the stock price changes by ΔS, the delta of the option changes by $0.065\Delta S$.

Gamma
Interpretation

- If gamma is small, delta changes slowly, and adjustments to keep a portfolio delta neutral need to be made only relatively infrequently.
- However, if gamma is highly negative or highly positive, delta is very sensitive to the price of the underlying asset.
  - It is then quite risky to leave a delta-neutral portfolio unchanged for any length of time.
- Gamma measures the curvature of the relationship between the option price and the stock price.

Gamma
Interpretation

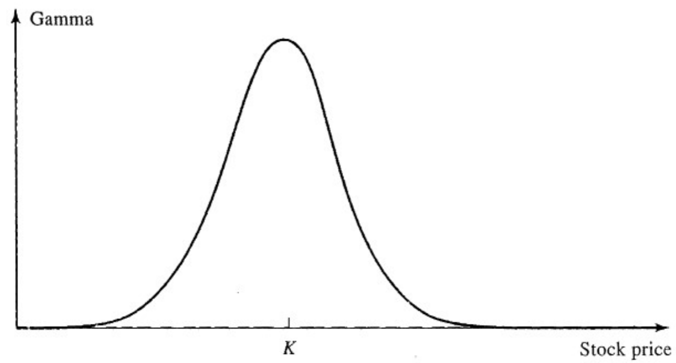


Hedging error introduced by nonlinearity.

- When the stock price moves from S to S' delta hedging assumes that the option price moves from C to C' when in fact it moves from C to C''.
- The difference between C' and C'' leads to a hedging error which is expressed by Γ.

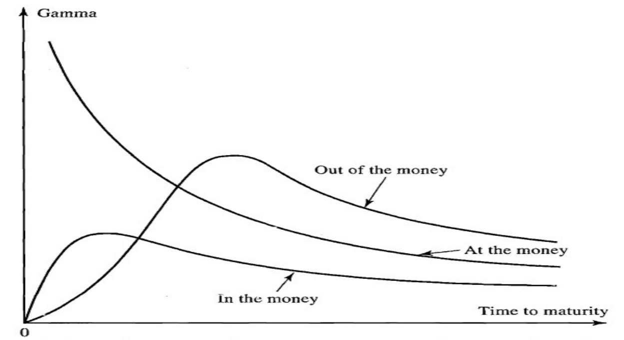
Gamma
Interpretation

- The gamma of a long position is always positive and (by computing the derivative of Γ we can show that it) varies with the stock price in the way indicated in the following figure.



Variation of gamma with stock price for an option.

Gamma
Interpretation



Variation of gamma with time to maturity for a stock option.

- For an at-the-money option, gamma increases as the time to maturity decreases.
- Short-life at-the-money options have very high gammas, which means that the value of the option holder's position is highly sensitive to jumps in the stock price.

Chapter 5: The Greek Letters
Outline

Vega
Definition

- Chapter 3 assumes that the volatility of the asset underlying a derivative is constant. In practice, volatilities change over time.

Definition

Vega (ν) (denoted as the greek letter "nu") measures the option's sensitivity to changes in the volatility of the underlying asset. It represents the amount that an option contract's price changes in reaction to a 1% change in the volatility of the underlying asset.

- The vega of a European call on a non-dividend-paying stock is then

$$\begin{aligned}\nu(C_t) &= \frac{\partial C_t}{\partial \sigma} = \frac{\partial}{\partial \sigma}\left(S_t\mathcal{N}(d_1) - Ke^{-r(T-t)}\mathcal{N}(d_2)\right)\\ &= S_t\mathcal{N}'(d_1)\frac{\partial d_1}{\partial \sigma} - Ke^{-r(T-t)}\mathcal{N}'(d_2)\frac{\partial d_2}{\partial \sigma}\end{aligned}$$

Vega
Definition

- According to (1) we have

$$S_t\mathcal{N}'(d_1) = Ke^{-r(T-t)}\mathcal{N}'(d_2)$$

- Using this here, we obtain

$$\begin{aligned}\nu(C_t) &= S_t\mathcal{N}'(d_1)\left(\frac{\partial d_1}{\partial \sigma} - \frac{\partial d_2}{\partial \sigma}\right) = S_t\mathcal{N}'(d_1)\frac{\partial}{\partial \sigma}(d_1 - d_2)\\ &= S_t\mathcal{N}'(d_1)\frac{\partial}{\partial \sigma}\left(\sigma\sqrt{(T-t)}\right) = S_t\mathcal{N}'(d_1)\sqrt{(T-t)}\end{aligned}$$

  - Similarly, the delta of a European put option on a non-dividend-paying stock is

$$\begin{aligned}\nu(P_t) &= \frac{\partial P_t}{\partial \sigma} = \frac{\partial}{\partial \sigma}\left(Ke^{-r(T-t)}\mathcal{N}(-d_2) - S_t\mathcal{N}(-d_1)\right)\\ &= S_t\mathcal{N}'(d_1)\sqrt{(T-t)} = \nu(C_t).\end{aligned}$$

Vega
Example

Example (A"')

Coming back to Example A, we have

$$\nu(C_0) = S_0 \mathcal{N}'(d_1)\sqrt{T}$$

with

$$S_0 = 49;\ d_1 = 0.0542;\ \text{and } T = 0.3846.$$

The option's vega is

$$\nu(C_0) = 49 \times \frac{1}{\sqrt{2\pi}} e^{-\frac{0.0542^2}{2}} \sqrt{0.3846} = 12.1$$

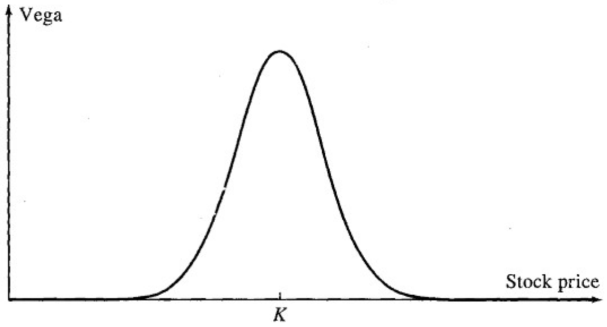
Thus a 1% increase in the volatility (from 20% to 21%) increases the value of the option by approximately $0.01 \times 12.1 = 0.121$.

Vega
Interpretation

- Volatility measures the amount and speed at which price moves up and down, and is often based on changes in recent, historical prices in a trading instrument.
- Vega changes when there are large price movements (increased volatility) in the underlying asset, and falls as the option approaches expiration.

Vega
Interpretation

- The vega of a long position in a European or American option is always positive.
- The general way in which vega varies with the stock price is shown in the next Figure



Variation of vega with stock price for an option.

Chapter 5: The Greek Letters
Outline

Rho
Definition

Definition

Rho (ρ) measures the option's sensitivity to changes in interest rate. It represents the amount that an option contract's price changes in reaction to a 1% change in the risk-free rate of interest with all else remaining the same.

- The rho of a European call on a non-dividend-paying stock is then

$$\begin{aligned}\rho(C_t) &= \frac{\partial C_t}{\partial r} = \frac{\partial}{\partial r}\left(S_t\mathcal{N}(d_1) - Ke^{-r(T-t)}\mathcal{N}(d_2)\right)\\ &= S_t\mathcal{N}'(d_1)\frac{\partial d_1}{\partial r} + (T-t)\,Ke^{-r(T-t)}\mathcal{N}(d_2)\\ &\quad -Ke^{-r(T-t)}\mathcal{N}'(d_2)\frac{\partial d_2}{\partial r}\end{aligned}$$

Rho
Definition

- According to (1), we have

$$S_t\mathcal{N}'(d_1) = Ke^{-r(T-t)}\mathcal{N}'(d_2)$$

- Using this, we obtain here

$$\begin{aligned}\rho(C_t) &= S_t\mathcal{N}'(d_1)\left(\frac{\partial d_1}{\partial r} - \frac{\partial d_2}{\partial r}\right) + (T-t)\,Ke^{-r(T-t)}\mathcal{N}(d_2)\\ &= S_t\mathcal{N}'(d_1)\frac{\partial}{\partial r}(d_1 - d_2) + (T-t)\,Ke^{-r(T-t)}\mathcal{N}(d_2)\\ &= (T-t)\,Ke^{-r(T-t)}\mathcal{N}(d_2).\end{aligned}$$

  - Similarly, the delta of a European put option on a non-dividend-paying stock is

$$\begin{aligned}\rho(P_t) &= \frac{\partial P_t}{\partial r} = \frac{\partial}{\partial r}\left(Ke^{-r(T-t)}\mathcal{N}(-d_2) - S_t\mathcal{N}(-d_1)\right)\\ &= -(T-t)\,Ke^{-r(T-t)}\mathcal{N}(-d_2).\end{aligned}$$

Rho
Example

Example ($A^{(4)}$)

Coming back to Example A, we have

$$\rho(C_0) = TKe^{-rT}\mathcal{N}(d_2)$$

with

$$K = 50;\ d_2 = -0.0698;\ r = 5\% \text{ and } T = 0.3846.$$

The option's rho is

$$\rho(C_0) = 0.3846 \times 50e^{-0.05\times 0.3846}\mathcal{N}(-0.0698) = 8.91$$

This means that a 1% increase in the risk-free rate (from 5% to 6%) increases the value of the option by approximately $0.01 \times 8.91 = 0.0891$.

Chapter 5: The Greek Letters
Outline

Extension
Asset that provides a yield

Greek letter	Call option	Put option
Delta	$e^{-qT}N(d_1)$	$e^{-qT}[N(d_1)-1]$
Gamma	$\frac{N'(d_1)e^{-qT}}{S_0\sigma\sqrt{T}}$	$\frac{N'(d_1)e^{-qT}}{S_0\sigma\sqrt{T}}$
Theta	$-S_0N'(d_1)\sigma e^{-qT}/(2\sqrt{T})$ $+qS_0N(d_1)e^{-qT}-rKe^{-rT}N(d_2)$	$-S_0N'(d_1)\sigma e^{-qT}/(2\sqrt{T})$ $-qS_0N(-d_1)e^{-qT}+rKe^{-rT}N(-d_2)$
Vega	$S_0\sqrt{T}N'(d_1)e^{-qT}$	$S_0\sqrt{T}N'(d_1)e^{-qT}$
Rho	$KTe^{-rT}N(d_2)$	$-KTe^{-rT}N(-d_2)$

Greek letters for European options on an asset that provides a yield at rate q.

Extension
Forward contract

- Consider a forward contract, with strike K and maturity T, i.e. with payoff at time t given by

$$F(t) = S(t) - Ke^{-r(T-t)}$$

- The Greeks of the forward contract are:

$$\Delta_F = \frac{\partial F}{\partial S} = 1$$

$$\Theta_F = \frac{\partial F}{\partial t} = -rKe^{-r(T-t)}$$

$$\Gamma_F = \frac{\partial^2 F}{\partial S^2} = 0$$

$$\nu_F = \frac{\partial F}{\partial \sigma} = 0$$

and

$$\rho_F = \frac{\partial F}{\partial r} = (T-t)\,Ke^{-r(T-t)}.$$