Industrial Organization

Master Quantitative Economics - 2023/2024
Chapter 3: Tacit Collusion

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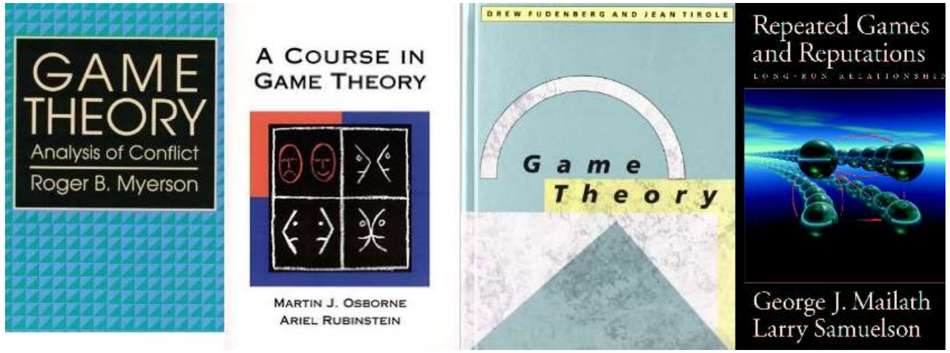
Chapter 3

Tacit Collusion
Outline

Introduction
Bibliography



Introduction
Issue

- Questions:
  - What is the price on a given market?
  - What are the profits?
  - What is the social surplus?

Introduction
Issue

- Answers from the previous chapter, it depends on:
  - How many firms are on the market
    - Monopoly, duopoly, oligopoly, ... , atomless firms.
  - Whether firms are competing on prices or on quantity.
  - Whether there are capacity constraints, decreasing returns to scale, ...
  - Whether there is a temporal dimension
    - Simultaneous moves, sequential moves...
  - Whether there is a good differentiation
    - Horizontal or Vertical.

Introduction
Issue

- In the previous chapter we have assumed that firms interact only once..
- In this chapter, we shall relax this assumption and allow firms to interact repeatedly.

Tacit Collusion
Outline

Repeated Games: Solving Prisoner's dilemma
Introduction

- Fundamental question of Economics and social sciences:
  - How to solve Prisoner's dilemma?
- Recall: Prisoner's dilemma.
  - Two suspects are put in separate cells.
  - If only one of them confess, he will reduce the sentence by four years and he will be used as a witness against the other, who will receive the maximal sentence.
  - If they both confess they reduce the sentence by one year each.
  - If none confess, due to lack of evidence they reduce the sentence by three years each.
  -

	D	C
D	$(1,1)$	$(4,0)$
C	$(0,4)$	$(3,3)$

Repeated Games: Solving Prisoner's dilemma
Introduction

	D	C
D	$(1,1)$	$(4,0)$
C	$(0,4)$	$(3,3)$

Question

What is the set of Nash equilibrium?

Answer

The set of Nash equilibrium is $\{(D,D)\}$.

Repeated Games: Solving Prisoner's dilemma
Finite horizon

Question

Is cooperation sustainable when the game is finitely repeated?

Answer

No!

Repeated Games: Solving Prisoner's dilemma
Introduction

	D	C
D	$(1,1)$	$(4,0)$
C	$(0,4)$	$(3,3)$

Question

What is the set of Pareto optima?

Answer

The set of Pareto optima is $\{(D,C);(C,D);(C,C)\}$.

- In particular, the unique Nash equilibrium (D,D) is Pareto dominated by (C,C).

Repeated Games: Solving Prisoner's dilemma
Finite horizon

Proposition

When the prisoner dilemma is finitely repeated, the only equilibrium is defection at every period.

- Assume the game is repeated T times, with T a finite number.
  - At period T, there is no future interaction and the behavior of this last period has no impact on the past moves.
    - So, each player plays his dominant strategy of the one-shot game: D.
  - At period $T-1$, the behavior of this period has no impact on past moves nor change the last periods behaviors.
    - So, again, each player plays his dominant strategy of the one-shot game: D.
  - By backward induction cooperation never takes place.

Question

What is infinite horizon?

- Our lives are finite.

Answer

***Infinite horizon** means that after each period the players believe that the game may continue for an additional period.*

- Conversely, **finite horizon** model is appropriate if the players clearly perceive a well-defined final period.

- Suppose the number of times the game will be played is a random variable.
  - After each period, the likelihood players will meet and play again is $q \in [0, 1]$.
  - The probability of play continuing for exactly t rounds is

$$q^{t-1}(1-q)$$

Proposition

If $q \geq \sqrt{2} - 1 \simeq 0.414$ then both players cooperating at every period is an equilibrium outcome.

Proof.

Consider the strategy that consists in cooperating (C) at every period until someone defect, and thereafter defecting (D) forever.

As long as both players plan to use this strategy they get each 3 per period.

Over the long run they get

$$\sum_{t=1}^{\infty} q^{t-1}(1-q)\, 3t$$

(...) □

Proof.

If either player i deviated from this strategy and chose to defect then his expected total future payoff from this particular period would be

$$4 + \sum_{t=2}^{\infty} q^{t-1}(1-q)\, t$$

The term

$$\sum_{t=1}^{\infty} q^{t-1}(1-q)\, t$$

can be deducted from the sum of a geometric sequence.(...) □

Repeated Games: Solving Prisoner's dilemma
Infinite horizon

Proof.

Indeed, from the sum of a geometric sequence, we know that

$$\sum_{t=1}^{\infty} q^t a = \frac{aq}{1-q}$$

By computing the derivative of each side of the previous equality, we obtain

$$\sum_{t=1}^{\infty} t q^{t-1} a = \frac{a}{(1-q)^2}$$

(...) □

Repeated Games: Solving Prisoner's dilemma
Infinite horizon

Proof.

So we have the two formula we need to compare the expected payoffs:

$$\sum_{t=1}^{\infty} q^{t-1} (1-q)\, at = \frac{a(1-q)}{(1-q)^2} = \frac{a}{1-q}$$

and

$$\sum_{t=2}^{\infty} q^{t-1} (1-q)\, at = \frac{a}{1-q} - (1-q)\, a$$

Now, by applying the first formula with $a = 3$, and the second one with $a = 1$, we deduce that the expected payoff is:
- $\frac{3}{1-q}$ along the equilibrium path; and
- $4 + \frac{1}{1-q} - (1-q)$ along the path of the deviation. (...) □

Repeated Games: Solving Prisoner's dilemma
Infinite horizon

Proof.

So, the deviation is profitable if and only if

$$\frac{3}{1-q} < 4 + \frac{1}{1-q} - (1-q)$$

that is

$$\frac{1 + (3+q)(1-q)}{1-q} < 0$$

which is equivalent to $q^2 + 2q - 1 < 0$ that is $q < \sqrt{2} - 1 \simeq 0.414$. □

Repeated Games: Solving Prisoner's dilemma
Conclusion

- The cooperative behavior is non sustainable at equilibrium when the game is repeated a finite number of times.
  - That is, when the players clearly perceive a well-defined final period.
- The cooperative behavior is sustainable at equilibrium when the game is repeated an infinite number of times.
  - That is, whenever after each period the players believe that the game may continue for an additional period.

Tacit Collusion
Outline

Repeated Games: Discounting
Definition

Definition

The **discount factor**, $\delta \in (0,1)$, is the factor by which a future payoff (e.g., cash flow) must be multiplied in order to obtain its present value.

Example

Firm's annually-compounded discount factor is typically

$$\delta = \frac{1}{(1+r)^T}$$

where r denotes the annually-compounded real rate of interest and T denotes the number of years.

Repeated Games: Discounting
Introduction

- The future does not have the same value as the present.
  - 1€ today is not equivalent to 1€ tomorrow.
- Trade-offs of today and the future are important in how I will behave today.

Repeated Games: Discounting
Definition

Example

Firm's continuously-compounded discount factor is typically

$$\delta = e^{-r\tau}$$

where r denotes the continuously-compounded real rate of interest and T denotes the lag of time between periods.

Repeated Games: Discounting
Definition

Definition

Given an infinite sequence of payoffs $u_1, u_2, \ldots$ for player i and discount factor δ, i's **future discounted reward** is $\sum_{t=0}^{T} \delta^t u_t$

- The higher δ, the more patient is the player.
  - $\delta = 1$: extremely patient player who value the future as of today.
  - $\delta = 0$: extremely impatient player who does not value the future at all.
- It is sometimes useful to look at the *average payoff* of the repeated game, to compare it with the stage game payoff.
  - This average payoff writes as

$$\sum_{t=0}^{T} (1-\delta)\,\delta^t u_t$$

Repeated Games: Discounting
Definition

- Two equivalent interpretations of the discount factor:
  - The agent cares more about his well-being in the near term than in the long term;
  - The agent cares about the future just as much as the present, but with probability $1-\delta$ the game will end in any given round.

Repeated Games: Discounting
Prisoner's dilemma with discounted payoff

- Consider the following Prisoner's dilemma where defection is more attractive by ε:

	D	C
D	$(1,1)$	$(4+\varepsilon,0)$
C	$(0,4+\varepsilon)$	$(3,3)$

Question

What is the discount factor δ above which cooperation is sustainable in the corresponding infinitely-repeated game?

Repeated Games: Discounting
Prisoner's dilemma with discounted payoff

Proposition

If $\delta \geq \frac{1+\varepsilon}{3+\varepsilon}$ cooperation at every periods is an equilibrium of the infinitely-repeated game.

Proof.

Consider the following strategy:

- Cooperate as long as everyone has in the past;

- Both players defect forever after if anyone ever deviates (Grim Trigger). (...)

□

Repeated Games: Discounting
Prisoner's dilemma with discounted payoff

Proof.

When both players play this strategy, by cooperating a player gets

$$\sum_{t=0}^{+\infty} \delta_t^t \times 3 = \frac{3}{1-\delta}$$

By defecting (deviating from cooperation) a player obtains

$$(4+\varepsilon) + \sum_{t=1}^{+\infty} \delta_t^t \times 1 = (4+\varepsilon) + \frac{\delta}{1-\delta}$$

The first payoff is higher than the second one if $\delta \geq \frac{1+\varepsilon}{3+\varepsilon}$. □

Repeated Games: Discounting
Prisoner's dilemma with discounted payoff

- The threshold above which cooperation is sustainable is clearly increasing with ε ($\frac{\partial}{\partial \varepsilon}\left(\frac{1+\varepsilon}{3+\varepsilon}\right) = \frac{2}{(3+\varepsilon)^2} > 0$).
- Basic logic to sustain cooperation at equilibrium:
  - Play something with relatively high payoffs, and
  - if anyone deviates punish by resorting to something that:
    - has lower payoffs (at least for that player);
    - and is credible: it is an equilibrium in the subgame.

Tacit Collusion
Outline

Repeated Games: Folk Theorem
Introduction

Question

What is a pure strategy in an infinitely-repeated game?

Answer

It is a choice of action at every decision point, that means here an action at every stage game ...which is an infinite number of actions!

- Some famous strategies:
  - **Trigger**: Start out cooperating. If the opponent ever defects, defect forever.
  - **Tit-for-tat**: Start out cooperating. If the opponent defected, defect in the next round. Then go back to cooperation.

Repeated Games: Folk Theorem
Introduction

Question

With an infinite number of pure strategies, what can we say about Nash equilibria?

Answer

We won't be able to construct an induced normal form and then appeal to Nash's theorem to say that an equilibrium exists.

Nash's theorem only applies to finite games.

Furthermore, with an infinite number of strategies, there could be an infinite number of pure-strategy equilibria!

- Instead, we can characterize a set of payoffs that are achievable under equilibrium, without having to enumerate the equilibria.
  - This is the idea of the *folk theorems*.

Repeated Games: Folk Theorem
Introduction

- In mathematics, the term *folk theorem* refers generally to any theorem that is believed and discussed, but has not been published.
- The idea of the folk theorem we are interested here is:
  - If players are sufficiently patient then any outcome that satisfies minimax conditions for every players is an equilibrium.
  - In order that the name of this theorem be more descriptive, Roger Myerson has recommended the terms *general feasibility theorem* in the place of *folk theorem*.
    - ⋆ See Myerson, Roger B. Game Theory, Analysis of conflict, Cambridge, Harvard University Press (1991)
- Intuition:
  - The fact that the game is repeated allows the players to agree on certain sequences of actions, and punish the players that deviate from that sequence.

Repeated Games: Folk Theorem
Introduction

- Folk theorems are partially converse claims: they say that, under certain conditions (that depends on the class of games), every payoff that is both *feasible* and *individually rational* can be realized as an equilibrium payoff profile in the repeated game.
  - We will define the terms *feasible* and *individually rational* later.
- There are various folk theorems.
  - Is the game finitely-repeated or infinitely-repeated?
  - How are the payoffs computed?
    - ⋆ E.g., arithmetic mean, limit of means, discounted, ...
  - What is the solution concept?
    - ⋆ E.g., Nash equilibrium, subgame-perfect, Coalition-Nash, ...

Repeated Games: Folk Theorem
Model

- Consider any n-player stage game $\Gamma =< N, \times_{i\in N} S_i, (u_i)_{i\in N} >$.
  - $N = \{1, ..., n\}$ set of players
  - S_i i's pure strategy (or action) space (in the stage game)
  - $u_i : S_1 \times ... \times S_n \rightarrow \mathbb{R}$ vN-M i's utility function.

Repeated Games: Folk Theorem
Model

- Players repeatedly play the same stage game over a (possibly infinite) time horizon $t = 0, 1, 2, \ldots, T$.
- In each period, player $i \in N$ takes some action $s_i \in S_i$.
- At the end of the period, player $i \in N$ observes his payoff $u_i(\mathbf{s})$
  - where $\mathbf{s} = (s_i, s_{-i})$ results from his action s_i and the actions of others s_{-i}.
- The repetition of the periods produce a history of moves.
  - A history up to time t is the sequence of realized action profiles before t.
  - We denote by $H^t := \left(\mathbf{s}^1, \mathbf{s}^2, ..., \mathbf{s}^{t-1}\right)$ the history at date t.

Repeated Games: Folk Theorem
Model

- Each history H^t define a new game $\Gamma\left(H^t\right)$ starting at H^t.
- Player i's strategy in the repeated game is a complete contingent action plan, which specifies a current action after any history.
  - Φ_i depends on H^T.
  - $$\begin{array}{cccc} \Phi_i^t: & \left(\times_{i\in N} S_i\right)^{t-1} & \mapsto & S_i \\ & H^t & \mapsto & \Phi_i^t\left(H^t\right) \end{array}$$

Repeated Games: Folk Theorem
Model

Definition (solution concept)

$(\Phi_i)_{i\in N}$ is a *Subgame-Perfect Nash equilibrium* if for every H^t the profile $\left(\Phi_i\left(H^t\right)\right)_{i\in N}$ is a Nash equilibrium of $\Gamma\left(H^t\right)$.

- Subgame perfect equilibrium specifies mutual best replies after any history.
- To game theory, we can apply the principle of optimality of dynamic programming: the **one-shot deviation principle**.
  - It was originally formulated by David Blackwell (1965).
    - Blackwell, D. (1965), "Discounted Dynamic Programming," *Annals of Mathematical Statistics* 36, 226–235.
  - Applied here, it says that a strategy profile of a finite extensive-form game is a subgame perfect equilibrium if and only if there exist no profitable one-shot deviations.
  - Ultimately, no player can profit from deviating from the strategy for one period and then reverting to the strategy.

Repeated Games: Folk Theorem
A first folk theorem

- Let $\mathbf{a} = (a_1, ..., a_n)$ be a Nash equilibrium of the stage game Γ.

Question

Does the infinite-repeated interaction allow players to achieve a higher average payoff along the equilibrium path than under $\mathbf{a}$?

E.g., in the prisoner's dilemma, can players achieve a higher payoff than under the repetition of defection?

Answer

Yes!

Repeated Games: Folk Theorem
A first folk theorem

- The first subgame perfect folk theorem shows that any payoff above the static Nash payoffs can be sustained as a subgame perfect equilibrium of the repeated game.

Theorem

If $\mathbf{a}' = (a'_1, ..., a'_n)$ is such that $u_i(\mathbf{a}') > u_i(\mathbf{a})$ for all $i \in N$, then there exists a discount factor $\bar{\delta}$, such that if $\delta_i \geq \bar{\delta}$ for all $i \in N$, then there exists a subgame perfect equilibrium of the infinite repetition of Γ that has $\mathbf{a}'$ played in every period on the equilibrium path.

Repeated Games: Folk Theorem
A first folk theorem

Proof.

[Hint] Use the following non-forgiving grim trigger:

- Play $\mathbf{a}'$ as long as everyone has in the past.

- If any player ever deviates, then play $\mathbf{a}$ forever after (Grim Trigger).

For δ_i sufficiently close to 1, it is better for each player $i \in N$ to obtain $u_i(\mathbf{a}')$ rather than deviate and get a high deviation payoff for one period, and then obtain $u_i(\mathbf{a})$ forever thereafter. □

Repeated Games: Folk Theorem
A first folk theorem

- The previous theorem relies on the use of non-forgiving grim trigger with punishment by the static Nash Equilibrium of the stage game.
  - So, punishment is credible.
- To provide incentive for cooperation in a more general setting, we need credible punishments.
  - This requires that it is always possible to single out individuals for punishment.
  - So, we need no two players having the same preferences over the stage game outcomes.
    - I.e., payoffs that are positive affine transformations of each other.
- Before stating our second folk theorem, we need some definitions.

Repeated Games: Folk Theorem
Definition

Definition (informal)

The **minimax value** of a player is the smallest value that the other players can force the player to receive, *without knowing* his actions.

Definition (formal)

The **minimax value** of player i is $\min_{s_{-i}} \max_{s_i} u_i(s_{-i}, s_i)$.

- Said differently, it is the largest value the player can be sure to get when he *knows* the actions of the other players.
  - The idea of minmax is the one of a maximal punishment applied to a player able to responds with an optimal defense.
  - The punished player knows the punishment (the action played by others) and best-responds to it.

Repeated Games: Folk Theorem
Definition

Example

- $\begin{pmatrix} & L & R \\ U & 1,0 & 2,6 \\ D & 4,-2 & -3,5 \end{pmatrix}$
  - Here, the smallest value that $P2$ can force $P1$ to receive is 2.
    - So, $P1$'s minmax is 2.
  - The smallest value that $P1$ can force $P2$ to receive is 5.
    - So, $P2$'s minmax is 5.

Repeated Games: Folk Theorem
Definition

- Any Nash equilibrium payoff in a repeated game must satisfy two properties:
  - **Individual rationality** (**IR**): the payoff must weakly dominate the minmax payoff profile of the constituent stage game.
    - I.e, the equilibrium payoff of each player must be at least as large as the minmax payoff of that player.
    - This is because a player achieving less than his minmax payoff always has incentive to deviate by simply playing his minmax strategy at every history.
  - **Feasibility**: the average payoff must be a convex combination of possible payoff profiles of the stage game.
    - This is because the average payoff in a repeated game is just a weighted average of payoffs in the basic games.
  - The set of the *feasible and individually rational payoff* corresponds to the set of physically achievable average payoff profiles in the repeated game where each player receives more than her minimax payoff.

Repeated Games: Folk Theorem
A second folk theorem

- Our second folk theorem states that any point in the set of the feasible and individually rational payoff can be an equilibrium outcome of the infinitely repeated game.

Theorem

In a n-player infinitely repeated game, any feasible and individually rational payoff profile can be achieved as the average payoff profile of a subgame perfect equilibrium when the discount factor δ is close enough to 1, provided that either $n = 2$, or $n \geq 3$ and no two players have identical interests.

- Although game theoretic predictions quite often depend on the fine details of the model, this result is a notable exception for its generality.

Repeated Games: Folk Theorem
Conclusion

- In an infinitely-repeated interaction, if players are sufficiently patient, any mutually beneficial outcome can be sustained in an equilibrium.
- The crucial condition in the folk theorem is a high discount factor.
  - For a given player, the shorter the period, the higher the discount factor
    - E.g., 1€ right now is almost equivalent to 1€ in two hours, but not to 1€ in two years.
  - Players who have daily interaction have a better scope for cooperation than those who interact only once a year.
  - An important message of the folk theorem is then that a high frequency of interaction is essential for the success of a long-term relationship.

Repeated Games: Folk Theorem
Conclusion

- For further reading on our first folk theorem in repeated games with discounting, see:
  - Friedman, J. (1971), "A non-cooperative equilibrium for supergames", *Review of Economic Studies* 38 (1): 1–12
- For further reading on our second folk theorem in repeated games with discounting, see:
  - Fudenberg, Drew; Maskin, Eric (1986). "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information". *Econometrica* 54 (3): 533; and
  - Abreu, D., P. Dutta and L. Smith (1994) “The Folk Theorem for Repeated Games: A NEU Condition,” *Econometrica*, 62(4), 939-948.

Tacit Collusion
Outline

Collusion in Prices
Model

- We consider the repeated version of the Bertrand model
- $N = \{1, 2\}$
- Constant and similar marginal cost to produce $C_i(q_i) = cq_i$.
- Firms (simultaneously) compete in price $T + 1$ times.
- Firm i's profit at date t, with $t = 0, 1, ..., T$, is denoted by: $\pi^i\left(p_t^i, p_t^j\right)$.

Collusion in Prices
Model

- Discounted value of firm i's profit:

$$\sum_{t=0}^{T} \delta^t \pi^i\left(p_t^i, p_t^j\right)$$

  - $\delta \in (0, 1)$: discount factor
  - The higher is δ, the more patient is the firm
  - E.g., $\delta = e^{-r\tau}$, where r is the instantaneous interest rate and τ is the real time between periods.

Collusion in Prices
Model

- $H_t := \left(p_0^1, p_0^2; p_1^1, p_1^2; ...; p_{t-1}^1, p_{t-1}^2\right)$ history at date t.
- Each history H_t define a new game $\Gamma(H_t)$ starting at H_t.
- Firm i's strategy Φ^i depends on H_T.
  - $\Phi_t^i(H_t) = p_t^i$
- (Φ^1, Φ^2) is a Subgame-Perfect Nash equilibrium if for every H_t $\left(\Phi_{H_t}^1, \Phi_{H_t}^2\right)$ is a Nash equilibrium of $\Gamma(H_t)$.

Collusion in Prices
Finite horizon

- Assume $T < +\infty$

Proposition

In finite horizon the only equilibrium is the Bertrand one: there is no collusion.

Proof.

We proceed by backward induction.

At the last period T, because the past prices do not affect the profits in period T, each firm ought to maximize its "static profit" $\pi^i\left(p_T^i, p_T^j\right)$ given its rival's price. So $p_T^1 = p_T^2 = c$.

Now, since prices choices at period T do not depend on what happens at period $T-1$, everything occurs as if $T-1$ was the last period.

Thus for any H_{T-1} we have $p_{T-1}^1 = p_{T-1}^2 = c$. □

Collusion in Prices
Infinite horizon

- Assume $T = +\infty$
- Denote p^m as the monopoly price.

Proposition

In infinite horizon, if $\delta \geq \frac{1}{2}$ then any $p \in [c, p^m]$ can be sustained as a (time-invariant) equilibrium.

Proof.

Let $\bar{p} \in [c, p^m]$. Consider the following trigger strategies.

Each firm plays, for every t:

$$p_t^i(H_t) = \begin{cases} \bar{p} & \text{if} \quad H_t = ((\bar{p}, \bar{p}); ...; (\bar{p}, \bar{p})) \\ c & \text{otherwise} \end{cases}$$

(...) □

Collusion in Prices
Infinite horizon

Proof.

By conforming to $\bar{p}$, each firm gets

$$\frac{\pi(\bar{p})}{2} \sum_{t=0}^{+\infty} \delta^t$$

By deviating at date k, a firm would get at most $\pi(\bar{p})$ at period k, so its deviating payoff is at most

$$\frac{\pi(\bar{p})}{2} \sum_{t=0}^{k-1} \delta^t + \pi(\bar{p}) \delta^k + 0 \sum_{t=k+1}^{+\infty} \delta^t$$

(...) □

Collusion in Prices
Infinite horizon

Proof.

So, such a deviation is profitable if

$$\frac{\pi(\bar{p})}{2}\sum_{t=k}^{+\infty}\delta^t \geq \pi(\bar{p})\delta^k \iff \sum_{t=k+1}^{+\infty}\delta^t \geq \delta^k$$
$$\iff \frac{\delta^{k+1}-\delta^{+\infty}}{1-\delta} \geq \delta^k \iff \delta^{k+1} \geq \delta^k - \delta^{k+1}$$
$$\iff \frac{\delta^{k+1}}{\delta^k} = \delta \geq \frac{1}{2}.$$

□

Collusion in Prices
Infinite horizon

- From previous Proposition, any symmetric per-period profit between 0 and π^m can be an equilibrium joint profit (each firm can earn half of this profit).
- The per-period profit writes as

$$(1-\delta)\sum_{t=0}^{+\infty}\delta^t\pi^i\left(p_t^i, p_t^j\right)$$

  - Observe that if $\pi^i\left(p_t^i, p_t^j\right) = \pi^i$ for every t, then

$$(1-\delta)\sum_{t=0}^{+\infty}\delta^t\pi^i\left(p_t^i, p_t^j\right) = (1-\delta)\pi^i\sum_{t=0}^{+\infty}\delta^t$$
$$= (1-\delta)\frac{1-\delta^{+\infty}}{(1-\delta)}\pi^i = \pi^i.$$

Collusion in Prices
Infinite horizon

Proposition (Folk Theorem)

In infinite horizon, any pair of profits $\left(\pi^1, \pi^2\right)$ such that $\pi^1 > 0$, and $\pi^2 > 0$ and $\pi^1 + \pi^2 \leq \pi^m$ is a per-period equilibrium payoff for δ sufficiently close to 1.

Proof.

Let $p \in [c, p^m]$ such that $\pi(p) = \pi^1 + \pi^2$, and let $\pi^1 = \alpha\pi(p)$, $\pi^2 = (1-\alpha)\pi(p)$.

Consider the ratio $\frac{\alpha}{1-\alpha}$. Let $\frac{m}{n}$ denote a rational (i.e., $m, n \in \mathbb{N}$) approximation of the real number $\frac{\alpha}{1-\alpha}$ (i.e., $\alpha = \frac{m}{m+n}$). (...) □

Collusion in Prices
Infinite horizon

Proof.

Consider the following strategies.

- During the m first periods firm: $\begin{cases} 1 & \text{charges} & p \\ 2 & \text{charges} & p' > p \end{cases}$;

- For the n subsequent periods firm: $\begin{cases} 1 & \text{charges} & p' > p \\ 2 & \text{charges} & p \end{cases}$;

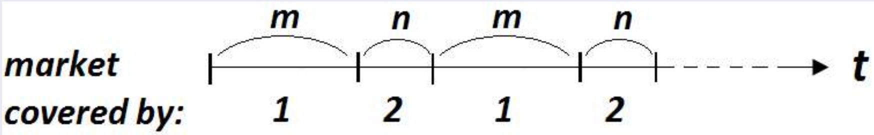
- For the m subsequent periods firm: $\begin{cases} 1 & \text{charges} & p \\ 2 & \text{charges} & p' > p \end{cases}$; ... and so on.

- If any one deviates the firms charge the marginal cost c forever.

(...) □

Collusion in Prices
Infinite horizon

Proof.



Since δ is close to 1, these strategies clearly form an equilibrium. (...) □

Collusion in Prices
Infinite horizon

Proof.

Furthermore, the per-period payoff for firm 1 is

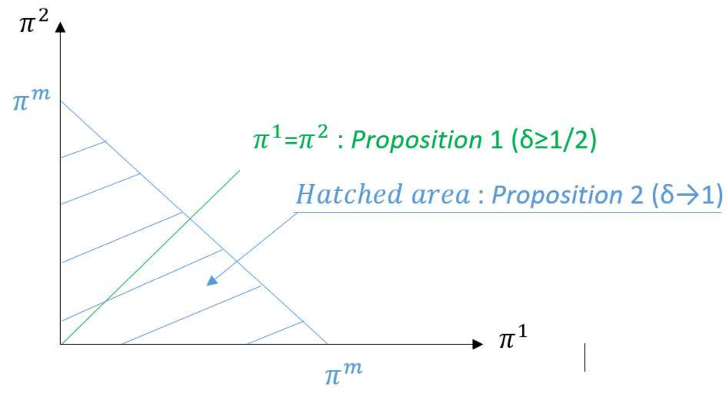
$$(1-\delta)\,\pi(p)\left[\left(1+\delta+...+\delta^{m-1}\right)+\left(\delta^{m+n}+...+\delta^{2m+n-1}\right)+...\right]$$
$$=\frac{1+\delta+...+\delta^{m-1}}{1+\delta+...+\delta^{n+m-1}}\pi(p)$$

which, for δ sufficiently close to 1 writes as

$$\frac{m}{m+n}\pi(p)\simeq\alpha\pi(p).$$

□

Collusion in Prices
Infinite horizon



Collusion in Prices
Infinite horizon

Proposition

For $\delta<\frac{1}{2}$ the only equilibrium profit is the Bertrand one: competitive zero profit.

Proof.

Clearly, both firms charging prices at marginal costs at every date regardless of competitor's behavior is an equilibrium.

Let us show that this equilibrium is unique.

Let $\delta<\frac{1}{2}$ and assume, *per contra*, there is an equilibrium with non zero profit for at least one firm.

Let $\bar{\pi}$ be the highest possible per-period equilibrium market profit.

The maximal punishment a firm can be subjected to, results in zero future profits. □

Proof.

There is a date k, where by deviating at this date, a firm earns at least from date k

$$(\bar{\pi} - \varepsilon)\,\delta^k + 0 \times \sum_{t=k+1}^{+\infty} \delta^t$$

Now, let i be the firm with the lowest equilibrium long-term payoff starting from date k.

From date k, by definition of $\bar{\pi}$, firm i earns at most

$$\frac{\bar{\pi}}{2} \sum_{t=k}^{+\infty} \delta^t$$

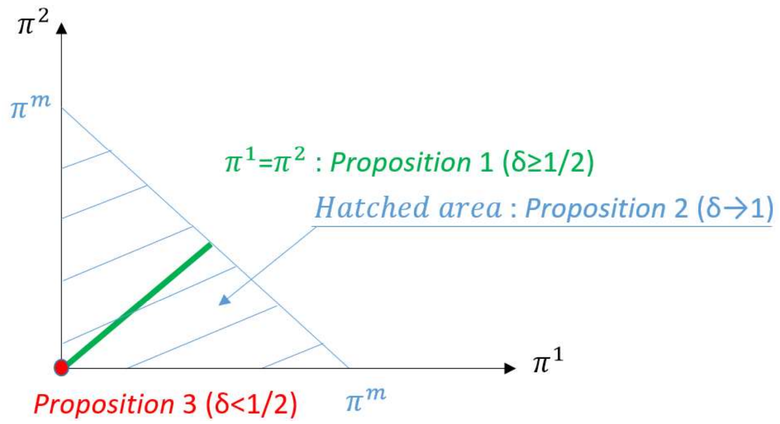
□

Proof.

Firm i's deviation at date k is then non profitable only if

$$\begin{aligned}
(\bar{\pi} - \varepsilon)\,\delta^k &\leq \frac{\bar{\pi}}{2} \sum_{t=k}^{+\infty} \delta^t = \frac{\bar{\pi}}{2} \frac{\delta^k - \delta^{+\infty}}{1-\delta} \sim \frac{\bar{\pi}}{2} \frac{\delta^k}{1-\delta} \\
&\iff (1-\delta)(\bar{\pi} - \varepsilon) \leq \frac{\bar{\pi}}{2} \\
&\iff \frac{\bar{\pi}}{2} - \varepsilon \leq \delta(\bar{\pi} - \varepsilon) \\
&\iff \delta \geq \frac{\frac{\bar{\pi}}{2} - \varepsilon}{\bar{\pi} - \varepsilon}
\end{aligned}$$

But RHS approaches $\frac{1}{2}$ as ε goes to zero, a *contradiction*. □



Conclusion

- When competition in price is finite there is no collusion (Bertrand result)
  - Backward induction: Selten Paradox.
- When competition in price is infinite, collusion depends on the discount factor.
  - If discount factor is low (impatient firms, high real interest rate), no collusion (Bertrand result)
  - If discount factor is medium-high, collusion on symmetric profits.
  - If discount factor is very high (very patient firms, very low real interest rate), collusion on any (possibly asymmetric) profits.

Tacit Collusion
Outline

Collusion and market concentration
Result

- When all firms charge the monopoly price (i.e., "fully collusive outcome") and share the market equally, the per firm profit is:

$$\frac{\pi^m}{n}\sum_{t=0}^{T}\delta^t = \frac{\pi^m}{n}\times\frac{1-\delta^T}{1-\delta} \underset{T\to+\infty}{\longrightarrow} \frac{\pi^m}{n\left(1-\delta\right)}$$

  - Clearly, this term is decreasing in n.
  - So, a large number of firms reduces the profit per firm and thus the cost of being punished for undercutting.
- This reasoning on the fully collusive outcome (monopoly) can be extended to any other collusive outcome.

Collusion and market concentration
Model

Question

What is the relation between collusion and the number of firms?

- $N = \{1, 2, , ..., n\}$
- Constant and similar marginal cost to produce $C_i\left(q_i\right) = cq_i$.
- Firms (simultaneously) compete in price $T+1$ times.

Collusion and market concentration
Result

Proposition

In infinite horizon, collusion is sustainable only if $\delta \geq \frac{n-1}{n}$.

Collusion and market concentration
Result

Proof.

Assume, *per contra*, there is an equilibrium that sustains collusion with $\delta < \frac{n-1}{n}$.

Let $\bar{\pi}$ be the highest possible per-period equilibrium market profit.

The maximal punishment a firm can be subjected to, results in zero future profits.

There is a date k, where by deviating at this date, a firm earns at least from date k

$$(\bar{\pi} - \varepsilon)\,\delta^k + 0 \times \sum_{t=k+1}^{+\infty} \delta^t$$

□

Collusion and market concentration
Result

Proof.

Now, let i be the firm with the lowest equilibrium long-term payoff starting from date k.

A strictly positive collusive profit requires the collusion of all firms.

From date k, by definition of $\bar{\pi}$, firm i earns at most

$$\frac{\bar{\pi}}{n}\sum_{t=k}^{+\infty} \delta^t$$

□

Collusion and market concentration
Result

Proof.

Firm i's deviation at date k is then non profitable only if

$$\begin{aligned}
(\bar{\pi} - \varepsilon)\,\delta^k \quad & \leq \quad \frac{\bar{\pi}}{n}\sum_{t=k}^{+\infty} \delta^t = \frac{\bar{\pi}}{n}\frac{\delta^k - \delta^{+\infty}}{1-\delta} \sim \frac{\bar{\pi}}{n}\frac{\delta^k}{1-\delta} \\
& \iff \quad (1-\delta)\,(\bar{\pi} - \varepsilon) \leq \frac{\bar{\pi}}{n} \\
& \iff \quad \bar{\pi}\left(\frac{n-1}{n}\right) - \varepsilon \leq \delta\,(\bar{\pi} - \varepsilon) \\
& \iff \quad \delta \geq \frac{\bar{\pi}\left(\frac{n-1}{n}\right) - \varepsilon}{\bar{\pi} - \varepsilon}
\end{aligned}$$

But RHS approaches $\frac{n-1}{n}$ as ε goes to zero, a *contradiction*. □

Tacit Collusion
Outline

1. Introduction
2. Repeated games
3. Collusion in Prices
4. Collusion and market concentration
5. Collusion and fluctuating demand
   - Introduction
   - Model
   - Result
   - Extension
   - Conclusion

Collusion and fluctuating demand
Introduction

Question

What is the impact of demand fluctuations on collusive behavior?

- Rotemberg, J., and G. Saloner (1986), "A Supergame-Theoretic Model of Business Cycles and Price Wars during Booms," *American Economic Review*, 76:390-407.
  - Random fluctuating demand.
- Haltiwanger, J. and J. Harrington (1991), "The impact of cyclical demand movements on collusive behavior," *Rand Journal of Economics*, 22:89-106.
  - Deterministic cyclical demand.

Answer

Collusion is less sustainable in markets that are subject to demand fluctuations.

Collusion and fluctuating demand
Introduction

- The idea, formally captured by Rotenberg and Saloner (1986) and Haltiwanger and Harrington (1991), is that when the market is at a peak, short-term gains from a deviation are maximal while the potential cost of retaliation is at a minimum.
- Hence, collusion is more difficult to sustain in those times.

Collusion and fluctuating demand
Model

- $N = \{1, 2\}$.
- Repeated version of the Bertrand model (simultaneous competition in price) but with fluctuating demand.
- Random fluctuating demand:
  - Demand shocks are independent and identically distributed across periods.
  - At each period t with probability $\frac{1}{2}$, the demand is low $q = D_L(p)$ (resp. high $q = D_H(p)$).
  - Full collusion in period of low (resp. high) demand yields to low (resp. high) monopoly market profits π_L^m (resp. π_H^m).
    - ⋆ Namely, for $\theta \in \{L, H\}$, we have $\pi_\theta^m := (p_\theta^m - c) D_\theta(p_\theta^m)$ where $p_\theta^m \in \arg\max (p - c) D_\theta(p)$
    - ⋆ $\pi_L^m < \pi_H^m$.

Collusion and fluctuating demand
Model

- At each period, firms learn the current state of demand before choosing their prices simultaneously.
  - So they get $\frac{\pi_\theta^m}{2}$ each.
- Constant and similar marginal cost to produce $C_i(q_i) = cq_i$.

Collusion and fluctuating demand
Result

- To simply the analysis, we focus on full collusion at every periods where the duopoly shares the market equally.

Proposition

In infinite horizon, full collusion is sustainable only if $\delta \geq \frac{2\pi_H^m}{3\pi_H^m+\pi_L^m}$.

Proof.

Assume, *per contra*, there is an equilibrium that sustains collusion with

$$\delta < \frac{2\pi_H^m}{3\pi_H^m + \pi_L^m}.$$

The maximal punishment a firm can be subjected to, results in zero future profits (trigger strategies profile). □

Collusion and fluctuating demand
Result

Proof.

Let k be the first date of high demand. By deviating at date k, given that the state is $\theta \in \{L, H\}$, a firm earns at least

$$\left(\pi_H^m - \varepsilon\right)\delta^k + 0 \times \sum_{t=k+1}^{+\infty} \delta^t$$

□

Collusion and fluctuating demand
Result

Proof.

From date k, full collusion yields

$$\begin{aligned} & \frac{\pi_H^m}{2}\delta^k + \left(\frac{1}{2}\frac{\pi_L^m}{2} + \frac{1}{2}\frac{\pi_H^m}{2}\right)\sum_{t=k+1}^{+\infty}\delta^t \\ = \; & \frac{1}{4\left(1-\delta\right)}\left(\pi_H^m\left(2-\delta\right) + \pi_L^m\delta\right) \end{aligned}$$

□

Collusion and fluctuating demand
Result

Proof.

So, deviating at date k is non profitable only if

$$\begin{aligned} \left(\pi_H^m - \varepsilon\right) \quad & \leq \quad \frac{1}{4\left(1-\delta\right)}\left(\pi_H^m\left(2-\delta\right) + \pi_L^m\delta\right) \\ & \Longleftrightarrow \quad 4\left(1-\delta\right)\left(\pi_H^m - \varepsilon\right) \leq \pi_H^m\left(2-\delta\right) + \pi_L^m\delta \\ & \Longleftrightarrow \quad 4\left(\pi_H^m - \varepsilon\right) - 2\pi_H^m \leq \delta\left(\pi_L^m - \pi_H^m + 4\left(\pi_H^m - \varepsilon\right)\right) \\ & \Longleftrightarrow \quad \delta \geq \frac{2\pi_H^m - 4\varepsilon}{3\pi_H^m + \pi_L^m - 4\varepsilon} \end{aligned}$$

But RHS approaches $\frac{2\pi_H^m}{3\pi_H^m+\pi_L^m}$ as ε goes to zero, a *contradiction*. □

Collusion and fluctuating demand
Result

- In the case where $\pi_H^m = \pi_L^m = \pi^m$, the threshold on δ writes as $\frac{2\pi^m}{3\pi^m + \pi^m} = \frac{1}{2}$.
  - This is congruent with the result obtained previously (cf. Section Collusion in Prices)
  - Compared to stable high demand, here the deviating firm faces the same reward from deviation but a lower punishment (its long-run profit loss contains some periods of low demand).
- In the proof, we define date k to be a period of high demand.
  - Indeed, collusion is more easily sustainable in period of low demand.
  - Namely, the threshold on δ writes as $\frac{2\pi_L^m}{3\pi_L^m + \pi_H^m} < \frac{2\pi_H^m}{3\pi_H^m + \pi_L^m}$.

Collusion and fluctuating demand
Result

- Observe that the threshold $\frac{2\pi_H^m}{3\pi_H^m + \pi_L^m}$ on δ for full collusion to be sustainable is:
  - increasing with π_H^m;
  - decreasing with π_L^m.
- So, the threshold increases with the magnitude of demand fluctuations $(\pi_H^m - \pi_L^m)$.
  - Cartels tend to break down when a big order arrives.

Extension

- We can show that the result extends to any collusion.
- It can be shown that for $\delta \in [1/2, \frac{2\pi_H^m}{3\pi_H^m + \pi_L^m}]$ some collusion is sustainable.
  - In the low state of demand, firms charge the low monopoly price (i.e., $p_L^* = p_L^m$).
  - In the high state of demand, firms charge a price below the high monopoly price (i.e., $p_H^* < p_H^m$).
- Rotemberg and Saloner (1986) interpret this as showing the existence of price war during booms.
  - Whether price in high state (p_H^*) is lower or higher than the monopoly price in the low demand state (p_L^*) depends on the demand function.

Extension

- Haltiwanger and Harrington (1991) shows that a similar analysis applies to more deterministic fluctuations, as for example in the case of seasonal or business cycles.
  - There again, undercutting rivals is more tempting when demand is high.
  - In addition, however, the perceived cost of future price wars is lower when the cycle is currently at its top and at the very beginning of recession.
    - When demand is still high but declining, retaliation will only occur later, when demand is low.
  - Overall, collusion remains more difficult to sustain than with random fluctuations.

Conclusion

- Collusion is more difficult (resp. easier) to sustain at the top (bottom) of the cycle.
  - Firms are then obliged to collude "less" (by lowering the collusive price) or even abandon any collusion when demand is high.
- As fluctuations gain in scale, collusion becomes more and more difficult to sustain, at least in those states where demand is especially high.
- Demand fluctuations hinder collusion, and more so when fluctuations are deterministic (as in the case of seasonal cycles) rather than random.

Tacit Collusion
Outline

Collusion and secret price cuts
Introduction

- Up to now, firm's past choice is perfectly observed by its rival.
- In practice, supervising the partners is difficult.
  - Effective prices may not be observable (discounts, quality, etc).
  - Must rely on the observation of its own realized market share or demand to detect any price undercutting by the rival.
    - ⋆ But a low market share may be due to the aggressive behavior of one's rival or to a slack in demand.
- Under uncertainty, mistakes are unavoidable and maximal punishments (eternal reversion to Bertrand behavior) need not be optimal.

Collusion and secret price cuts
Introduction

Question
Is collusion sustainable under secret price cutting?

- Porter, R. (1983), "A Study of Cartel Stability: The Joint executive Committee, 1880-1886," *Bell Journal of Economics*, 14: 301-314.
- Green, E. and R. Porter (1984), "Non-Cooperative Collusion under Imperfect Price Information," *Econometrica*, 52:87-100.
  - In their model, Green and Porter assume quantity competition.
  - Here, we will go through a version of the model where firms compete in prices.
  - The essence is the same.

Collusion and secret price cuts
Model

- $N = \{1, 2\}$
- A firm that does not sell at some date is unable to observe whether the absence of demand is due to the realization of the low-demand state or to its rival's lower price.
  - The realizations of demand are unobservable.
  - The realizations of profits are observable.
- In each period, there are two possible realizations of demand (states of nature), i.i.d..
  - With probability α, there is no demand for the product sold by the duopolists (i.e., $D(p) = 0$ for every p)
  - With probability $(1 - \alpha)$, there is a positive demand $D(p)$ (the "high-demand" state).

Collusion and secret price cuts
Result

We look for an equilibrium with the following strategies:

- There is a collusive phase and a punishment phase.
- The game begins in the collusive phase.
  - Both firms charge p^m until one firm makes a zero profit.
- The occurrence of a zero profit triggers a punishment phase.
  - Here both firms charge c for exactly T periods, where T can *a priori* be finite or infinite.
- At the end (if any) of the punishment phase, the firms revert to the collusive phase.

Collusion and secret price cuts
Result

- We want to look for a length T of the punishment phase such that the expected present value of profits for each firm is maximal subject to the constraint that the associated strategies form a SPNE.
- Let V^c (resp. V^p) denote the present discounted value of a firm's profit from date t on assuming that at date t the game is in the collusive (resp. punishment) phase.
- V^c satisfies the following "Bellman" equation:

$$V^c = (1 - \alpha)\left(\frac{\pi^m}{2} + \delta V^c\right) + \alpha\delta V^p$$

  - with probability $(1 - \alpha)$ the demand is high, the firm profit is $\frac{\pi^m}{2}$, and the game remains in the collusive phase for the next period.
  - with probability α there is no demand for that period and the game will be in the punishment phase for the next period.

Collusion and secret price cuts
Result

- V^p satisfies the following "Bellman" equation:

$$V^p = \delta^T V^c$$

  - which is the present discounted value of profits at the beginning of the punishment phase.
- Using V^p in the expression of V^c we obtain

$$V^c = \frac{(1 - \alpha)\frac{\pi^m}{2}}{1 - (1 - \alpha)\delta - \alpha\delta^{T+1}}$$

Collusion and secret price cuts
Result

- Since strategies need to be a SPNE, we need to include incentive compatibility constraints ruling out profitable deviations in both phases.
  - It is easy to see that there are no profitable one shot deviations in the punishment phase.
  - The equilibrium then reduce to incentive constraint which states that no firm would wish to undercut in the collusive phase:

$$\begin{aligned} V^c &\geq (1-\alpha)\left(\pi^m + \delta V^p\right) + \alpha \delta V^p \\ \iff \quad & \frac{\pi^m}{2} + \delta V^c \geq \pi^m + \delta V^p \end{aligned}$$

- This expresses the trade-off for each firm.
  - If a firm undercuts, it gets $\pi^m > \frac{\pi^m}{2}$, but it automatically triggers the punishment phase, which yields valuation V^p instead of V^c.
- To deter undercutting, V^p must be sufficiently lower than V^c
  - This means that the punishment must last long enough

Collusion and secret price cuts
Result

- The previous incentive constraint rewrites as

$$\delta\left(V^c - V^p\right) \geq \frac{\pi^m}{2}$$

so using V^p in the expression of V^c, we obtain

$$(2\alpha - 1)\,\delta^{T+1} + 2\,(1-\alpha)\,\delta \geq 1$$

- This is not satisfied when:
  - $T = 0$: non negligible punishments are required.
  - $\alpha \geq \frac{1}{2}$: the intuition is that the temptation to undercut increases when the expected gain from the future collusion decreases.

Collusion and secret price cuts
Result

- The highest profit for the firms is then obtained by solving

$$\max_{s.t.\ (2\alpha-1)\delta^{T+1}+2(1-\alpha)\delta\geq 1} V^c$$

- Since V^c is a decreasing function of T, the firms optimal choice consists in selecting the smallest (or closer integer to) T that satisfies the constraint.
- Assuming that $2\,(1-\alpha)\,\delta \geq 1$, so that the constraint is satisfied for $T \to +\infty$, there exists a (finite) optimal length of punishment T^* (if it is not an integer just take the lowest higher integer of T^*).

$$T^* = \frac{\ln\left(\frac{2(1-\alpha)\delta - 1}{1-2\alpha}\right)}{\ln \delta} - 1$$

Collusion and secret price cuts
Conclusion

- This model predicts periodic price wars, contrary to the perfect observation models.
- Price wars are involuntary, in that they are triggered not by a price cut but by an unobservable slump in demand.
- Contrary to the Rotemberg-Saloner model, here price wars are triggered by a recession.
- Under imperfect information, the fully collusive outcome cannot be sustained.
  - It could be sustained only if the firms kept on colluding (charging the monopoly price) even when making small profits, because even under collusion small profits can occur as a result of low demand.
  - However, a firm that is confident that its rival will continue cooperating even if its profit is low has every incentive to (secretely) undercut - price undercutting yields a short-term gain and creates no long-run loss.
  - Thus, full collusion is inconsistent with the deterrence of price cuts.

Collusion and secret price cuts
Conclusion

- Oligopolists are likely to recognize the threat to collusion posed by secrecy, and take steps to eliminate it.
- E.g., Industry trade associations
  - Collect detailed information on the transactions executed by the members.
  - Allows it members to cross-check price quotations.
  - Imposes standarization agreements to discourage price-cutting when products have multiple attributes.

Tacit Collusion
Outline

Conclusion

- Collusion is sustainable when the short-term gains from stealing the rival's market share and profit is lower than the cost of future prices wars.
- Cartel is much like a repeated Prisoner's Dilemma
  - Need to easily observe each other's plays and react (quickly) to punish undesired behavior;
  - Need patient players who value the long run (wars don't help!);
  - Need a stable set of players and some stationarity helps;
    - ⋆ constantly changing sources of production can hurt, but growing demand can help...

Conclusion

- Under finite horizon of any previous model, collusion is not sustainable.
  - Due to backward induction argument.
- Under infinite horizon
  - Collusion is easier to sustain when:
    - ⋆ firms are sufficiently patient (the more patient they are, the easier is collusion)
    - ⋆ the number of competitors is low.
    - ⋆ the market is transparent.
    - ⋆ the interaction is frequent.
    - ⋆ the demand is stable.

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