Industrial Organization

Master Quantitative Economics - 2023/2024

Chapter 2: Quality and Product Differentiation

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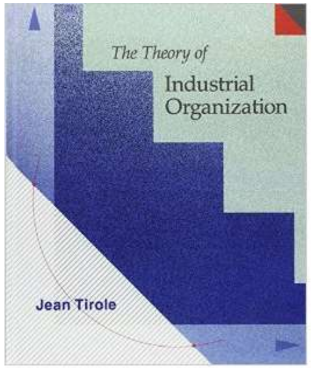
Chapter 2

Quality and Product Differentiation

Outline

Introduction

Bibliography



English ed.: MIT; French ed.: Economica

Introduction

Jean Tirole (Nobel 2014)

Affiliation: Toulouse School of Economics (TSE), Toulouse, France

- **Prize motivation:** "for his analysis of market power and regulation"
- **Field:** industrial organization, microeconomics

Introduction
Issue

- Questions:
  - What is the price on a given market?
  - What are the profits?
  - What is the social surplus?
- Answers from the previous chapter, it depends on:
  - How many firms are on the market
    - Monopoly, duopoly, oligopoly, ... , atomless firms.
  - Whether firms are competing on prices or on quantity.
  - Whether there are capacity constraints, decreasing returns to scale, ...
  - Whether there is a temporal dimension
    - Simultaneous moves, sequential moves...

Introduction
Issue

- In the previous chapter we have assumed that goods, produced by different firms, are homogenous, that is perfect substitutes.
- In this chapter, we shall relax this assumption and allow firms to produce differentiated goods.
  - We say that two goods are **differentiated good** if they are substitutes but not perfect substitutes.

Introduction
Issue

- Product differentiation is the process of distinguishing a product or service from others, to make it more attractive to a particular target market.
  - This involves differentiating it from competitors' products as well as a firm's own products.
  - It is a way to extract from price competition pressure.
  - It leads to imperfect competition, namely monopolistic competition.

Introduction
Issue

- The brand differences are usually minor.
  - They can be merely a difference in packaging or an advertising theme.
  - The physical product need not change, but it may.
  - Differentiation is due to buyers perceiving a difference

Introduction
Issue

- The concept of product differentiation was proposed by Edward Chamberlin in his 1933 *Theory of Monopolistic Competition*.

Edward Chamberlin (1899 – 1967)

Introduction
Issue

- 2008 Nobel-Prize winner Paul Krugman and others built the foundations of the *New Theory of International Trade* by
  - combining such theories of industrial structure with production functions that assumed significant :
    - ⋆ economies of scale (average cost of producing decreases with output volume); and
    - ⋆ economies of scope (cheaper to produce a range of products together than to produce each one of them on its own).

Introduction
Issue

- There are two broad categories of differentiation based on a single characteristic.
  - Horizontal: products are different according to features that cannot be ordered in an objective way.
    - ⋆ At the same price some consumers will buy one and some will buy other, it really depends on their preferences.
    - ⋆ E.g., differentiation in colors, in styles, in shapes, in flavours, in tastes, ...
  - Vertical: products are different according to features that can be ordered according to their objective quality from the highest to the lowest.
    - ⋆ All consumers would prefer one to the other if they were sold at the same price.
    - ⋆ E.g., BMW vs Fiat, Pharmaceutical branded good vs generics, Regular cell phone versus the latest version of Iphone.

Quality and Product Differentiation
Outline

Quality and Product Differentiation
Outline

Horizontal Differentiation
Introduction

- Breakfast cereals may have lots of sugar or may be healthier.
  - We may order them one a line according to their sugar content.
  - Consumers have different tastes for sugar and therefore at same prices they will buy those cereals that are closer to their taste.
- We will depict this situation using the analogy of spatial competition.
  - From a firm's point of view, location works as a choice of product characteristics.

Quality and Product Differentiation
Outline

1. Introduction
2. Horizontal Differentiation
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     - Results: Location choice
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     - Introduction
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     - Conclusion
   - Circular city
     - Introduction

Linear City
Introduction

- Hotelling, Harold (1929): "Stability in Competition". *Economic Journal* 39(153):41-57
  - Hotelling (1929) rebelled against the assumption of homogeneous products because of its implication, in the Bertrand duopoly model, that all demand would switch from one supplier to another in response to an infinitesimal difference in price.
  - He assumed linear transportation cost and obtained a *Principle of Minimum Differentiation*.

Linear City
Introduction

- d'Aspremont, C., Gabszewicz, J.-J. and Thisse, J.-F.(1979): "On Hotelling's 'Stability in competition'" *Econometrica*, 47: 1145-1150
  - Show that Hotelling's 1929 paper is invalid.
    - When transportation cost is linear, nothing can be said about the tendency of both sellers to agglomerate at the center of the market.
  - When transportation cost is quadratic, an equilibrium does exist.
    - We then obtain a *Principle of Maximum Differentiation*.
- Observe that the *Principle of Minimum Differentiation* is still valid when there is no price competition (i.e., it is the unique Nash equilibrium of the location game).

Linear City
Model

- We consider a linear city of length 1: $[0;1]$
- Consumers are uniformly distributed along the city.
- $N = \{1,2\}$
- Two stages game:
  - Firms choose their locations simultaneously.
  - Firms compete in price simultaneously.
- Constant and similar marginal cost to produce $C_i(q_i) = cq_i$.

Linear City
Model

- We assume the market is covered
  - All consumers buy exactly one unit of the good.
- As in d'Aspremont et al. (1979, we assume transportation costs are quadratic:
  - A consumer located in $x \in [0;1]$ who buys from a firm located in $a \in [0;1]$ who charges price p pays:

$$p + t(x-a)^2$$

Linear City
Results: price competition

- Let us solve the game by backward induction
  - We take firms' location as given and look for Nash equilibrium in prices.
  - We assume firm 1 is located in $a \in [0;1]$, and firm 2 is located in $(1-b) \in [0;1]$.
    - The distance between firm 1 (resp. 2) and point 0 (resp. 1) is a (resp. b).
    - Firm 1 is to the left of firm 2: $a \leq 1-b$, i.e., $1-a-b \geq 0$.

Linear City
Results: price competition

- Let us identify the consumer $\tilde{x} \in [0;1]$ who is indifferent from buying from firm 1 and 2:

$$
\begin{aligned}
& \textit{Firm } 1 \underset{\tilde{x}}{\sim} \textit{Firm } 2 \\
\iff & p_1 + t(\tilde{x} - a)^2 = p_2 + t(\tilde{x} - (1-b))^2 \\
\iff & \tilde{x} = \frac{p_2 - p_1 + t\left((1-b)^2 - a^2\right)}{2t(1-b-a)} \\
\iff & \tilde{x} = a + \frac{1-a-b}{2} + \frac{p_2 - p_1}{2t(1-b-a)}
\end{aligned}
$$

with the interpretation that
- a represents firm 1's turf;
- $\frac{1-a-b}{2}$ is half of consumers between firms 1 and 2; and
- $\frac{p_2-p_1}{2t(1-b-a)}$ is the sensitivity of demand of the price differential.

Linear City
Results: price competition

- Clearly,

$$D_1(a,b,p_1,p_2) = \tilde{x} \qquad \textit{and} \qquad D_2(a,b,p_1,p_2) = 1 - \tilde{x}$$

that is

$$D_1(a,b,p_1,p_2) = a + \frac{1-a-b}{2} + \frac{p_2 - p_1}{2t(1-b-a)}$$

and

$$D_2(a,b,p_1,p_2) = b + \frac{1-a-b}{2} + \frac{p_1 - p_2}{2t(1-b-a)}$$

Linear City
Results: price competition

- Firm i's program writes as

$$
\begin{aligned}
& \max_{p_i \in \mathbb{R}^+} (p_i - c) D_i(a,b,p_i,p_j) \\
= & \max_{p_i \in \mathbb{R}^+} (p_i - c)\left(\alpha_i + \frac{1-a-b}{2} + \frac{p_j - p_i}{2t(1-b-a)}\right)
\end{aligned}
$$

with $\alpha_i = \begin{cases} a \text{ if } i = 1 \\ b \text{ if } i = 2 \end{cases}$

- F.O.C

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial p_i} &= D_i(a,b,p_1,p_2) + (p_i - c)\frac{\partial D_i(a,b,p_1,p_2)}{\partial p_i} \\
&= \left(\alpha_i + \frac{1-a-b}{2} + \frac{p_j - p_i}{2t(1-b-a)}\right) - \frac{(p_i - c)}{2t(1-b-a)} \\
&= \alpha_i + \frac{1-a-b}{2} + \frac{p_j + c - 2p_i}{2t(1-b-a)}
\end{aligned}
$$

Linear City
Results: price competition

- F.O.C.

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial p_i} &= 0 \\
\iff & p_i = t(1-b-a)\left(\alpha_i + \frac{1-a-b}{2}\right) + \frac{p_j + c}{2} = p_i^*(p_j)
\end{aligned}
$$

Linear City
Results: price competition

- Nash equilibrium:

$$p_i = p_i^*\left(p_j^*\left(p_i\right)\right)$$

that is

$$\begin{aligned} p_i &= t\left(1-b-a\right)\left(\alpha_i + \frac{1-a-b}{2}\right) + \frac{c}{2} + \frac{p_j^*\left(p_i\right)}{2} \\ &= t\left(1-b-a\right)\left(\alpha_i + \frac{1-a-b}{2}\right) + \frac{c}{2} \\ &\quad + \frac{t\left(1-b-a\right)\left(\alpha_j + \frac{1-a-b}{2}\right) + \frac{p_i+c}{2}}{2} \end{aligned}$$

Linear City
Results: price competition

- So,

$$\begin{aligned} p_i &= \left(\frac{3c}{4} + t\left(1-b-a\right)\left(\alpha_i + \frac{\alpha_j}{2} + \frac{3}{4}\left(1-b-a\right)\right)\right)\frac{4}{3} \\ &= c + t\left(1-b-a\right)\left(1-a-b+\frac{4}{3}\alpha_i + \frac{2}{3}\alpha_j\right) \\ &= c + t\left(1-b-a\right)\left(1 + \frac{\alpha_i - \alpha_j}{3}\right) \end{aligned}$$

Therefore,

$$p_1^* = c + t\left(1-b-a\right)\left(1 + \frac{a-b}{3}\right)$$

and

$$p_2^* = c + t\left(1-b-a\right)\left(1 + \frac{b-a}{3}\right).$$

Linear City
Results: location choice

- Let

$$p_i^*\left(a,b,p_j\right) \in \arg\max_{p_i} \pi_i\left(a,b,p_i,p_j\right)$$

- Nash:

$$p_i^*\left(a,b,p_j^*\left(a,b,p_i^*\right)\right) := p_i^*\left(a,b\right)$$

- Let

$$\pi_i\left(a,b\right) := \pi_i\left(a,b,p_1^*\left(a,b\right),p_2^*\left(a,b\right)\right)$$

Linear City
Results: location choice

- Firm i selects the location $\alpha_i = \begin{cases} a \text{ if } i=1 \\ b \text{ if } i=2 \end{cases}$ that maximizes its profits:

$$\max_{\alpha_i} \pi_i\left(a,b\right) = \max_{\alpha_i}\left(p_i^*\left(a,b\right)-c\right) D_i\left(a,b,p_i^*\left(a,b\right),p_j^*\left(a,b\right)\right)$$

- We shall use the Envelope Theorem.

Linear City
Results: location choice

- Recall on the Envelope Theorem
  - Let $f(x,y)$.
  - Define

$$y^*(x) \in \arg\max_y f(x,y) \quad and \quad f^*(x) := f(x, y^*(x))$$

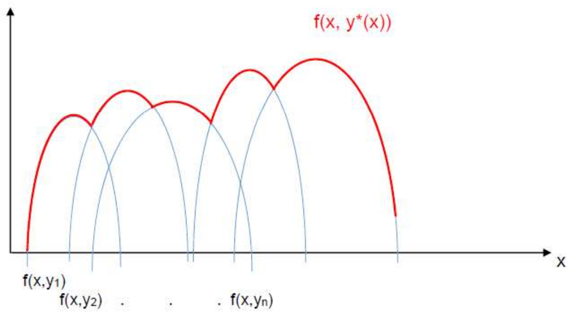


Figure 5.6

Linear City
Results: location choice

- Recall on the Envelope Theorem
  - Let $f(x,y)$.
  - Define

$$y^*(x) \in \arg\max_y f(x,y) \quad and \quad f^*(x) := f(x, y^*(x))$$

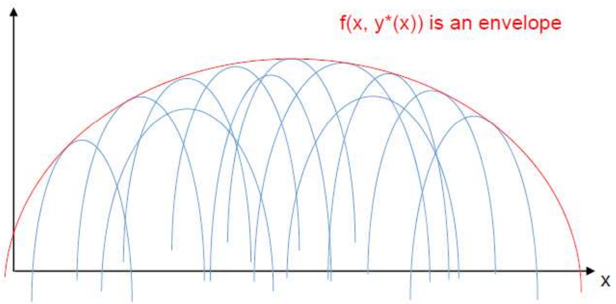


Figure 5.7

Linear City
Results: location choice

- Recall on the Envelope Theorem
  - $$\frac{df^*(x)}{dx} = \left[\frac{\partial f(x, y^*(x))}{\partial x}\right] + \left[\frac{\partial f(x, y^*(x))}{\partial y^*(x)} \frac{dy^*(x)}{dx}\right]$$
  - RHS: 1^{st} term is the direct effect; 2^{nd} term is the indirect effect.
  - Envelope Theorem: the indirect effect is locally (i.e., for small variation) negligible:

$$\frac{df^*(x)}{dx} = \frac{\partial f(x, y^*(x))}{\partial x}$$

Linear City
Results: location choice

- Envelope Theorem: the indirect effect is locally (i.e., for small variation) negligible:

$$\frac{df^*(x)}{dx} = \frac{\partial f(x, y^*(x))}{\partial x}$$

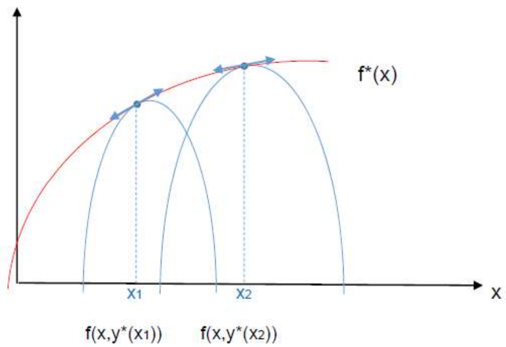


Figure 5.8

Linear City
Results: location choice

- By the Envelope Theorem, to compute $\frac{d\pi_i}{da}$ we can ignore the derivative

$$\frac{\partial \pi_1}{\partial p_1} \times \frac{dp_1}{da}$$

since firm 1 maximizes w.r.t. price in the 2^{nd} period.

- That is

$$\begin{aligned}\frac{d}{da}\pi_1\left(a, b, p_1^*\left(a, b\right), p_2^*\left(a, b\right)\right) &= \frac{\partial \pi_1}{\partial a} + \frac{\partial \pi_1}{\partial p_1^*} \times \frac{dp_1^*}{da} + \frac{\partial \pi_1}{\partial p_2^*} \times \frac{dp_2^*}{da} \\ &= \frac{\partial \pi_1}{\partial a} + \frac{\partial \pi_1}{\partial p_2^*} \times \frac{dp_2^*}{da}\end{aligned}$$

because from the Envelope Theorem we get $\frac{\partial \pi_1}{\partial p_1} \sim 0$.

Linear City
Results: location choice

- So,

$$\frac{d\pi_1}{da} = \left(p_1^* - c\right)\left(\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2^*} \times \frac{dp_2^*}{da}\right)$$

with

$$\frac{\partial D_1}{\partial a} = \frac{1}{2} + \frac{p_2^* - p_1^*}{2t\left(1 - b - a\right)^2} = \frac{3 - 5a - b}{6\left(1 - a - b\right)^2}$$

which is positive when $a < \frac{1}{2}$ so firm 1 will want to move toward the center to increase its market share given the price structure.

Linear City
Results: location choice

- Also, we have

$$\frac{\partial D_1}{\partial p_2^*} \times \frac{dp_2^*}{da} = \frac{1}{2t\left(1 - a - b\right)} t\left(-\frac{4}{3} + \frac{2a}{3}\right) = \frac{a - 2}{3\left(1 - a - b\right)} < 0$$

so the associated decrease in product differentiation (increase in a) forces firm 2 to reduce its price.

- Hence,

$$\frac{d\pi_1}{da} = \left(p_1^* - c\right)\left(\frac{-1 - 3a - b}{6\left(1 - a - b\right)}\right) < 0$$

so firm 1 always wants to move leftward if it is to the left of firm 2.

- Therefore, the equilibrium in locations exhibits maximal differentiation: $(a^*, 1 - b^*) = (0, 1)$.
  - The corresponding prices are $p_1^* = p_2^* = c + t$.

Linear City
Results: location choice

Question

Is such maximal differentiation (i.e., $(a^*, 1 - b^*) = (0, 1)$) socially optimal?

Answer

No!

- Indeed, for given locations, as long as the market is covered, the pricing structure does not affect the sum of consumers' surplus and profits.
  - Thus, the social planner would minimize the consumers transportation costs.

Linear City
Results: location choice

- When the density is uniform and costs are quadratics, socially optimal locations are equidistant on either side of the segment, that is $a^{FB} = \frac{1}{4}$ and $1 - b^{FB} = \frac{3}{4}$.
  - The corresponding prices would be $p_1^* = p_2^* = c$.

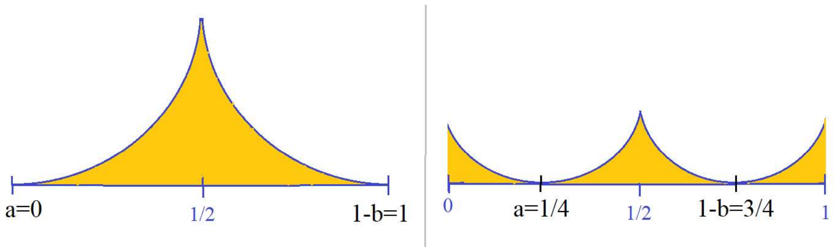


Figure 2.1. LHS (resp. RHS): transportation cost for $(a^*, 1 - b^*) = (0, 1)$ (resp. $(a^{FB}, 1 - 1 - b^{FB}) = (\frac{1}{4}, \frac{3}{4})$).

Linear City
Conclusion

- d'Aspremont et al. (1979) show that if, keeping all other aspects of Hotellingís specification, transportation cost is made quadratic rather than linear in distance, duopolists will choose maximum differentiation (rather than minimum differentiation).
- The social planner would instead chooses locations that are equidistant on either side of the segment.

Linear City
Conclusion

- Although firms like to differentiate to soften price competition they also all want to locate where the demand is.
  - I.e., near the center of the linear city.
- By fictitiously extending the linear city from $[0, 1]$ to $[-1, 2]$ with no consumers outside the interval $[0, 1]$ we can show that the new equilibrium location is: $(a^*, 1 - b^*) = (-\frac{1}{4}, \frac{5}{4})$.
  - (Just solve $\frac{d\pi_1}{da} = (p_1^* - c)\left(\frac{-1-3a-b}{6(1-a-b)}\right) = 0$)

Quality and Product Differentiation
Outline

Market transparency
Introduction

- Schultz, Christian: “Market transparency and product differentiation”. *Economics Letters*, 2004, vol. 83, issue 2, pages 173-178.
  - Product characteristics as well as prices are not obvious to all consumers.
    - E.g., experience good, good’s characteristic and the pricing are “complicated” as is the case with insurance policies, internet access or mobile phones.
  - Hence consider a Hotelling market where some consumers are uninformed about the firms’ locations, and only learn these when buying.

Market transparency
Model

- We considers a Hotelling market with a continuum of consumers along the line $[0, 1]$.
  - A consumer buys exactly one unit of the good.
  - There are two firms, A and B.
  - First firms simultaneously choose locations, a and $1 - b$, on the unit interval, w.l.o.g. $a \leq 1 - b$.
  - Second firms simultaneously compete in price.

Market transparency
Model

- There are two different types of consumers that are uniformly distributed on locations:
  - a fraction ϕ are informed about both firms’ prices and locations, while a fraction $1 - \phi$ are uninformed.
  - The variable ϕ is our measure of market transparency, the higher is ϕ, the more transparent is the market.
  - We will concentrate on symmetric equilibria where half of the uninformed consumers consume from firm A.

Market transparency
Model

- A parameter $t > 0$ measures the “intensity” of product differentiation, we denote it the transportation cost and we assume that it is quadratic.
- We assume both firms’ costs are equal to zero.
- Denote p_i the firm i’s price and D the firm A’s demand.

Market transparency
Results

- The location $\tilde{x}$ of the informed consumer who is indifferent between buying from firm A and B satisfies

$$p_a + t(\tilde{x} - a)^2 = p_b + t(\tilde{x} - (1 - b))^2$$

which is equivalent to

$$\tilde{x}(a, b, p_a, p_b) = \frac{1 + a - b}{2} + \frac{p_b - p_a}{2t(1 - a - b)}$$

Market transparency
Results

- The firm A's demand writes as

$$D = \phi\tilde{x}(a, b, p_a, p_b) + \frac{1 - \phi}{2}$$

We assume that the market is covered so that firm B's demand is

$$1 - D = \phi(1 - \tilde{x}(a, b, p_a, p_b)) + \frac{1 - \phi}{2}$$

Market transparency
Results

- At the competing stage, the firms' maximization programs write as

$$\max_{p_a} \left\{ \pi_a(a, b, p_a, p_b) \equiv p_a \left(\phi\tilde{x}(a, b, p_a, p_b) + \frac{1 - \phi}{2} \right) \right\}$$

and

$$\max_{p_b} \left\{ \pi_b(a, b, p_a, p_b) \equiv p_b \left(\phi(1 - \tilde{x}(a, b, p_a, p_b)) + \frac{1 - \phi}{2} \right) \right\}$$

Market transparency
Results

- The price that maximizes firm A's program must satisfies the F.O.C.

$$\frac{d\pi_a}{dp_a}(a, b, p_a, p_b) = 0$$

that is

$$\frac{d}{dp_a}(p_a D) = D + p_a \frac{dD}{dp_a} = 0$$

which is

$$D + \phi p_a \frac{\partial \tilde{x}}{\partial p_a} = 0.$$

Market transparency
Results

- The analogous of the previous equation for firm B writes as

$$1 - D - \phi p_b \frac{\partial \tilde{x}}{\partial p_b} = 0$$

- Adding both equations and using the fact that

$$\frac{\partial \tilde{x}}{\partial p_a} = -\frac{\partial \tilde{x}}{\partial p_b}$$

we get

$$1 + \phi(p_a + p_b)\frac{\partial \tilde{x}}{\partial p_a} = 0$$

So

$$p_a^*(a, b, p_b) = \frac{2t(1 - a - b)}{\phi} - p_b$$

Market transparency
Results

- To determine what is the location at equilibrium we have to study how vary the firms' profits with location.

$$\begin{aligned} \frac{d\pi_a}{da}(a, b, p_a, p_b) &= \frac{dp_a}{da} + p_a \frac{dD}{da}(a, b, p_a, p_b) \\ &= \frac{dp_a}{da} + p_a\left(\frac{\partial D}{\partial a} + \frac{\partial D}{\partial p_a}\frac{dp_a}{da} + \frac{\partial D}{\partial p_b}\frac{dp_b}{da}\right) \end{aligned}$$

Market transparency
Results

- To obtain the Nash equilibrium in price, we can solve the following system of equations

$$\begin{cases} p_a^*(a, b, p_b) = \frac{2t(1-a-b)}{\phi} - p_b \\ D + \phi p_a \frac{\partial \tilde{x}}{\partial p_a} = 0 \end{cases}$$

which leads to

$$\begin{cases} p_a^*(a, b) = \frac{(1-a-b)(\phi(1+a-b)+2+(1-\phi))}{3\phi} t \\ p_b^*(a, b) = \frac{(1-a-b)(4-\phi(1+a-b)-(1-\phi))}{3\phi} t \end{cases}$$

- The prices are decreasing in ϕ: increasing transparency makes price setting more competitive for given locations.

Market transparency
Results

- By the envelop theorem, $\frac{dp_a}{da} = 0$, so the previous equation writes as

$$\begin{aligned} \frac{d\pi_a}{da}(a, b, p_a, p_b) &= p_a\left(\frac{\partial D}{\partial a} + \frac{\partial D}{\partial p_b}\frac{dp_b}{da}\right) \\ &= p_a\left(\phi\frac{\partial \tilde{x}}{\partial a} + \phi\frac{\partial \tilde{x}}{\partial p_b}\frac{dp_b}{da}\right) \\ &= p_a\phi\left(\frac{\partial \tilde{x}}{\partial a} + \frac{\partial \tilde{x}}{\partial p_b}\frac{dp_b}{da}\right) \end{aligned}$$

- According to the values of $\tilde{x}$, $p_a^*(a, b)$, $p_b^*(a, b)$, we have $\frac{d\pi_a}{da}(a, b, p_a^*, p_b^*) = 0$ (and $\frac{d\pi_b}{db}(a, b, p_a^*, p_b^*) = 0$) for

$$a^* = b^* = \frac{7\phi - 9}{8\phi} < 0.$$

Market transparency
Results

- When firms can locate outside the city and can choose their location on the whole real line they decide to locate in

$$(a^*, 1 - b^*) = (\frac{7\phi - 9}{8\phi}, \frac{\phi + 9}{8\phi})$$

- The equilibrium prices and profits are

$$p_a^* = p_b^* = \frac{3}{4}\frac{3-\phi}{\phi^2}t \quad \text{and} \quad \pi_a^* = \pi_b^* = \frac{3}{8}\frac{3-\phi}{\phi^2}t$$

  - They both decrease in ϕ: firms dislike transparency.
  - It is straightforward that a^* and b^* are increasing with ϕ.
- Therefore we can conclude that increasing market transparency leads to less product differentiation, lower prices and lower profits.

Market transparency
Conclusion

- In a Hotelling market with endogenous choice of product characteristics, increasing market transparency on the consumer side leads to:
  - less product differentiation;
  - lower prices; and
  - lower profits.
- This is welfare improving for all consumers and total surplus increases.

Circular city
Outline

Circular city
Introduction

- Steven C. Salop (1979): “Monopolistic Competition with Outside Goods”, *The Bell Journal of Economics*, Vol. 10, No. 1 , 141-156.
  - The linear city model assumes that there are two firms in the market.
    - ⋆ This suggests some kind of entry barriers to block potential entrants entering the market.
  - Salop (1979) uses a circular city model to study entry and location choice when there are no barriers to entry other than fixed cost or entry costs.
  - The circular city model is “easier” in that the product space is completely homogenous (no location is a priori better than another).

Circular city
Model

- Two stages game:
  - First, firms simultaneously choose whether or not to enter in the market.
  - Second, firms simultaneously choose their prices given their location.
- Finally, the consumers (knowing the locations and prices of all firms) choose from which firm to buy one unit of the good.

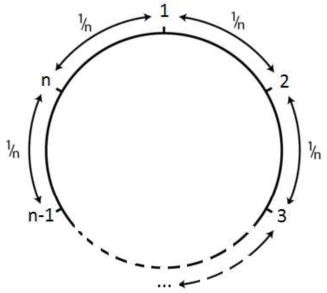
Circular city
Model

- A continuum of consumers in a market are uniformly distributed on a circle with a perimeter equal to 1.
  - The consumers either buy one good or no good at all.
  - The parameters of the model are such that, in equilibrium, all consumers indeed buy.
  - The consumers have linear transportation costs t.
- Each firm only locates in one location
  - Fixed cost of entry f, and marginal cost c (same for all firms).
  - Firm i's profit when facing demand D_i:

$$\begin{cases} (p_i - c)\, D_i - f & \text{if it enters} \\ 0 & \text{otherwise} \end{cases}$$

Circular city
Model

- Maximum differentiation is exogenously imposed.
  - Purpose of Salop's model is to look at extent of entry rather than product choice.
  - The firms that choose to enter are (exogenously and automatically) located equidistant from one another on the circle.



Circular city
Results

Question

How many firms will enter the market?

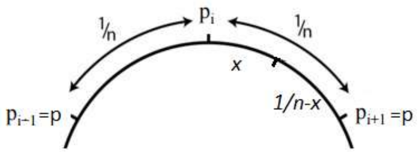
- Let there be n firms entered the market in the first stage.
- We look for the symmetric Nash equilibrium price p in the second-stage.
  - Note that any firm has only two real competitors: the two closest nearby ones.

Circular city
Model

- Let us determine demand D_i when there are n firms.
  - Suppose firm i chooses price p_i.
  - A consumer located at the distance $x \in (0, \frac{1}{n})$ from firm i is indifferent between purchasing from firm i and purchasing from i's closest neighbor if:

$$\text{firm } i \underset{x}{\sim} i\text{'s closest neighbor}$$

$$\iff \quad p_i + tx = p + t(\frac{1}{n} - x)$$



Circular city
Model

- So,

$$x = \frac{p + \frac{t}{n} - p_i}{2t}$$

- Since i serves its right and left sides (the firm captures all consumers in a $2x$ segment of the circle), we have

$$D(p_i, p) = 2x = \frac{p + \frac{t}{n} - p_i}{t}$$

- So firm i's problem is

$$\max_{p_i} (p_i - c)\left(\frac{p + \frac{t}{n} - p_i}{t}\right) - f$$

Circular city
Model

- FOC and setting $p_i = p$ yields

$$p = c + \frac{t}{n}$$

  - this result is similar to the one we found for linear city when we take $n = 2$.
  - Observe that from $p_i = p$ the indifferent consumer's location is

$$x = \frac{1}{2n}$$

  which is just between two firms.

Circular city
Model

- Since it is a free entry model, the zero-profit condition determines the number of firms:

$$\frac{p-c}{n} - f = 0 \iff \frac{t}{n^2} - f = 0$$

- So,

$$n^* = \sqrt{\frac{t}{f}} \quad \textit{and} \quad p^* = c + \sqrt{tf}\ .$$

- Hence
  - An increase in f causes a decrease in n^*, and an increase in p^*
    - ⋆ The lower the entry cost the more competitive is the market.
    - ⋆ At the limit, we have $\lim_{f \to 0} n^* = \infty$, and $\lim_{f \to 0} p^* = c$, that is perfect competitive market.
  - An increase in t causes an increase in p^*, and thereby an increase in n^*.
    - ⋆ From firms' point of view, the increase in t is an increase in the possibility of differentiation.

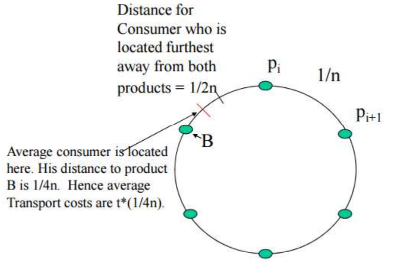
Circular city
Model

- Observe that the equilibrium price is above marginal cost, nevertheless, firms are making zero profit due to the fixed cost. In other words, a larger than zero price-cost margin does not imply positive profit.

Circular city
Model

- The social planner's problem is to minimize the total fixed cost (nf) plus the average transportation cost.
  - the total fixed cost is nf;
  - the average transportation cost is

$$t\left(2n\int_0^{\frac{1}{2n}} x dx\right) = 2tn\left[\frac{x^2}{2}\right]_0^{\frac{1}{2n}} = \frac{2tn}{8n^2} = \frac{t}{4n}$$



Circular city
Model

- Social planner's problem then writes

$$\min_n nf + \frac{t}{4n}$$

- So we get

$$n^{FB} = \frac{1}{2}\sqrt{\frac{t}{f}} = \frac{1}{2}n^*$$

- The market outcome yields socially too much product differentiation (too many firms have entered).
  - Private motivation for entry: Business stealing.
  - Social motivation for entry: Saving transportation costs.

Circular city
Conclusion

- Free entry can lead to excessive variety.
- Maximal product differentiation is assumed.
  - Economides, Nicholas, 1989. "Symmetric equilibrium existence and optimality in differentiated product markets," *Journal of Economic Theory*, Elsevier, vol. 47(1), pages 178-194, February.
    - Endogenize through a 3–stage game where firms first choose to enter, second choose varieties (maximal differentiation is not imposed), third compete in prices.
    - Equidistant firms location is obtained as a result.

Circular city
Conclusion

- Excessive entry can result when two conditions hold:
  - entrants' products are substitutes for existing firms' products, so that entry steals business from incumbents; and
  - firm's average costs are decreasing with output (which is the case with a fixed cost of entry)
- An extreme example with perfect substitutes, fixed prices and fixed costs, illustrates this clearly.
  - A second entrant garners half of the market and halves of the incumbent's output.
  - Consumers derive no additional benefit from the new entrant's product
  - Resource usage on fixed costs is now doubled.
  - So the social surplus is reduced.

Circular city
Conclusion

- Berry, Steven and Joel Waldfogel (1999) : "Free Entry and Social Inefficiency in Radio Broadcasting", *RAND Journal of Economics*, vol. 30, issue 3, pages 397-420
  - Empirical support on the U.S. Radio broadcasting industry.
  - Relative to the social optimum, they find that the welfare loss (to firms and advertisers) of free entry is 45% of revenue.

Quality and Product Differentiation
Outline

Horizontal Differentiation
Conclusion

- The greater the degree of product differentiation, the greater the degree of market power.
  - The greater the value of search costs or switching costs (what is modeled through transportation cost in this chapter), the greater the sellers' market power tends to be.
- Firms want to differentiate to soften price competition but they also want to locate where the demand is.
  - Minimum differentiation when there is no price competition.
  - Maximal differentiation to counteract price competition.
- Free entry can lead to excessive variety.
  - Implication: from a social point of view, firms' costly attempts to differentiate themselves are wasteful.

Quality and Product Differentiation
Outline

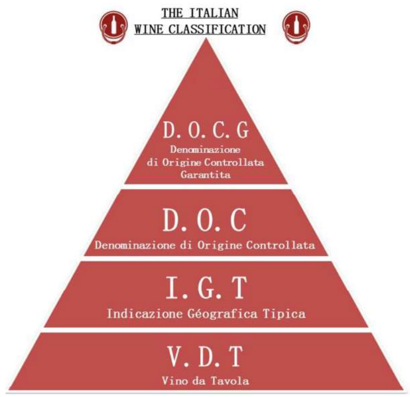
Quality and Product Differentiation
Outline

1. Introduction
2. Horizontal Differentiation
3. Vertical differentiation
   - Introduction
   - Model
   - Results
   - Conclusion
4. References

Vertical Differentiation
Introduction

- **Vertical differentiation** is about being better than your competitors while **horizontal differentiation** is about being different.
- E.g., Horizontal differentiation wrt wines
  - Colour, brand, Country of origin, Grape variety, etc.
- E.g., Vertical differentiation wrt wines
  - Brands with strong reputation, AOC wines, quality of grapes, etc.

Vertical Differentiation
Introduction



Vertical Differentiation
Introduction

- Gabszewicz, J. & Thisse, J. -F. (1979): “Price competition, quality and income disparities,” *Journal of Economic Theory*, vol. 20, issue 3, pages 340-359
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Vertical Differentiation
Model

- Duopoly: $N = \{1, 2\}$
  - Same marginal cost $c \in \mathbb{R}^+$.
- Timing: two-stage game.
  - 1. Simultaneously choice of quality
    - $s_i \in [\underline{s}, \bar{s}]$, $(\underline{s}, \bar{s}) \in \mathbb{R}^2_+$, $i \in N$
    - Quality is costless
  - 2. Price competition given these qualities
- Each consumer consumes 0 or 1 unit of a good.

Vertical Differentiation
Model

- A consumer with taste parameter $\theta > 0$ has the following preferences:

$$u_\theta = \begin{cases} \theta s - p & \text{if} \quad \text{he buys the good of quality } s \text{ at price } p \\ 0 & \text{if} \quad \text{he does not buy} \end{cases}$$

- Taste parameter for quality θ has a cumulative distribution function $F(\theta) \in [0; \infty)$
  - $F(\theta)$: fraction of consumers with a taste parameter lower (or equal) to θ.

Vertical Differentiation
Model

Question

How does write the demand functions?

- If single quality s is offered at price p, the demand for the good is equal to the number of consumers with taste θ such that $\theta s \geq p$ ($\Longleftrightarrow \theta \geq \frac{p}{s}$).

$$D(p, s) = 1 - F\left(\frac{p}{s}\right).$$

- if two qualities with $s_1 < s_2$ and $p_1 \geq p_2$ then s_1 is not consumed.

Vertical Differentiation
Model

- if two qualities with $s_1 < s_2$ and $p_1 < p_2$ then:
  - high quality s_2 is preferred to low quality s_1 if $\theta s_2 - p_2 \geq \theta s_1 - p_1$ ($\Longleftrightarrow \theta \geq \frac{p_2 - p_1}{s_2 - s_1} := \hat{\theta}$).

$$D_2\left(s_1, s_2, p_1, p_2\right) = 1 - F\left(\hat{\theta}\right)$$

  - low quality s_1 is preferred to no consumption if $\theta s_1 - p_1 \geq 0$ ($\Longleftrightarrow \theta \geq \frac{p_1}{s_1} := \underline{\theta}$).

$$D_1\left(s_1, s_2, p_1, p_2\right) = F\left(\hat{\theta}\right) - F\left(\underline{\theta}\right)$$

Vertical Differentiation
Model

- Assume F is uniform (and has full support) on $[\underline{\theta},\overline{\theta}]$ with $\overline{\theta} = \underline{\theta} + 1$
- So we get

$$\begin{aligned} D_1\left(s_1, s_2, p_1, p_2\right) &= F\left(\hat{\theta}\right) - F\left(\underline{\theta}\right) = \frac{\hat{\theta} - \underline{\theta}}{\overline{\theta} - \underline{\theta}} = \hat{\theta} - \underline{\theta} \\ &= \frac{p_2 - p_1}{s_2 - s_1} - \underline{\theta} \end{aligned}$$

and

$$\begin{aligned} D_2\left(s_1, s_2, p_1, p_2\right) &= 1 - F\left(\hat{\theta}\right) = 1 - \frac{\hat{\theta} - \underline{\theta}}{\overline{\theta} - \underline{\theta}} \\ &= \overline{\theta} - \hat{\theta} = \overline{\theta} - \frac{p_2 - p_1}{s_2 - s_1}. \end{aligned}$$

Vertical Differentiation
Results

- Let us solve the game by backward induction.
- Fix two qualities s_1 and s_2 and let us solve the price competition.
- In Nash equilibrium, each firm maximizes

$$\pi^i = \left(p_i - c\right) D_i\left(s_1, s_2, p_1, p_2\right)$$

with respect to p_i.

Vertical Differentiation
Results

- The reaction functions are

$$p_1^*\left(p_2\right) = \frac{p_2 + c - \underline{\theta}\left(s_2 - s_1\right)}{2}$$

and

$$p_2^*\left(p_1\right) = \frac{p_1 + c + \overline{\theta}\left(s_2 - s_1\right)}{2}$$

Vertical Differentiation
Results

- The Nash equilibrium satisfies

$$p_i^*\left(p_j^*\left(p_i^*\right)\right)=p_i^*$$

which implies

$$p_1^*=c+\frac{(\overline{\theta}-2\underline{\theta})(s_2-s_1)}{3}$$

and

$$p_2^*=c+\frac{(2\overline{\theta}-\underline{\theta})(s_2-s_1)}{3}>p_1^*$$

Vertical Differentiation
Results

- This yiels demands for firm 1

$$D_1=\max\left\{\frac{\overline{\theta}-2\underline{\theta}}{3},0\right\}$$

which is positive if the amount of consumers heterogeneity is sufficiently large: $\overline{\theta}>2\underline{\theta}$.
  - In case of low consumer heterogeneity ($\overline{\theta}<2\underline{\theta}$) intense price competition drives the low-quality firm out.
  - The intuition is that a low quality firm cannot compete with the higher quality firm.
- And for firm 2

$$D_2=\frac{2\overline{\theta}-\underline{\theta}}{3}$$

Vertical Differentiation
Results

- The profits then write

$$\pi^1(s_1,s_2)=\frac{(\overline{\theta}-2\underline{\theta})^2(s_2-s_1)}{9}$$

and

$$\pi^2(s_1,s_2)=\frac{(2\overline{\theta}-\underline{\theta})^2(s_2-s_1)}{9}>\pi^1(s_1,s_2)$$

- Observe that Bertrand's result is obtained when there is no differentiation (i.e., $s_1=s_2$)

Vertical Differentiation
Results

- Now, we can solve the quality choice.

Proposition

If the choice of the quality is costless then maximal differentiation (i.e., $s_1^*=\underline{s}$, $s_2^*=\bar{s}$) is the unique pure-strategy Nash equilibrium.

- If we relax the assumption $s_2>s_1$, there is another Nash equilibrium: $s_1^*=\bar{s}$, $s_2^*=\underline{s}$.

Vertical Differentiation
Results

Proof.

If $s_1 < s_2$ then from

$$\frac{\partial \pi^1(s_1,s_2)}{\partial s_1} < 0 \quad and \quad \frac{\partial \pi^2(s_1,s_2)}{\partial s_2} > 0$$

we obtain the result.

If $s_1 = s_2$ then both firms make zero profits. At least one firm has then an incentive to differentiate from the other to realize positive profit, a contradiction. □

Vertical Differentiation
Conclusion

- As for the spatial model (horizontal differentiation) firms try to relax price competition through product differentiation.
- Even if quality is costless to produce, the low quality firm gains from reducing its quality to the minimum because it softens price competition.
- If sequential entry, the unique Nash equilibrium is the first chooses $\bar{s}$ and the second chooses $\underline{s}$.
  - But then race to be first...

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