Game Theory with Economic and Finance Applications Magistère BFA 2 - Fall 2024

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Chap.1 Simultaneous games

Simultaneous game

A simultaneous game is defined by:

• A finite set of *n* players $N = \{1, 2, ..., n\}$

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- S₁, ..., S_n

Game Theory

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- Notation:
 - A strategy profile $s = (s_1, ..., s_n)$ specifies a strategy for each player.

Game Theory

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Simultaneous game			
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- Notation:
 - A strategy profile $s = (s_1, ..., s_n)$ specifies a strategy for each player.
 - Denote $s_{-i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_n)$ the strategy of *i*'s opponents

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Chap.1 Simultaneous games

Players?		

Beauty contest

• Players?

- ▶ all students *N* = {1, ..., 79}
- Strategy sets?

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Beauty contest			

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• Without showing your neighbor what you are doing, write down on a form either the letter α or the letter β .

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Payoff functions?

• $u_i = 1$ if closest to half the average, 0 otherwise.

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Grade game

- Without showing your neighbor what you are doing, write down on a form either the letter α or the letter β .
 - Think of this as a 'grade bid'.

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 - We will randomly pair your form with one other form.

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- Without showing your neighbor what you are doing, write down on a form either the letter *α* or the letter *β*.
 - Think of this as a 'grade bid'.
 - We will randomly pair your form with one other form.
 - ▶ Neither you nor your pair will ever know with whom you were paired.
- Here is how grades may be assigned for this course:

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Game Theory

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 - if both you and your pair put α , then you both will get grade B^- ;

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 - if both you and your pair put α , then you both will get grade B^- ;
 - if you put β and your pair puts α, then you will get grade C, and your pair grade A;
 - if both you and your pair put β , then you will both get grade B^+ .

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 - if you put β and your pair puts α, then you will get grade C, and your pair grade A;

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Grade game

		Your pair	r
		αlpha	β
You	αlpha	(B-, B-)	(A, C)
rou	β	(C, A)	(B+, B+)

Vocabulary

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Vocabulary

• The possible choices, α or β , are called '**strategies**'.

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- The possible choices, α or β , are called '**strategies**'.
- ► The grades e.g., (A, C)-, are 'outcomes'.
- Q.: What do you play?

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Grade game

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- Number of students having played α (resp. β):
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 - > To answer this, we first need to know what that person cares about.
 - What 'payoff' does each outcome yield for this person?

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Game Theory

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Grade game Possible payoffs: selfish players

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 - This depends on the preferences (and moral sentiments?) of the players, not just you but also your opponents.

• Assume every player is selfish, only caring about her own grade then (assuming she prefers A to B etc.) the payoffs associated to the outcome might be as follows:

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 - What 'payoff' does each outcome yield for this person?
- Game theory can not tell us what payoffs to assign to outcomes.
 - This depends on the preferences (and moral sentiments?) of the players, not just you but also your opponents.
- But game theory has a lot to say about how to play the game once payoffs are known.

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Grade game Possible payoffs: selfish players

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- Assume every player is selfish, only caring about her own grade then (assuming she prefers A to B etc.) the payoffs associated to the outcome might be as follows:
 - A = 3 points; $B^+ = 1$ points; $B^- = 0$ points; and C = -1 points.

Game Theory

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- So, the payoffs matrix writes as:

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- A.: If your pair chooses *α*, then you choosing *α* yields a higher payoff than you choosing *β*.

Game Theory

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- Q.: What should you choose in this case?
- A.: If your pair chooses *α*, then you choosing *α* yields a higher payoff than you choosing *β*.
 - If your pair chooses β, then again, you choosing α yields a higher payoff than you choosing β.
- So, you should always choose α because the payoff from α is strictly higher than that from β regardless of others' choices.

Game Theory

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Definitions

Dominant and dominated strategies

Definition (Informal)

A strategy is **dominated** if, regardless of what any other players do, the strategy earns a player a smaller payoff than some other strategy.

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Definition (Informal)

A strategy is **dominated** if, regardless of what any other players do, the strategy earns a player a smaller payoff than some other strategy.

Game Theory

Definition (Formal)

A strategy s_i is (**strictly**) **dominated** if there exits some $s'_i \in S_i$ such that for all $s_{-i} \in S_{-i}$ we have $u_i(s'_i, s_{-i}) > u_i(s_i, s_{-i})$

Game Theory

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Definitions

Dominant and dominated strategies

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A strategy is **dominated** if, regardless of what any other players do, the strategy earns a player a smaller payoff than some other strategy.

Game Theory

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As a "rational" player, you should never play a strictly dominated strategy.

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Game Theory

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In the Grade game, α is a dominant strategy.

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As a "rational" player, you should always play a dominant strategy (if you have one to play).

Game Theory

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As a "rational" player, you should always play a dominant strategy (if you have one to play).

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Grade game Possible payoffs: selfish players

		Your pair		
		αlpha β		
You	αlpha	(0, 0)	(3, -1)	
Tou	β	(-1, 3)	(1, 1)	

Game Theory

• Unfortunately, the reasoning is the same for your pair:

		Your pair		
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 - given these payoffs, she will also choose α .

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- Unfortunately, the reasoning is the same for your pair:
 - given these payoffs, she will also choose α .
- You will end up both getting B^- even though there is a possible outcome (B^+, B^+) that is better for both of you.
 - ▶ To use some economics jargon: the outcome (B^-, B^-) is Pareto inefficient.

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Grade game Possible payoffs: selfish players

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		Your pair		
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• Games like this one are called *Prisoners' Dilemmas*.

Lesson						
Rational play by rational players can lead to bad outcomes.						
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- Games like this one are called *Prisoners' Dilemmas*.
 - The *jointly* preferred outcome (B⁺, B⁺) arises when each chooses its *individually* worse strategy (i.e., β).

Lesson				
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Grade game Possible payoffs: indignant altruistic players

• Suppose that each person cares not only about her own grade but also about the grade of the person with whom she is paired.

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 - For example, each player likes getting an A but she feels guilty that this is at the expense of her pair getting a C.

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Grade game Possible payoffs: indignant altruistic players

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 - $\star\,$ The guilt lowers her payoff from 3 to -1.

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 - ★ The guilt lowers her payoff from 3 to -1.
 - * Conversely, if she gets a C because her pair gets an A, indignation reduces the payoff from -1 to -3.

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- A.: If your pair chooses α , then you choosing α yields a higher payoff than you choosing β .

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- Q.: What should you choose in this case?
- A.: If your pair chooses α , then you choosing α yields a higher payoff than you choosing β .
 - If your pair chooses β , however, then you choosing β yields a higher payoff than you choosing α .
 - In this case, no strategy is dominated.

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Grade game Possible payoffs: indignant altruistic players

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 - The best choice depends on what you think your pair is likely to do.

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Grade game

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Possible payoffs: indignant altruistic players

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- Q.: What should you choose in this case?
- A.: If your pair chooses *α*, then you choosing *α* yields a higher payoff than you choosing *β*.
 - If your pair chooses β, however, then you choosing β yields a higher payoff than you choosing α.
 - In this case, no strategy is dominated.
 - The best choice depends on what you think your pair is likely to do.
 - Later in the course, we will examine games like this called 'co-ordination games'.

Game Theory

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Grade game Possible payoffs: indignant altruistic players

Lesson

To figure out what actions you should choose in a game, a good first step is to figure out what are your payoffs (what do you care about) and what are other players' payoffs. • Suppose you are a selfish player playing with an indignant altruistic player.

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Grade game

Possible payoffs: selfish player vs indignant altruistic player

Game Theory

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Game Theory

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• Suppose you are a selfish player playing with an indignant altruistic player.

			Your pair	
			αlpha	β
	You	αlpha	(0, 0)	(3, -3)
•	Tou	β	(-1, -1)	(1, 1)

• Q.: What should you choose in this case?

• Suppose you are an indignant altruistic player playing with a selfish player.

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Grade game

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Possible payoffs: indignant altruistic player vs selfish player

 Suppose you are an indignant altruistic player playing with a selfish player.

			Your pair	
			αlpha	β
	Vou	αlpha	(0, 0)	(-1, -1)
•	You	β	(-3, 3)	(1, 1)

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Grade game Possible payoffs: selfish player vs indignant altruistic player

• Suppose you are a selfish player playing with an indignant altruistic player.

		Your pair		
		αlpha	β	
Vau	αlpha	(0, 0)	(3, -3)	
You	β	(-1, -1)	(1, 1)	

- Q.: What should you choose in this case?
- A.: Your strategy α strictly dominates your strategy β .

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• Suppose you are an indignant altruistic player playing with a selfish player.

		Your pair	
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Grade game Possible payoffs: indignant altruistic player vs selfish player

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	Vou	αlpha	(0, 0)	(-1, -1)
•	You	β	(-3, 3)	(1, 1)

- Q.: What should you choose in this case?
- A.: Neither of your strategies dominates the other.
 - But, your pair's strategy α strictly dominates her strategy β .

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Possible payoffs: indignant altruistic player vs selfish player

• Suppose you are an indignant altruistic player playing with a selfish player.

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			αlpha	β
	Vau	αlpha	(0, 0)	(-1, -1)
•	You	β	(-3, 3)	(1, 1)

- Q.: What should you choose in this case?
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Grade game

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Game Theory

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Game Theory

• In which case, you should play α .

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Grade game Possible payoffs: indignant altruistic player vs selfish player

Lesson

If you do not have a dominated strategy, put yourself in your opponents' shoes to try to predict what they will do.

Example

In their shoes, you would not choose a dominated strategy.

• What do real people do in Prisoners' Dilemmas?

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Grade game Conclusion

- What do real people do in Prisoners' Dilemmas?
 - Only about % of the class chose β in the grade game.

Game Theory

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 - Only about % of the class chose β in the grade game.
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 - Does this mean that Dauphine students are smarter than normal folk?
 - Not necessarily. It could just be that Dauphine students are selfish.

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Grade game			
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Prisoner's dilemma			

Game Theory

• Conductor of orchestra under Stalin era.

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Prisoner's dilemma

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- Choose between: to stay silent/to denounce.
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Game Theory

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Prisoner's dilemma

- 4 possible outcomes (conductor's years in jail):
 - 1 He denounces and he is not denounced: 1 year;

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Prisoner's dilemma

• 4 possible outcomes	(conductor's years in jail):
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Prisoner's dilemma

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 - 1 He denounces and he is not denounced: 1 year;
 - **2** He stays silent and he is not denounced: 3 years;

Game Theory

- 4 possible outcomes (conductor's years in jail):
 - 1 He denounces and he is not denounced: 1 year;
 - 2 He stays silent and he is not denounced: 3 years;
 - 3 He denounces and he is denounced: 10 years;

Prisoner's dilemma

• Strategy sets $S_1 = S_2 = \{\text{Denounce, Stay silent}\}$

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Prisoner's dilemma

- 4 possible outcomes (conductor's years in jail):
 - 1 He denounces and he is not denounced: 1 year;
 - 2 He stays silent and he is not denounced: 3 years;
 - 3 He denounces and he is denounced: 10 years;
 - 4 He denounces but he is denounced: 25 years.

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Prisoner's dilemma

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 - $u_i(Denounce, Denounce) = -10;$

Prisoner's dilemma

- Strategy sets $S_1 = S_2 = \{\text{Denounce, Stay silent}\}$
- Payoffs, for i = 1, 2:
 - ► u_i (Denounce, Denounce) = -10;
 - $u_i(\text{Stay silent}, \text{Stay silent}) = -3;$
 - $u_1(\text{Stay silent}, \text{Denounce}) = u_2(\text{Denounce}, \text{Stay silent}) = -25$; and

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Prisoner's dilemma	Prisoner's dilemma
• Strategy sets $S_1 = S_2 = \{Denounce, Stay silent\}$	• Strategy sets $S_1 = S_2 = \{Denounce, Stay silent\}$
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	• $u_1(Stay \ silent, Denounce) = u_2(Denounce, Stay \ silent) = -25;$ and
	• $u_1(Denounce, Stay silent) = u_2(Stay silent, Denounce) = -1.$

Game Theory

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Represent the game in table:

		Player	2
		Denounce	Stays Silent
	Denounce	(-10,-10)	(-1,-25)
Player 1	Stays Silent	(-25,-1)	(-3,-3)

Prisoner's dilemma

• The same holds for Tchaikovsky namesake.

• Individually rational strategy :

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Prisoner's dilemma

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Game Theory

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Prisoner's dilemma

• Later, when they meet in the Gulag, they compare stories and realize that they have been had.

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Prisoner's dilemma

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Prisoner's dilemma

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- Later, when they meet in the Gulag, they compare stories and realize that they have been had.
- If only they had the opportunity to meet and talk things over before they were interrogated, they could have agreed that neither would give in.

Game Theory

- Later, when they meet in the Gulag, they compare stories and realize that they have been had.
- If only they had the opportunity to meet and talk things over before they were interrogated, they could have agreed that neither would give in.
- However, once separated, each one get a better deal by double-crossing the other.

• Each superpower prefers the outcome where others, are disarmed while he is keeping his arsenal "just in case".

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Prisoner's dilemma Examples of prisoner's dilemma: Nuclear race

• Each superpower prefers the outcome where others, are disarmed while he is keeping his arsenal "just in case".

Game Theory

• To be disarmed while others keep their arsenal is inconceivable.

Game Theory

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Prisoner's dilemma

• Later, when they meet in the Gulag, they compare stories and realize that they have been had.

Game Theory

- If only they had the opportunity to meet and talk things over before they were interrogated, they could have agreed that neither would give in.
- However, once separated, each one get a better deal by double-crossing the other.
- Problem: As in the Grade game, the *jointly* preferred outcome arises when each chooses its *individually* worse strategy.

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Game Theory

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Game Theory

• ... then secretly breaking the pact.

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- We shall study how to solve such avenues.

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Prisoner's dilemma			
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Another example of prisoner's dilemma: Apps for Ipads

• The Web is a non-commercial entity that enables information spread and commerce as nothing that has come before.

- The Web is a non-commercial entity that enables information spread and commerce as nothing that has come before.
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Another example of prisoner's dilemma: Apps for Ipads

• The Web is a non-commercial entity that enables information spread and commerce as nothing that has come before.

Game Theory

Game Theory

- His success relies on two salient characteristics.
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 - If it is converted to HTML, we all can see it (and even save or print it).
- Connectivity.

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Game Theory

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Game Theory

- It becomes part of the whole system.
- Furthermore, linking allows us to vote for what we think is important. Links, after all, form the basis of how search engines like Google help us find what we're looking for.

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Prisoner's dilemma Another example of prisoner's dilemma: Apps for Ipads

• Today, universality and connectivity of the Web are threatened by *closed Internet applications* or "apps" that are designed to be proprietary, like those on devices such as *iPads* or *iPhones* and, to a lesser extent, on web sites like *Facebook*.

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- There is a clear benefit to universality and connectivity.
- However, individual corporations stand to benefit if they can rig the game towards proprietary solutions (i.e. screw their buddy).

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- There is a clear benefit to universality and connectivity.
- However, individual corporations stand to benefit if they can rig the game towards proprietary solutions (i.e. screw their buddy).
- If that happens, it will hurt consumers and threatens free enterprise and innovation.

Game Theory

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Equilibrium and efficiency

• Prisoner's dilemma represents the classic conflict between:

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Chap.1 Simultaneous games

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- Prisoner's dilemma represents the classic conflict between:
 - individual incentives of players

Equilibrium and efficiency

- Prisoner's dilemma represents the classic conflict between:
 - individual incentives of players
 - joint payoff maximization
- In the equilibrium of a game, the total payoff is typically not maximized.

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Equilibrium and efficiency				Equilibrium and efficiency			

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Game Theory

- Prisoner's dilemma represents the classic conflict between:
 - individual incentives of players
 - joint payoff maximization
- In the equilibrium of a game, the total payoff is typically not maximized.
- In this sense, equilibria are typically "inefficient for players"
- Examples: pricing by firms, international negotiations, arms races

To go to the representation of the game need to make some assumptions:

- if both countries pay 2 % of GDP, no damage on climate
- if only one does, damage is 1.5 % of GDP

Climate Change: Stern Report (2006)

• if none pay, damage is 3 %

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Climate Change: Stern Report (2006)



- Estimates from Stern 2006 report:
 - 4 degrees increase, the damage would be around 3% of GDP

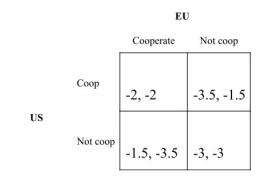
Game Theory

- $\blacktriangleright\,$ 8 degrees increase, damage estimated between 11 to 20 $\%\,$
- Estimates of costs: 1 to 2 % of GDP to limit the rise to 2 3 degrees.

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Climate Negotiations

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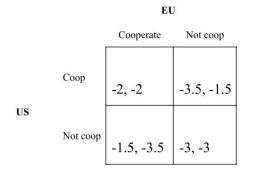
Game Theory

• Is there any strictly dominated strategy?

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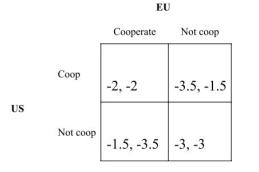
Chap.1 Simultaneous games



- Is there any strictly dominated strategy?
- Yes! "Cooperate" is strictly dominated by "Not cooperate".

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Climate Negotiations



- Is there any strictly dominated strategy?
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Game Theory

► Here, "Not cooperate" is a strictly dominant strategy.

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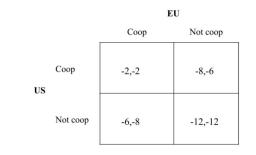
Climate Negotiations: More Dramatic Effects

In 2013, Stern declared to *The Guardian*: "I got it wrong on climate change – it's far, far worse"

- if both countries pay 2 % of GDP, no damage on climate
- if only one does, damage is 6 % of GDP
- if none pay, damage is 12 %

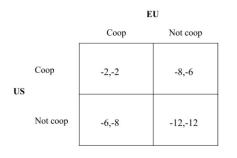
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Climate Negotiations: More Dramatic Effects



• Is there any strictly dominated strategy?

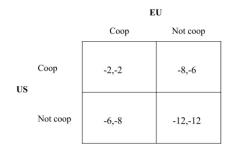
Climate Negotiations: More Dramatic Effects



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Climate Negotiations: More Dramatic Effects



- Is there any strictly dominated strategy?
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 - Here, "Cooperate" is a strictly dominant strategy.

1 Simultaneous games	
2 Elimination of domina	ated strategies
3 Experimental eviden	ce: Iterated strict dominance
4 Nash Equilibrium	
5 More strategies	
6 Multiple equilibria	
7 Focal Point	
B Experimental evidence	ce: Nash equilibrium
Mixed strategies	
D Empirical evidence: I	nixed strategies
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Iterated strict dominance

Simultaneous games

Outline

 It may happen that there is no dominant strategy but stil there are dominated strategies.

	L	М	R
U	(2 , 2)	(1, 1)	(4,0)
D	(1,2)	(4, 1)	(3,5)

Game Theory

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- Consider the following game:

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- Is there any dominant strategy?
 - No.

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Iterated strict dominance

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Iterated strict dominance

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- Is there any dominant strategy?
 - No.
- Is there any strictly dominated strategy?

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Game Theory

- Is there any dominant strategy?
 - No.
- Is there any strictly dominated strategy?
 - Yes: M.

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Iterated strict dominance

Iterated strict dominance:

- 1 Column M dominated by column L: eliminate M
- 2 Once M eliminated, row D dominated by row U: eliminate D

Iterated strict dominance Iterated strict dominance: ate M 1 Column M dominated by column L: eliminate M 2 Once M eliminated, row D dominated by row U: eliminate D 3 Once M and D eliminated, column R dominated	 < ▷ ▷ < 큔 ▷ < Ξ ▷ < Ξ ▷ < Ξ ▷ E ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○	୬ ୯.୧ 37 / 133	Jérôme MATHIS (LEDa - Univ. Paris-Dauphin	Game Theory	 ・・・・・ (日本)・・・ (日本)・・・ 日本) Chap.1 Simultaneous games 	e ৩৭৫ 38 / 133
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 Once M eliminated, row D dominated by row U: eliminate D 			Iterated strict dominance:			
	e M		1 Column M dominated by	column L: elimi	nate M	
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Iterated strict dominance:

Iterated strict dominance

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Game Theory

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Iterated strict dominance

Iterated strict dominance:

- 1 Column M dominated by column L: eliminate M
- 2 Once M eliminated, row D dominated by row U: eliminate D
- 3 Once M and D eliminated, column R dominated
- Iterated strict dominance leads to outcome (U,L)

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Iterated strict dominance

Simultaneous games
 Elimination of dominated strategies
 Experimental evidence: Iterated strict dominance
 Nash Equilibrium
 More strategies
 Multiple equilibria
 Focal Point
 Experimental evidence: Nash equilibrium
 Mixed strategies
 Empirical evidence: mixed strategies

Game Theory

Nagel (AER, 1995): Testing the Beauty Contest

• Groups of 15 -18 subjects each

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Simultaneous games

Outline

- The same group played for four periods
- After each round the response cards were collected
- All chosen numbers, the mean, and half the mean were announced
- The prize to the winner of each round was 20 DM (about \$13)
- After four rounds, each player received the sum of his gains of each period

Game Theory

۲	Iterated	strict	dominance	applied to	the	beauty	contest.

- What is the unique equilibrium?
- Prediction correct? Even if repeated?

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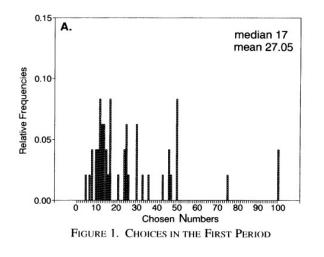
Chap.1 Simultaneous games

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Chap.1 Simultaneous games

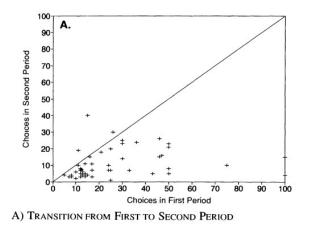
Nagel (AER, 1995): Testing the Beauty Contest **First-Period Choices**



• 6 % of the subjects chose numbers greater than 50

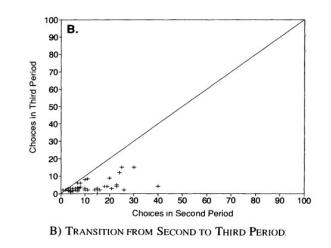
and 8 % chose 50.		(日) (四) (日) (日) (日) (日) (日) (日) (日) (日) (日) (日	≣ ∽৭৫
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Nagel (AER, 1995): Testing the Beauty Contest Choices from periods 1 to 2



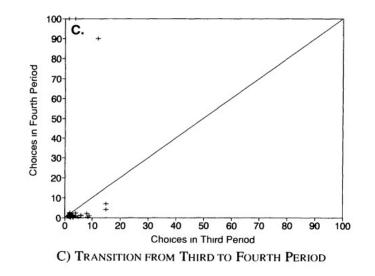
• A plot under the bisecting line indicates that the subject chose a lower number in period 2 than in period 1 < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □

Nagel (AER, 1995): Testing the Beauty Contest Choices from periods 2 to 3



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Nagel (AER, 1995): Testing the Beauty Contest Choices from periods 3 to 4



Game Theory

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Nagel (AER, 1995): Testing the Beauty Contest Conclusion

• The process is driven by iterative, naive best replies rather than by an elimination of dominated strategies.

Nagel (AER, 1995): Testing the Beauty Contest Conclusion

- The process is driven by iterative, naive best replies rather than by an elimination of dominated strategies.
- The process of iteration is finite and not infinite.
- There is a moving target, which approaches zero.

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Nagel (AER, 1995): Testing the Beauty Contest Conclusion

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Nagel (AER, 1995): Testing the Beauty Contest Conclusion

• The process is driven by iterative, naive best replies rather than by an elimination of dominated strategies.

Game Theory

Game Theory

- The process of iteration is finite and not infinite.
- There is a moving target, which approaches zero.
- Over time the chosen numbers approach the *equilibrium* or converge to it.

Nagel (AER, 1995): Testing the Beauty Contest Conclusion

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- (Many) people don't play equilibrium because they are confused.

Nagel (AER, 1995): Testing the Beauty Contest Conclusion

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- (Many) people don't play *equilibrium* because they are confused.
- (Many) people don't play *equilibrium* because doing so (here, choosing 0) doesn't win;
 - ► rather they are cleverly anticipating the behavior of others, with noise.

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 Over time the chosen numbers appro- comported to it 	bach the <i>equilibrium</i> or	5 More strategies	
converge to it.		6 Multiple equilibria	
• (Many) people don't play <i>equilibrium</i>		Focal Point	
 (Many) people don't play equilibrium choosing 0) doesn't win; 	because doing so (here,	B Experimental evidence: Nash equilibrium	
······································		 Mixed strategies 	
		Empirical evidence: mixed strategies	
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• You and a competitor are to set up an ice cream parlour on the beach

- You and a competitor are to set up an ice cream parlour on the beach
- Once built, the location of the parlour is fixed for the season
- People are evenly distributed over the one kilometer long beach, and buy from the nearest vendor

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			Nash equilibrium Location game			
e to set up an ice	e cream parlour on the		 You and a competitor an beach 	e to set up an ic	e cream parlour on the	
f the parlour is f	ixed for the season		 Once built, the location of 	of the parlour is	fixed for the season	
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	e to set up an ice	Game Theory Chap.1 Simultaneous games	e to set up an ice cream parlour on the	Game Theory Chap.1 Simultaneous games 48 / 133 Jérôme MATHIS (LEDa - Univ. Paris-Dauphin Nash equilibrium Location game • to set up an ice cream parlour on the • to set up an ice cream parlour on the • f the parlour is fixed for the season • Once built, the location of • People are evenly district	Game Theory Chap.1 Simultaneous games 48 / 133 Jérôme MATHIS (LEDa - Univ. Paris-Dauphin Game Theory Nash equilibrium Location game Nash equilibrium Location game Image: Chap.1 Simultaneous games Mash equilibrium Location game e to set up an ice cream parlour on the f the parlour is fixed for the season • You and a competitor are to set up an ice beach • Once built, the location of the parlour is	Game Theory Chap.1 Simultaneous games 48 / 133 Jerôme MATHIS (LEDa - Univ. Paris-Dauphin) Game Theory Chap.1 Simultaneous games Nash equilibrium Location game Nash equilibrium Location game e to set up an ice cream parlour on the f the parlour is fixed for the season • You and a competitor are to set up an ice cream parlour on the beach • Once built, the location of the parlour is fixed for the season • People are evenly distributed over the one kilometer long beach

• Ice creams are sold at fixed price

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Nash equilibrium Location game

- You and a competitor are to set up an ice cream parlour on the beach
- Once built, the location of the parlour is fixed for the season
- People are evenly distributed over the one kilometer long beach, and buy from the nearest vendor
- Ice creams are sold at fixed price
- You decide simultaneously on your location

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Nash equilibrium Location game

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• Do you have a dominant strategy?



- Do you have a dominant strategy?
- Where do you go?

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Nash equilibrium Location game



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• To solve this game you need some belief about what the other player will do

Nash equilibrium Location game

- To solve this game you need some belief about what the other player will do
- What you do depends on what you think he will do
- What you do depends on what you think he thinks you will do

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Nash equilibrium Location game		
 To solve this game you player will do 	need some belie	f about what the other

• What you do depends on what you think he will do

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Nash equilibrium Location game

- To solve this game you need some belief about what the other player will do
- What you do depends on what you think he will do
- What you do depends on what you think he thinks you will do
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Game Theory

Nash equilibrium Location game

- To solve this game you need some belief about what the other player will do
- What you do depends on what you think he will do
- What you do depends on what you think he thinks you will do
- What you do depends on what you think he thinks you think he will do

Nash equilibrium Definition

Definition (Informal)

A **Nash equilibrium** is an outcome where given what the other is doing, neither wants to change his own move.

Said differently, a Nash equilibrium is a strategy profile where :

- there is no unilateral profitable deviation; or
- each player's action is the best response to that of the other.

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Nash equilibrium Location game

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- To solve this game you need some belief about what the other player will do
- What you do depends on what you think he will do
- What you do depends on what you think he thinks you will do
- What you do depends on what you think he thinks you think he will do
- ...
- Need equilibrium concept to solve these iterations

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Nash equilibrium Definition

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Definition (Formal)

A strategy profile $(s_1^*, s_2^*, ..., s_n^*)$ is a **Nash equilibrium** if for every *i* and every $s_i^{\prime} \in S_i$ we have $u_i(s_i^*, s_{-i}^*) \ge u_i(s_i^{\prime}, s_{-i}^*)$.

Game Theory

Think of two players. Denote the Nash equilibrium {s₁^{*}, s₂^{*}}. Nash equilibrium means:

Definition (Formal)

A strategy profile $(s_1^*, s_2^*, .., s_n^*)$ is a **Nash equilibrium** if for every *i* and every $s_i' \in S_i$ we have $u_i(s_i^*, s_{-i}^*) \ge u_i(s_i', s_{-i}^*)$.

- Think of two players. Denote the Nash equilibrium $\{s_1^*, s_2^*\}$. Nash equilibrium means:
 - If player 1 plays s^{*}₁, best player 2 can do is play s^{*}₂

• The *best response* to other player's strategy is the strategy for you that maximizes your payoff given what the others play

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	p			
Nash equilibrium		Nash equilibrium		
Definition		Best responses		
Definition (Formal)				
A strategy profile $(s_1^*, s_2^*,, s_n^*)$ is a Nash equ	ilibrium if for every <i>i</i> and	The best response to of	her player's strat	egy is the strategy for you
every $\mathbf{s}'_i \in S_i$ we have $u_i(\mathbf{s}^*_i, \mathbf{s}^*_{-i}) \ge u_i(\mathbf{s}'_i, \mathbf{s}^*_{-i})$		that maximizes your pay	off given what th	ne others play
 Think of two players. Denote the Nash equencies of two players. 	uilibrium $\{s_1^*, s_2^*\}$. Nash	 The best response to a strategies that maximize (maximizes u_i(s'_i, s_{-i})) 	•••••••	ne opponents is the set of /en that the others plays s_{-i}
If player 1 plays s [*] ₁ , best player 2 can do i	s play s ₂ *			
If player 2 plays s [*] ₂ , best player 1 can do i	s play s ₁			

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Nash equilibrium **Best responses**

- The best response to other player's strategy is the strategy for you that maximizes your payoff given what the others play
- The best response to a strategy s_{-i} by the opponents is the set of strategies that maximize your payoffs given that the others plays s_{-i} (maximizes $u_i(s'_i, s_{-i})$)
- Nash equilibrium as we defined it is a fixed point of best responses

		Player 2	
		Denounce	Stays Silent
	Denounce	(-10,-10)	(-1,-25)
Player 1	Stays Silent	(-25,-1)	(-3,-3)

BR1(P2 plays « Denounce »)=

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Nash equilibrium Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Discos 1	Denounce	(-10,-10)	(-1,-25)
Player 1	Stays Silent	(-25,-1)	(-3,-3)

Game Theory

BR1(P2 plays « Denounce »)={Denounce};

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Nash equilibrium Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
2	Denounce	(-10,-10)	(-1,-25)
Player 1	Stays Silent	(-25,-1)	(-3,-3)

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		Player 2	
		Denounce	Stays Silent
Diaman 1	Denounce	(-10,-10)	(-1,-25)
Player 1	Stays Silent	(-25,-1)	(-3,-3)

BR1(P2 plays « Denounce »)={Denounce}; BR1(P2 plays « Stays Silent »)=

		Player 2	
		Denounce	Stays Silent
	Denounce	(-10,-10)	(-1,-25)
Player 1	Stays Silent	(-25,-1)	(-3,-3)

BR1(P2 plays « Denounce »)={Denounce}; BR1(P2 plays « Stays Silent »)={Denounce};

Similarly:

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BR2(P1 plays « Denounce »)={Denounce}; BR2(P1 plays « Stays Silent »)={Denounce}.

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Nash equilibrium Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Disusa 1	Denounce	(-10,-10)	(-1,-25)
Player 1	Stays Silent	(-25,-1)	(-3,-3)

BR1(P2 plays « Denounce »)={Denounce}; BR1(P2 plays « Stays Silent »)={Denounce};

	
Nach aquilibrium	
Nash equilibrium	

Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
	Denounce	(-10,-10)	(-1,-25)
Player 1	Stays Silent	(-25,-1)	(-3,-3)

Game Theory

Game Theory

The unique Nash equilibrium is:

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Chap.1 Simultaneous games

		Player	2
		Denounce	Stays Silent
Diama 1	Denounce	(-10,-10)	(-1,-25)
Player 1	Stays Silent	(-25,-1)	(-3,-3)

The unique Nash equilibrium is: {Denounce, Denounce}.

- Is (0,1) a Nash equilibrium (i.e., both position themselves at the extremes of the beach)?
- Is (1/4,3/4) a Nash equilibrium?

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	Nash equilibrium			
	Back to the beach locatio	n game		
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- Is (0,1) a Nash equilibrium (i.e., both position themselves at the extremes of the beach)?
- Is (1/4,3/4) a Nash equilibrium?
- Ο...

Nash equilibrium Back to the beach location game
• Is (0.4) - Nach an ilibrium (i.e., both position the machine at the

Game Theory

Is (0,1) a Nash equilibrium (i.e., both position themselves at the extremes of the beach)?

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Game Theory

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• You are working for Armani

- You are working for Armani
- Main competitor is Ralph Lauren, with shop next door
- It is the end of the season, so unsold clothes are worthless

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Nash equilibrium Fashion pricing			
You are working for Arm	ani		
Main competitor is Ralph	n Lauren, with s	hop next door	

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Nash equilibrium Fashion pricing

- You are working for Armani
- Main competitor is Ralph Lauren, with shop next door
- It is the end of the season, so unsold clothes are worthless
- Should you have sale or keep prices at normal high level?

Game Theory

Nash equilibrium Fashion pricing

- You are working for Armani
- Main competitor is Ralph Lauren, with shop next door
- It is the end of the season, so unsold clothes are worthless
- Should you have sale or keep prices at normal high level?
- RL has similar dilemma..

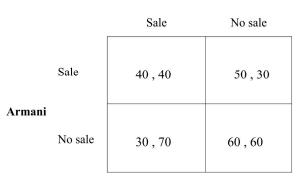
Nash equilibrium Fashion pricing

- You are working for Armani
- Main competitor is Ralph Lauren, with shop next door
- It is the end of the season, so unsold clothes are worthless
- Should you have sale or keep prices at normal high level?
- RL has similar dilemma..
- If only one shop has sale, that shop attracts some of the other shop's customers and possibly some new customers
- You and RL make independent and simultaneous decisions

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Nash equilibrium Fashion pricing

RL



Game Theory

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Nash equilibrium Fashion pricing

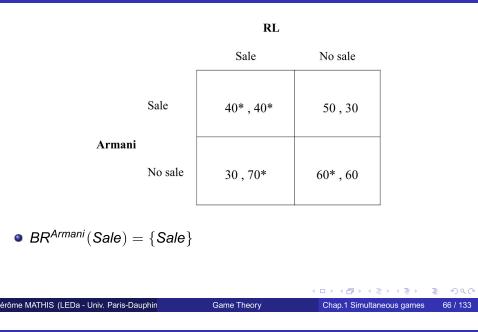
- You are working for Armani
- Main competitor is Ralph Lauren, with shop next door
- It is the end of the season, so unsold clothes are worthless
- Should you have sale or keep prices at normal high level?
- RL has similar dilemma..
- If only one shop has sale, that shop attracts some of the other shop's customers and possibly some new customers

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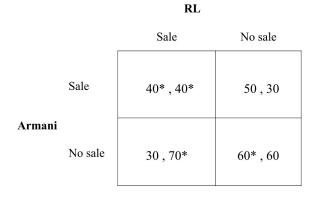
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Chap.1 Simultaneous games

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Nash equilibrium Fashion pricing



• $BR^{RL}(Sale) = \{Sale\}$

		RL		
		Sale	No sale	
	Sale	40* , 40*	50,30	
Armani	No sale	30,70*	60* , 60	
 BR^{Armani}(Sale) = BR^{RL}(Sale) = {S There is a unique 	Sale}	quilibrium: { <i>Sal</i>	e, Sale}	ह > < ह > ह - 9q (°
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Nash equilibrium **Dominant strategy**

• Fashion pricing game is also solvable by iterated deletion of dominated strategy.

Property

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If a game is solvable by iterated deletion of dominated strategies, then the solution is a Nash equilibrium.

Game Theory

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 Fashion pricing game is also solvable by iterated deletion of dominated strategy.

Property

If a game is solvable by iterated deletion of dominated strategies, then the solution is a Nash equilibrium. • Some justifications of the concept:

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Nash equilibrium Dominant strategy

• Fashion pricing game is also solvable by iterated deletion of dominated strategy.

Property

If a game is solvable by iterated deletion of dominated strategies, then the solution is a Nash equilibrium.

Property

If all players have a dominant strategy, then the only Nash Equilibrium is one where all players play their dominant strategy.

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Nash equilibrium Interpretation

- Some justifications of the concept:
 - Introspection: correct conjectures about opponent's play

Game Theory

- Some justifications of the concept:
 - Introspection: correct conjectures about opponent's play
 - Self enforcing agreement: if players communicate and agree initially they will not deviate

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Nash equilibrium			
•			
Interpretation			

Simulation of dominated strategies
Elimination of dominated strategies
Experimental evidence: Iterated strict dominance
Nash Equilibrium
More strategies
Multiple equilibria
Focal Point
Experimental evidence: Nash equilibrium
Mixed strategies
Empirical evidence: mixed strategies

More strategies

Simultaneous games

Outline

- Some justifications of the concept:
 - Introspection: correct conjectures about opponent's play
 - Self enforcing agreement: if players communicate and agree initially they will not deviate
 - Result of learning: situation that arises repeatedly

• Up till now only games with 2 players and 2 choices

Game Theory

More strategies

- Up till now only games with 2 players and 2 choices
- We examine now:

More strategies

- Up till now only games with 2 players and 2 choices
- We examine now:
 - Games with more choices (but finite number)
 - Games with continuous strategy space

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More strategies: 3x3

		COLUMN		
		Left	Middle	Right
	Up	0, 1	9, 0	2, 3
ROW	Straight	5,9	7, 3	1,7
	Down	7, 5	1 0 , 10	3, 5

Iterated strict dominance:

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More strategies

- Up till now only games with 2 players and 2 choices
- We examine now:
 - Games with more choices (but finite number)

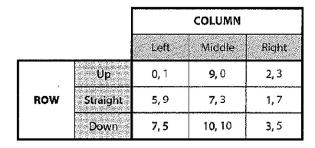
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		COLUMN			
		Left	Middle	Right	
ROW	Up	0, 1	9, 0	2, 3	
	Straight	5,9	7, 3	1,7	
	Down	7, 5	1 0 , 10	3, 5	

- Iterated strict dominance:
- 1 "Down" is a strictly dominant strategy: eliminate "Up" and "Straight"

Game Theory

More strategies: 3x3



- Iterated strict dominance:
- 1 "Down" is a strictly dominant strategy: eliminate "Up" and "Straight"
- 2 Once "Up" and "Straight" are eliminated, "Middle" is a dominant strategy: eliminate "Left" and "Right".
- Iterated strict dominance leads to outcome (Down.Middle)

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More strategies: 4x3

		COLUMN			
		West	Center	East	
ROW	North	2, 3	8, 2.	7,4	
	Up	3, 0	4, 5	б, 4	
	Down	10, 4	6, 1	3, 9	
	South	4, 5	2, 3	5,2	

No strategy can be eliminated (as long as we restrict to pure dominance).

Game Theory

More strategies: 3x3

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		COLUMN			
		Left	Middle	Right	
	Up	0, 1	9 , 0	2, 3	
ROW	Straight	5,9	7, 3	1,7	
	Down	7, 5	10, 10	3,5	

- Iterated strict dominance:
- 1 "Down" is a strictly dominant strategy: eliminate "Up" and "Straight"

Game Theory

2 Once "Up" and "Straight" are eliminated, "Middle" is a dominant strategy: eliminate "Left" and "Right".

More strategies: 4x3

		COLUMN			
		West	Center	East	
ROW	North	2, 3	8, 2.	7,4	
	Up	3, 0	4, 5	б, 4	
	Down	10, 4	6, 1	3, 9	
	South	4, 5	2, 3	5, 2	

- No strategy can be eliminated (as long as we restrict to pure dominance).
- We shall see later on how to solve this game (use of mixed strategies).

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More strategies: infinite number Competition in an oligopoly

• Two firms *i* and *j* compete in quantity they produce (called Cournot competition).

Game Theory

- Two firms *i* and *j* compete in quantity they produce (called Cournot competition).
- We consider here a situation where they make their choice simultaneously

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More strategies: infinite number Competition in an oligopoly

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Game Theory

Game Theory

- We consider here a situation where they make their choice simultaneously
- Strategy of player *i* is quantity *q_i*

More strategies: infinite number Competition in an oligopoly

- Two firms *i* and *j* compete in quantity they produce (called Cournot competition).
- We consider here a situation where they make their choice simultaneously
- Strategy of player *i* is quantity *q_i*
- Given the choice of quantities produced (q_i, q_j), there is a resulting price that emerges in the market: what we call a demand function

• Each unit of good is of course costly to produce

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More strategies: infinite number Competition in an oligopoly

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- Two firms *i* and *j* compete in quantity they produce (called Cournot competition).
- We consider here a situation where they make their choice simultaneously
- Strategy of player *i* is quantity *q_i*
- Given the choice of quantities produced (q_i, q_j), there is a resulting price that emerges in the market: what we call a demand function
- In this case we consider a very simple demand function: price on the market is given by P = 1 - q_i - q_j

More strategies: infinite number Competition in an oligopoly: Objective of firms

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- Each unit of good is of course costly to produce
- Here we assume that each unit costs *c* to produce so that the total cost of production for firm *i* is given by C(q_i) = cq_i

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More strategies: infinite number Competition in an oligopoly: Objective of firms

- Each unit of good is of course costly to produce
- Here we assume that each unit costs *c* to produce so that the total cost of production for firm *i* is given by C(q_i) = cq_i
- If player *i* knows what player *j* does, choice is easy, it just maximizes profits, i.e. price×quantity - cost:

• In practice you don't know for sure what the other one will do

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More strategies: infinite number Competition in an oligopoly: Objective of firms

- Each unit of good is of course costly to produce
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Game Theory

- If player *i* knows what player *j* does, choice is easy, it just maximizes profits, i.e. price×quantity - cost:
 - ▶ In other words, firm *i*, if firm *j* produces q_i , chooses q_i to maximize

$$Pq_i - C(q_i) = (1 - q_i - q_j)q_i - cq_i$$

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Competition in an oligopoly: Nash equilibrium as solution

• In practice you don't know for sure what the other one will do

Game Theory

• Depends on belief of what the others will do

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More strategies: infinite number Competition in an oligopoly: Nash equilibrium as solution

- In practice you don't know for sure what the other one will do
- Depends on belief of what the others will do
- No obvious choice: i.e no dominant strategy

More strategies: infinite number Competition in an oligopoly: Best responses

• To determine the Nash equilibrium, consider firm *i*. It takes the quantity of firm *j* as given and maximizes her own profits by choosing optimally q_i .

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More strategies: infinite number Competition in an oligopoly: Nash equilibrium as solution

Game Theory

- In practice you don't know for sure what the other one will do
- Depends on belief of what the others will do
- No obvious choice: i.e no dominant strategy
- So we look for the Nash Equilibrium

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More strategies: infinite number Competition in an oligopoly: Best responses

• To determine the Nash equilibrium, consider firm *i*. It takes the quantity of firm *j* as given and maximizes her own profits by choosing optimally q_i .

Game Theory

 Problem facing player i, given that opponent produces q_j is to maximize

$$\Pi(q_i) = q_i[1 - (q_i + q_j) - c] = -q_i^2 + q_i(1 - q_j - c)$$

Game Theory

More strategies: infinite number Competition in an oligopoly: Best responses

- To determine the Nash equilibrium, consider firm *i*. It takes the quantity of firm *j* as given and maximizes her own profits by choosing optimally q_i .
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$$\Pi(q_i) = q_i[1 - (q_i + q_j) - c] = -q_i^2 + q_i(1 - q_j - c)$$

• Reminder: to find a maximum, equalize the derivative to zero

$$\Pi'(q_i) = 0$$

 $-2q_i + (1 - q_j - c) = 0$

More strategies: infinite number Competition in an oligopoly: Nash equilibrium

• A Nash equilibrium is a pair (q_i, q_i) such that q_i is a best response to q_i while q_i is itself a best response to q_i .

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More strategies: infinite number Competition in an oligopoly: Best responses

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Game Theory

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So best response is

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$$BR_i(q_j) = \frac{1-c}{2} - \frac{q_j}{2}$$

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More strategies: infinite number Competition in an oligopoly: Nash equilibrium

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Game Theory

Game Theory

•
$$q_i = BR(q_j) = \frac{1-c}{2} - \frac{q_j}{2}$$
 and $q_j = BR(q_i) = \frac{1-c}{2} - \frac{q_i}{2}$

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More strategies: infinite number Competition in an oligopoly: Nash equilibrium

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$$q_i = BR(q_j) = \frac{1-c}{2} - \frac{q_j}{2}$$
 and $q_j = BR(q_i) = \frac{1-c}{2} - \frac{q_i}{2}$

Replace and get:

$$q_i = \frac{1-c}{2} - \frac{1}{2} \left[\frac{1-c}{2} - \frac{q_i}{2} \right]$$

More strategies: infinite number Competition in an oligopoly: Nash equilibrium

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$$P q_i = BR(q_j) = \frac{1-c}{2} - \frac{q_j}{2} \text{ and } q_j = BR(q_i) = \frac{1-c}{2} - \frac{q_i}{2}$$

Replace and get:

$$q_i = \frac{1-c}{2} - \frac{1}{2} \left[\frac{1-c}{2} - \frac{q_i}{2} \right]$$

check yourself that the unique solution is

$$q_i = q_j = (1-c)/3$$

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More strategies: infinite number Competition in an oligopoly: Nash equilibrium

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More strategies: infinite number Competition in an oligopoly: Nash equilibrium

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 - ► $q_i = BR(q_j) = \frac{1-c}{2} \frac{q_j}{2}$ and $q_j = BR(q_i) = \frac{1-c}{2} \frac{q_j}{2}$
 - Replace and get:

$$q_i = \frac{1-c}{2} - \frac{1}{2} \left[\frac{1-c}{2} - \frac{q_i}{2} \right]$$

check yourself that the unique solution is

$$q_i = q_j = (1-c)/3$$

Solution

The unique Nash equilibrium is for each firm to choose quantity $q = \frac{(1-c)}{3}$.

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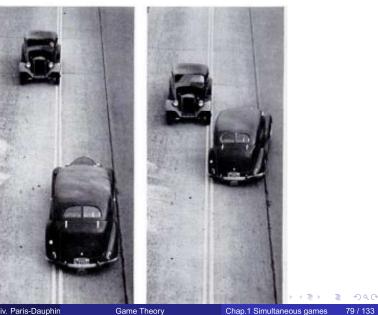
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Simultaneous games Outline

- Multiple equilibria

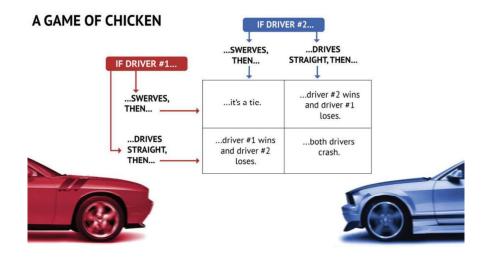
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Multiple equilibria



Game Theory

Multiple equilibria



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Multiple equilibria

There may be multiple Nash equilibria.

Example, the game of chicken (aka hawk-dove).

		Player 2	
		Straight	Swerve
-	Straight	(Crash, Crash)	(Win, Lose)
Player 1	Swerve	(Lose, Win)	(Tie, Tie)

Game Theory

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Chap.1 Simultaneous games

Multiple equilibria

There may be multiple Nash equilibria.

		Player 2		
		Straight	Swerve	
Player 1	Straight	(Crash, Crash)	(Win, Lose)	
	Swerve	(Lose, Win)	(Tie, Tie)	

Example the same of chicken (ake house)

BR1(P2 plays « Straight »)=

Multiple equilibria

There may be multiple Nash equilibria.

Example, the game of chicken (aka hawk-dove) .

		Player 2	
		Straight	Swerve
Straight	Straight	(Crash, Crash)	(Win, Lose)
Player 1	Swerve	(Lose, Win)	(Tie, Tie)

Game Theory

BR1(P2 plays « Straight »)={Swerve}; BR1(P2 plays « Swerve »)=



Multiple equilibria

There may be multiple Nash equilibria.

Example, the game	of chicken (aka hav	wk-dove)
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		Straight	Swerve
-	Straight	(Crash, Crash)	(Win, Lose)
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BR1(P2 plays « Straight »)={Swerve};

Multiple equilibria

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Multiple equilibria

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BR1(P2 plays « Straight »)={Swerve}; BR1(P2 plays « Swerve »)={Straight}; Nash equilibria: {(Straight, Swerve); (Swerve, Straight)}

Multiple equilibria



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Multiple equilibria



Game Theory

Multiple equilibria



Multiple equilibria Prisoner's dilemma: playing with your cousin

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Mixed strategies		
B Experimental evidence:	Nash equilibriu	m
7 Focal Point		
6 Multiple equilibria		
5 More strategies		
Nash Equilibrium		
3 Experimental evidence:	Iterated strict do	ominance
2 Elimination of dominate	d strategies	
Simultaneous games		

Focal Point

Simultaneous games

Outline

• How to select one equilibrium from multiple equilibria?

		Player 2	
		Denounce	Stays Silent
Diawar 1	Denounce	(-10,-10)	(-6,-25)
Player 1	Stays Silent	(-25,-6)	(-3,-3)

• What is the set of Nash equilibrium?

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Focal Point

- How to select one equilibrium from multiple equilibria?
- Usually, the selection proceeds from social norms.

Focal Point

- How to select one equilibrium from multiple equilibria?
- Usually, the selection proceeds from social norms.
- On which side of the road to drive?
 - Dominant strategy: on the side used by other drivers.

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Focal Point			

- How to select one equilibrium from multiple equilibria?
- Usually, the selection proceeds from social norms.
- On which side of the road to drive?

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Focal Point

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Game Theory

• Which side to choose? No side is better than other.

Focal Point

- How to select one equilibrium from multiple equilibria?
- Usually, the selection proceeds from social norms.
- On which side of the road to drive?
 - Dominant strategy: on the side used by other drivers.
- Which side to choose? No side is better than other.
 - ► UK, Australia, Japan: left-side.

- Choosing a date.
- Choosing a place to meet next week in Paris.

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Focal Point			

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Essel Delet			
Focal Point			

Game Theory

• Choosing a date.

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- The weather can modify the rdv location.

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- Choosing a place to meet next week in Paris.
- The weather can modify the rdv location.
- Sunspot can make people moving from one equilibrium to another.

Bank run is a move from one equilibrium to another.

Self-fulfilling prophecy.

northern rock



Northern Rock bank run on September 2007. People queuing outside a branch in London to withdraw their savings due to fallout from the subprime crisis. -100

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Game Theory

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Focal Point Bank run from depositors



On March 21, 2013, people queue at an ATM outside a closed Laiki Bank branch in capital Nicosia, Cyprus.

"There are rumours that Laiki Bank (the Greek name for the Popular Bank) will never open again. I want to take out as much as I can," said a depositor.

Game Theory

"It's all about cash now. Only a gambler will take cheques in this situation," said a depositor.

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Focal Point

- Choosing a date.
- Choosing a place to meet next week in Paris.
- The weather can modify the rdv location.
- Sunspot can make people moving from one equilibrium to another.
- Extrinsic fluctuations can cause financial crises.

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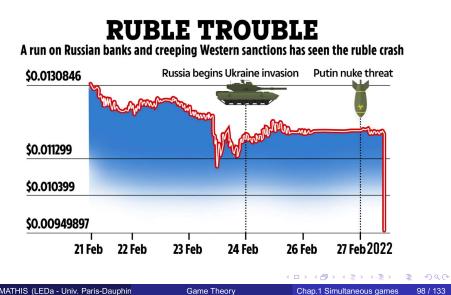
Long lines at Russia's ATMs as citizens rush to withdraw cash amid escalating EU sanctions on 27 Feb 2022

Game Theory

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Focal Point Bank run from depositors



Focal Point Bank run from depositors



Game Theory

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Focal Point Bank run from depositors



Deople wait outside the Silicon Valley Bank headquarters in Santa Clara, California, to withdraw funds after the federal government intervened upon the collapse of the bank. Photograph: Brittany Hosea = nar

Game Theory

Bank run from bondholders

On March 2008, a bank run began on the securities and banking firm *Bear Stearns*. The non deposit-taking bank had financed huge long-term investments by selling short-maturity bonds, making it vulnerable to panic on the part of its bondholders.

Credit officers of rival firms began to say that Bear Stearns would not be able to make good on its obligations. Within two days, Bear Stearns's capital base of \$17 billion had dwindled to \$2 billion in cash. By the next morning, the Fed decided to lend Bear Stearns money (the first time since the Great Depression that it had lent to a nonbank).



Stocks sank, and that day JPMorgan Chase began to buy Bear Stearns as part of a governmentsponsored bailout.

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Simultaneous games Outline

- Simultaneous games
- 2 Elimination of dominated strategies
- Experimental evidence: Iterated strict dominance
- 4 Nash Equilibrium
- 5 More strategies
- 6 Multiple equilibria
- Focal Point
- Experimental evidence: Nash equilibrium
- 9 Mixed strategies
- 10) Empirical evidence: mixed strategies

Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

 Ensminger (Oxford University Press, 2004): Testing Nash equilibrium in *Public good game*

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- *N* players are grouped and each given an amount X (10 for example).

Game Theory

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- For example: N = 4. If you have 10, you give 4 and the others give 20 in total. Total is 24, and each gets 0.5 of that. So you will get 10 4 + 0.5 * 24 = 18.

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• What is a public good?

Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

- What is a public good?
- A **public good** is a good that is both *non-excludable* and *non-rivalrous*
 - non-excludable: non-paying consumers cannot be prevented from accessing it

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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

Game Theory

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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

Game Theory

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Game Theory

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 - E.g., fish stocks, forest, fresh air, national defense, street lighting ...
 - non-rivalrous: one person's consumption of the good does not affect another
 - E.g., cinemas, parks, satellite television, fresh air, national defense
- Need for public provision because these goods will tend to be privately under provided

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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

Game Theory

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Game Theory

	Excludable	Non-excludable
Rivalrous	Private goods food, clothing, cars, personal electronics	Common goods (Common-pool resources) fish stocks, timber, coal
Non-rivalrous	Club goods cinemas, private parks, satellite television	Public goods free-to-air television, air, national defense

Game Theory

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• Ensminger (2004): experiment in small society in Kenya

Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

- Ensminger (2004): experiment in small society in Kenya
- Players grouped by 4 (anonymously) and given 50 shillings
- Can choose to keep the amount or contribute part or the whole of it to a public good

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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

- Ensminger (2004): experiment in small society in Kenya
- Players grouped by 4 (anonymously) and given 50 shillings
- Can choose to keep the amount or contribute part or the whole of it to a public good
- Amount contributed doubled by experimenter and divided among the 4 players: so got back 50 percent of the total

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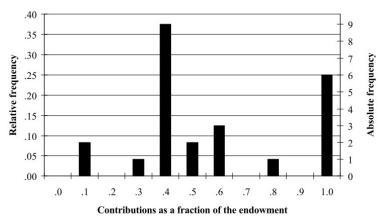
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Chap.1 Simultaneous games

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Distribution of offers in the 4-person public goods game (N=24, endowment=50 Kenyan shillings with doubling of contributions by experimenter)



Results: on average contributions were 60 percent of endowments
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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

Players give more than in the NE (where contributions should be zero)

Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

- Players give more than in the NE (where contributions should be zero)
- Example player who gives 20 out of his 50 in a group where other three give 75 total, gets a payoff of:

50-20+0.5*95=77.5

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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

Game Theory

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• If the same player had given 0, he would get

50 + 0.5 * 75 = 87.5

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What explains this?

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Experimental evidenc	o: Nach ogi	uilibrium	
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Ensminger (Oxford Univer	sity Press, 20	04): Public good gar	me

• How do we interpret these deviations?

Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

- How do we interpret these deviations?
 - 1 Players are not rational and cannot compute what is best for them

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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

Game Theory

- How do we interpret these deviations?
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Game Theory

2 Players do not adopt a pure selfish stance:

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- How do we interpret these deviations?
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 - \star aversion for inequality
 - 3 Social norms

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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

Theory

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Experimental evidence: Nash equilibrium

Ensminger (Oxford University Press, 2004): Public good game

Game Theory

Altruism: care not only about your own payoff but also payoff of others

Game Theory

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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

- Altruism: care not only about your own payoff but also payoff of others
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- Specifically, suppose two players, you contribute X and other contributes Y
- Own payoff of player *i* is $P_i = 100 X + 0.8 * (Y + X)$

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Chap.1 Simultaneous games

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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

Game Theory

- Altruism: care not only about your own payoff but also payoff of others
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Experimental evidence: Nash equilibrium Ensminger (Oxford University Press, 2004): Public good game

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Game Theory

Can solve for Nash equilibrium

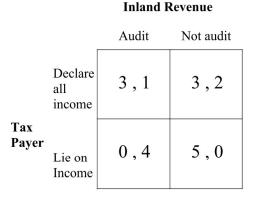
Simultaneous games Outline

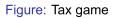
Simultaneous games	
Elimination of dominated strategies	
B Experimental evidence: Iterated strict dom	inance
4 Nash Equilibrium	
5 More strategies	
Multiple equilibria	
Focal Point	
B Experimental evidence: Nash equilibrium	
Initial Strategies	
Empirical evidence: mixed strategies	
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Game Theory

Mixed strategies

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• Does this game have any (pure) strategy equilibrium?

Game Theory Chap.1 Simul

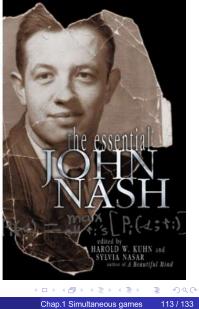
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Chap.1 Simultaneous games

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Theorem (Nash, 1950):

Every finite game has a strategy equilibrium.



Mixed strategies

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 In previous classes, all the games we saw had a Nash equilibrium in what is called "pure strategy": i.e where all players play one action for sure

Game Theory

Mixed strategies

- In previous classes, all the games we saw had a Nash equilibrium in what is called "pure strategy": i.e where all players play one action for sure
- In this game, if you know what the other player is going to choose, the strategy that makes you better off makes him worse off

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Mixed strategies

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Game Theory

 In this game, if you know what the other player is going to choose, the strategy that makes you better off makes him worse off

Game Theory

• No equilibrium in pure strategies

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- In this game, if you know what the other player is going to choose, the strategy that makes you better off makes him worse off
- No equilibrium in pure strategies
- Other example: penalty kicks (most sports in fact)

Mixed strategies

 In the tax game no Nash Equilibrium where the player plays an action for sure: what is called pure strategy

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Mixed strategies

- In the tax game no Nash Equilibrium where the player plays an action for sure: what is called pure strategy
- Exists other types of strategies, where the players randomize over actions: called *mixed strategy*

Game Theory

Game Theory

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Mixed strategies

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- In this game, if you know what the other player is going to choose, the strategy that makes you better off makes him worse off
- No equilibrium in pure strategies
- Other example: penalty kicks (most sports in fact)
- Intuitively the only outcome is an outcome where the other player does not know for sure what you are going to play: players randomize

Game Theory

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Mixed strategies

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Definition

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Chap.1 Simultaneous games

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Mixed strategies

- In the tax game no Nash Equilibrium where the player plays an action for sure: what is called pure strategy
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Game Theory

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Mixed strategies

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- It is as if you were giving these probabilities to a machine that picked accordingly and told you what strategy you should play
- Example a strategy in tax game for tax authority could be: "audit" with probability 0.4 and "not audit" with probability 0.6

Game Theory

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• Payoff for a player is a weighted average of payoff of each action where the weight is the probability: called expected payoff

- Payoff for a player is a weighted average of payoff of each action where the weight is the probability: called expected payoff
- Suppose for instance that the tax payer plays a mixed strategy: "declare" with probability 0.2 and "lie" with 0.8.
- Then the payoff of the tax authority if it plays "audit" is:

$$0.2 \times 1 + 0.8 \times 4 = 3.4$$

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Mixed strategies			
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Mixed strategies			
Mixed strategies			
Nash equilibrium			

• Definition of Nash Equilibrium remains the same: combination of strategies such that if other players play their Nash equilibrium strategies, you also want to play your Nash equilibrium strategy

Game Theory

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Mixed strategies Nash equilibrium

- Definition of Nash Equilibrium remains the same: combination of strategies such that if other players play their Nash equilibrium strategies, you also want to play your Nash equilibrium strategy
- Remember strategy is defined for a mixed strategy by a combination of probabilities

Mixed strategies

• To determine an equilibrium in mixed strategies, we will always use the following essential property:

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Mixed strategies			
Nash equilibrium			

- Definition of Nash Equilibrium remains the same: combination of strategies such that if other players play their Nash equilibrium strategies, you also want to play your Nash equilibrium strategy
- Remember strategy is defined for a mixed strategy by a combination of probabilities
- So the probabilities are not any probabilities: they are defined at the equilibrium

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Mixed strategies

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Game Theory

• To determine an equilibrium in mixed strategies, we will always use the following essential property:

Property (Indifference)

In equilibrium, the players are indifferent (i.e get the same payoff) from all the strategies they play with positive probability.

Mixed strategies

Example

What is the Nash equilibrium of the Tax payer game?

	L	R
U	(3, 1)	(3, 2)
	(0, 4)	

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Mixed strategies

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• To determine an equilibrium in mixed strategies, we will always use the following essential property:

Property (Indifference)

In equilibrium, the players are indifferent (i.e get the same payoff) from all the strategies they play with positive probability.

• For example, if the tax payer in equilibrium plays "declare" with probability 0.2 and "lie" with 0.8, then his payoff if he played "declare" for sure and his payoff if he played "lie" for sure should be equal.

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Mixed strategies

Example

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What is the Nash equilibrium of the Tax payer game? L R

 $\begin{array}{ccc} U & (3,1) & (3,2) \\ D & (0,4) & (5,0) \end{array}$

Solution

Nash equilibrium is such that:

Player 1 plays U with probability 4/5 and D with probability 1/5

Player 2 plays L with probability 2/5 and R with probability 3/5

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• To check that this is an equilibrium, we need to show that no player can do better by switching strategy if the other plays his Nash equilibrium strategy.

Mixed strategies

- To check that this is an equilibrium, we need to show that no player can do better by switching strategy if the other plays his Nash equilibrium strategy.
- Fix player 2 at his Nash equilibrium strategy: plays L with probability 2/5 and R with probability 3/5.
- We need to check that Player 1 is ready to play U with probability 4/5 and D with probability 1/5, i.e that he is indifferent between U and D:

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Mixed strategies

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• Fix player 2 at his Nash equilibrium strategy: plays L with probability 2/5 and R with probability 3/5.

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Mixed strategies

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Game Theory

• Payoff from U: $\frac{2}{5} \times 3 + \frac{3}{5} \times 3 = 3$

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- Fix player 2 at his Nash equilibrium strategy: plays L with probability 2/5 and R with probability 3/5.
- We need to check that Player 1 is ready to play U with probability 4/5 and D with probability 1/5, i.e that he is indifferent between U and D:
 - ▶ Payoff from U: $\frac{2}{5} \times 3 + \frac{3}{5} \times 3 = 3$ ▶ Payoff from D: $\frac{2}{5} \times 0 + \frac{3}{5} \times 5 = 3$

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Chap.1 Simultaneous games

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Mixed strategies

• To check that this is an equilibrium, we need to show that no player can do better by switching strategy if the other plays his Nash equilibrium strategy.

Game Theory

- Fix player 2 at his Nash equilibrium strategy: plays L with probability 2/5 and R with probability 3/5.
- We need to check that Player 1 is ready to play U with probability 4/5 and D with probability 1/5, i.e that he is indifferent between U and D:

 - ▶ Payoff from U: $\frac{2}{5} \times 3 + \frac{3}{5} \times 3 = 3$ ▶ Payoff from D: $\frac{2}{5} \times 0 + \frac{3}{5} \times 5 = 3$
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Game Theory

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Mixed strategies Finding it

If the strategy is not given to you but you want to find it, just assume it is of the type:

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 - Player 1 plays U with probability p and D with 1 p
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$$3 \times q + 3 \times (1-q) = 0 \times q + 5 \times (1-q) \Rightarrow q = \frac{2}{5}$$

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• Player 2 has to be indifferent between L and R, so:

$$1 \times p + 4 \times (1 - p) = 2 \times p + 0 \times (1 - p) \Rightarrow p = \frac{4}{5}$$

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Mixed strategies

 Idea of people randomizing before making decisions can appear unnatural.

Mixed strategies

- Idea of people randomizing before making decisions can appear unnatural.
- Mixed strategies are commonly used:

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Mixed strategies

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Game Theory

Game Theory

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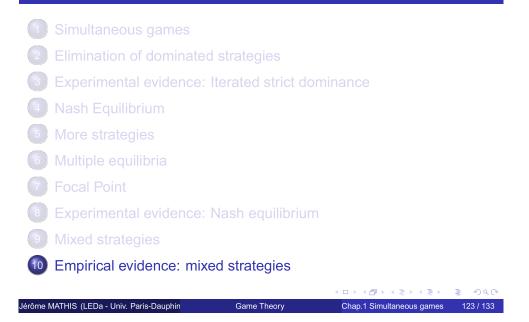
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Game Theory

Players are indifferent, but the probabilities with which they randomize are very well defined: they leave the other players indifferent

Simultaneous games Outline



Chiappori et al. (2002): testing mixed strategies

Game Theory

Mixed strategies can appear unnatural

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• Well defined environment

• Mixed strategies can appear unnatural

Need empirical evidence

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Game Theory

Game Theory

- Well defined environment
- Number of players: 2

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- Need empirical evidence
- Chiappori, Levitt and Groseclose (AER, 2002) using empirical evidence from the French and Italian first-leagues containing 459 penalty kicks over a period of 3 years (1997-2000).

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- Well defined environment
- Number of players: 2
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Chiappori et al. (2002): testing mixed strategies

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Chiappori et al. (2002): testing mixed strategies

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Chiappori et al. (2002): testing mixed strategies

Game Theory

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 - The maximum speed the ball can reach exceeds 125 mph. At this speed, the ball enters the goal about two-tenths of a second after having been kicked.

Game Theory

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- Number of players: 2
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- No ambiguity on preferences of players
- Players play simultaneously
 - The maximum speed the ball can reach exceeds 125 mph. At this speed, the ball enters the goal about two-tenths of a second after having been kicked.
- The structure of this game is such that there is no pure-strategy equilibrium.

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Chiappori et al. (2002): testing mixed strategies

• Seems clear that strikers and goalies randomize.

Chiappori et al. (2002): testing mixed strategies

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- But do probabilities played correspond to the theory?

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Chiappori et al. (2002): testing mixed strategies

Game Theory

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Chiappori et al. (2002): testing mixed strategies

Game Theory

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Chiappori et al. (2002): testing mixed strategies

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- But do probabilities played correspond to the theory?
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 - Do the probabilities with which each player randomize leave the other player indifferent?
- If the indifference property holds, the kicker's scoring probability should be the same whether he kicks L, C or R, and the goalkeeper's probability of averting a goal should the same whether he dives L, C or R.
 - If the players were not indifferent, then it would pay them to adjust their probabilities towards more frequent selection of the strategy with the higher scoring probability (in the case of the kicker) or the strategy with the higher probability of averting a goal (in the case of the goalkeeper).

Game Theory

Game Theory

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Chiappori et al. (2002): testing mixed strategies

• Important elements of the theory:

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 - Kicking at the center when the keeper stays is very damaging for the kicker (the scoring probability is zero)

Chiappori et al. (2002): testing mixed strategies

- Important elements of the theory:
 - Kicking at the center when the keeper stays is very damaging for the kicker (the scoring probability is zero)
 - A right-footed kicker (about 85 percent of the population) will find it easier to kick to his left (his "natural side") than his right; and vice versa for a left-footed kicker.
 - ★ For simplicity, for shots involving left-footed kickers, the direction will be reversed so that shooting left correspond to the "natural side" for all kickers.

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 - ★ For simplicity, for shots involving left-footed kickers, the direction will be reversed so that shooting left correspond to the "natural side" for all kickers.
- Probability of scoring in the middle is lower than on the sides if goalie does not go the correct way

Game Theory

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Indeed, we have:

TABLE 1—OBSERVED SCORING PROBABILITIES, BY FOOT AND SIDE

	Go	alie	
Kicker	Correct side	Middle or wrong side	
Natural side ("left") Opposite side ("right")	63.6 percent 43.7 percent	94.4 percent 89.3 percent	

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Game Theory

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Opposite side ("right")

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	Go	alie	
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- The scoring probability when the goalie is mistaken varies between 89 percent and 95 percent (depending on the kicking foot and the side of the kick), whereas it ranges between 43 percent and 64 percent when the goalkeeper makes the correct choice.
- Also, the scoring probability is always higher on the kicker's natural side.

Game Theory

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Prediction

The indifference property leads to several testable propositions:

(i) The right-footed kicker selects L (his natural side) more often than R;

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Prediction

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(i) The right-footed kicker selects L (his natural side) more often than R;

(ii) The goalkeeper selects L more often than R;

Prediction

The indifference property leads to several testable propositions:

(i) The right-footed kicker selects L (his natural side) more often than R;

(ii) The goalkeeper selects L more often than R;

(iii) The goalkeeper selects L more often than the right-footed kicker;

(iv) The kicker selects C more often than the goalkeeper.

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Game Theory

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 It is straightforward to show that departures from these propositions lead to violations of the indifference property:

Game Theory

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(i) The right-footed kicker selects L (his natural side) more often than R;

(ii) The goalkeeper selects L more often than R;

(iii) The goalkeeper selects L more often than the right-footed kicker;

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- It is straightforward to show that departures from these propositions lead to violations of the indifference property:
 - In the case of (i), if the right-footed kicker selects L and R with equal probability, the goalkeeper would not be indifferent between L and R, because he would avert a goal more often by selecting R (diving to the kicker's weaker side).

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Game Theory

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 - In the case of (i), if the right-footed kicker selects L and R with equal probability, the goalkeeper would not be indifferent between L and R, because he would avert a goal more often by selecting R (diving to the kicker's weaker side).
 - In the case of (ii), if the goalkeeper selects L and R with equal probability, the right-footed kicker would not be indifferent between L and R, because he would score more often by selecting L (kicking on his stronger side).

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 - In the case of (ii), if the goalkeeper selects L and R with equal probability, the right-footed kicker would not be indifferent between L and R, because he would score more often by selecting L (kicking on his stronger side).
 - Selecting C is highly damaging for the kicker if the goalkeeper also selects C. For the kicker to be indifferent between C and either L or R, in accordance with (iv), the goalkeeper must only select C very rarely.

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TABLE 3—	-OBSERVED	MATRIX	OF	SHOTS	TAKEN
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Game Theory

		Kicker		
Goalie	Left	Middle	Right	Total
Left	117	48	95	260
Middle	4	3	4	11
Right	85	28	75	188
Total	206	79	174	459

 Predictions (i) & (ii): the kicker and the goalie are both more likely to go L than R.

Game Theory

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TABLE 3—OBSERVED MATRIX OF SHOTS TAKEN

		Kicker		
Goalie	Left	Middle	Right	Total
Left	117	48	95	260
Middle	4	3	4	11
Right	85	28	75	188
Total	206	79	174	459

- Predictions (i) & (ii): the kicker and the goalie are both more likely to go L than R.
 - This prediction is confirmed: in the data, 260 jumps are made to the (kicker's) left, and only 188 to the right.

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 - This prediction is confirmed: in the data, 260 jumps are made to the (kicker's) left, and only 188 to the right.
 - The same pattern holds for the kicker, although in a less spectacular way (206 against 174).

Game Theory

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 Prediction (iii): The goalkeeper selects L more often than the right-footed kicker.

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Total	206	79	174	459	

- Prediction (iii): The goalkeeper selects L more often than the right-footed kicker.
 - The result emerges very clearly in the data: goalies play "left" 260 times (56.6 percent of kicks), compared to 206 (44.9 percent) instances for kickers.

Game Theory

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• Prediction (iv): The kicker selects C more often than the goalkeeper.

Game Theory

The result emerges very clearly in the data: kickers play "center" 79 times in the sample, compared to only 11 times for goalies.