Game Theory with Economic and Finance Applications

Magistère BFA 2 - Fall 2024

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Chap.1 Simultaneous games

Simultaneous games
Outline

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- Notation:
  - A strategy profile $s = (s_1, ..., s_n)$ specifies a strategy for each player.
  - Denote $s_{-i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_n)$ the strategy of i's opponents

Beauty contest

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  - $S_i = [0, 100]$ for all $i \in N$
- Payoff functions?
  - $u_i = 1$ if closest to half the average, 0 otherwise.

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  - if you put β and your pair puts α, then you will get grade C, and your pair grade A;
  - if both you and your pair put β, then you will both get grade B^+.

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  - The grades - e.g., (A, C)-, are '**outcomes**'.
- Q.: What do you play?

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  - What 'payoff' does each outcome yield for this person?
- Game theory can not tell us what payoffs to assign to outcomes.
  - This depends on the preferences (and moral sentiments?) of the players, not just you but also your opponents.
- But game theory has a lot to say about how to play the game once payoffs are known.

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Dominant and dominated strategies

Definition (Informal)

A strategy is **dominated** if, regardless of what any other players do, the strategy earns a player a smaller payoff than some other strategy.

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- A.: If your pair chooses α, then you choosing α yields a higher payoff than you choosing β.
  - If your pair chooses β, then again, you choosing α yields a higher payoff than you choosing β.
- So, you should always choose α because the payoff from α is strictly higher than that from β **regardless of others' choices**.

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A strategy s_i is **(strictly) dominated** if there exits some $s_i' \in S_i$ such that for all $s_{-i} \in S_{-i}$ we have $u_i(s_i', s_{-i}) > u_i(s_i, s_{-i})$

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As a “rational” player, you should never play a strictly dominated strategy.

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  - given these payoffs, she will also choose α.
- You will end up both getting B^- even though there is a possible outcome (B^+, B^+) that is better for both of you.
  - To use some economics jargon: the outcome (B^-, B^-) is Pareto inefficient.

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- Games like this one are called *Prisoners' Dilemmas*.

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Rational play by rational players can lead to bad outcomes.

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  - The *jointly* preferred outcome (B^+, B^+) arises when each chooses its *individually* worse strategy (i.e., β).

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Possible payoffs: indignant altruistic players

- Suppose that each person cares not only about her own grade but also about the grade of the person with whom she is paired.
  - For example, each player likes getting an A but she feels guilty that this is at the expense of her pair getting a C.

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  - If your pair chooses β, however, then you choosing β yields a higher payoff than you choosing α.
  - In this case, no strategy is dominated.

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  - Later in the course, we will examine games like this called 'co-ordination games'.

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Lesson

To figure out what actions you should choose in a game, a good first step is to figure out what are your payoffs (what do you care about) and what are other players' payoffs.

Grade game
Possible payoffs: selfish player vs indignant altruistic player

- Suppose you are a selfish player playing with an indignant altruistic player.

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Possible payoffs: indignant altruistic player vs selfish player

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  - Therefore, if you know she is rational then you know she'll play α.

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  - Therefore, if you know she is rational then you know she'll play α.
  - In which case, you should play α.

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Lesson

If you do not have a dominated strategy, put yourself in your opponents' shoes to try to predict what they will do.

Example

In their shoes, you would not choose a dominated strategy.

Grade game
Conclusion

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  - Does this mean that Dauphine students are smarter than normal folk?
  - Not necessarily. It could just be that Dauphine students are selfish.

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- 4 possible outcomes (conductor's years in jail):
  1. He denounces and he is not denounced: 1 year;

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  3. He denounces and he is denounced: 10 years;

Prisoner's dilemma

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  1. He denounces and he is not denounced: 1 year;
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  3. He denounces and he is denounced: 10 years;
  4. He denounces but he is denounced: 25 years.

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  - $u_1(Stay\ silent, Denounce) = u_2(Denounce, Stay\ silent) = -25$; and
  - $u_1(Denounce, Stay\ silent) = u_2(Stay\ silent, Denounce) = -1$.

Prisoner's dilemma

Represent the game in table:

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

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Examples of prisoner's dilemma: Nuclear race

- Each superpower prefers the outcome where others, are disarmed while he is keeping his arsenal "just in case".

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- If only they had the opportunity to meet and talk things over before they were interrogated, they could have agreed that neither would give in.
- However, once separated, each one get a better deal by double-crossing the other.
- Problem: As in the Grade game, the *jointly* preferred outcome arises when each chooses its *individually* worse strategy.

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  - ... then secretly breaking the pact.
- We shall study how to solve such avenues.

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- **Connectivity.**
  - Once a page is on the Web, it is theoretically connected to every other page.
  - It becomes part of the whole system.
  - Furthermore, linking allows us to vote for what we think is important. Links, after all, form the basis of how search engines like Google help us find what we're looking for.

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- There is a clear benefit to universality and connectivity.
- However, individual corporations stand to benefit if they can rig the game towards proprietary solutions (i.e. screw their buddy).
- If that happens, it will hurt consumers and threatens free enterprise and innovation.

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- In this sense, equilibria are typically "inefficient for players"
- Examples: pricing by firms, international negotiations, arms races

Climate Change: Stern Report (2006)

AP / Kieran Doherty

- Estimates from Stern 2006 report:
  - 4 degrees increase, the damage would be around 3% of GDP
  - 8 degrees increase, damage estimated between 11 to 20 %
- Estimates of costs: 1 to 2 % of GDP to limit the rise to 2 – 3 degrees.

Climate Change: Stern Report (2006)

To go to the representation of the game need to make some assumptions:

- if both countries pay 2 % of GDP, no damage on climate
- if only one does, damage is 1.5 % of GDP
- if none pay, damage is 3 %

Climate Negotiations

		EU	
		Cooperate	Not coop
US	Coop	-2, -2	-3.5, -1.5
	Not coop	-1.5, -3.5	-3, -3

- Is there any strictly dominated strategy?

Climate Negotiations

US \ EU	Cooperate	Not coop
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- Is there any strictly dominated strategy?
- Yes! “Cooperate” is strictly dominated by “Not cooperate”.

Climate Negotiations: More Dramatic Effects

In 2013, Stern declared to *The Guardian*: “I got it wrong on climate change – it’s far, far worse”

- if both countries pay 2 % of GDP, no damage on climate
- if only one does, damage is 6 % of GDP
- if none pay, damage is 12 %

Climate Negotiations

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Simultaneous games
Outline

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Jérôme MATHIS (LEDa - Univ. Paris-Dauphin Game Theory Chap.1 Simultaneous games 35 / 133

Iterated strict dominance

- It may happen that there is no dominant strategy but stil there are dominated strategies.

$$\begin{array}{cccc} & L & M & R \\ U & (2,2) & (1,1) & (4,0) \\ D & (1,2) & (4,1) & (3,5) \end{array}$$

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- Is there any dominant strategy?
  - No.
- Is there any strictly dominated strategy?
  - Yes: M.

Iterated strict dominance

Iterated strict dominance:

1. Column M dominated by column L: eliminate M
2. Once M eliminated, row D dominated by row U: eliminate D

Iterated strict dominance

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- Iterated strict dominance leads to outcome (U,L)

Iterated strict dominance

- Iterated strict dominance applied to the beauty contest.
- What is the unique equilibrium?
- Prediction correct? Even if repeated?

Simultaneous games
Outline

Nagel (AER, 1995): Testing the Beauty Contest

- Groups of 15 -18 subjects each
- The same group played for four periods
- After each round the response cards were collected
- All chosen numbers, the mean, and half the mean were announced
- The prize to the winner of each round was 20 DM (about $13)
- After four rounds, each player received the sum of his gains of each period

Nagel (AER, 1995): Testing the Beauty Contest
First-Period Choices

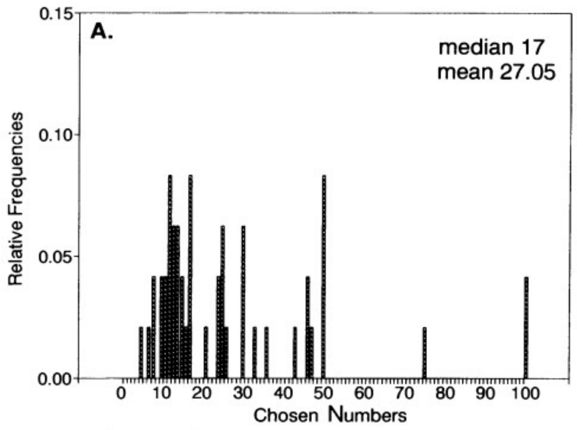
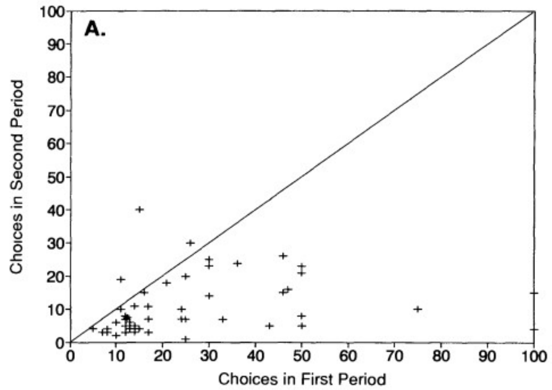


FIGURE 1. CHOICES IN THE FIRST PERIOD

- 6 % of the subjects chose numbers greater than 50
- and 8 % chose 50.

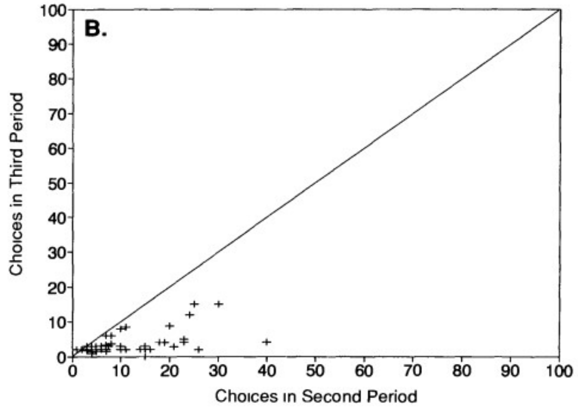
Nagel (AER, 1995): Testing the Beauty Contest
Choices from periods 1 to 2



A) TRANSITION FROM FIRST TO SECOND PERIOD

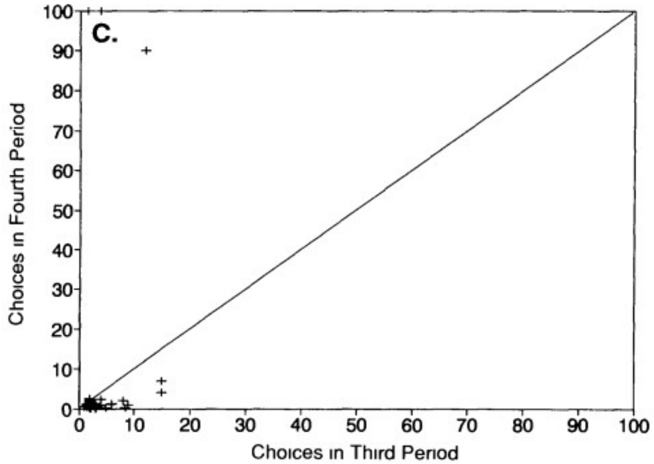
- A plot under the bisecting line indicates that the subject chose a lower number in period 2 than in period 1

Nagel (AER, 1995): Testing the Beauty Contest
Choices from periods 2 to 3



B) TRANSITION FROM SECOND TO THIRD PERIOD

Nagel (AER, 1995): Testing the Beauty Contest
Choices from periods 3 to 4



C) TRANSITION FROM THIRD TO FOURTH PERIOD

Nagel (AER, 1995): Testing the Beauty Contest
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Nagel (AER, 1995): Testing the Beauty Contest
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- There is a moving target, which approaches zero.

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- (Many) people don't play *equilibrium* because they are confused.
- (Many) people don't play *equilibrium* because doing so (here, choosing 0) doesn't win;
  - rather they are cleverly anticipating the behavior of others, with noise.

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Simultaneous games
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Nash equilibrium
Location game

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- Once built, the location of the parlour is fixed for the season
- People are evenly distributed over the one kilometer long beach, and buy from the nearest vendor

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- Ice creams are sold at fixed price
- You decide simultaneously on your location

Nash equilibrium
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- Do you have a dominant strategy?
- Where do you go?

Nash equilibrium
Location game

- Do you have a dominant strategy?

Nash equilibrium
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- ...

Nash equilibrium
Definition

Definition (Informal)

A **Nash equilibrium** is an outcome where given what the other is doing, neither wants to change his own move.

Said differently, a Nash equilibrium is a strategy profile where :

- there is no unilateral profitable deviation; or

- each player's action is the best response to that of the other.

Nash equilibrium
Location game

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- What you do depends on what you think he will do
- What you do depends on what you think he thinks you will do
- What you do depends on what you think he thinks you think he will do
- ...
- Need equilibrium concept to solve these iterations

Nash equilibrium
Definition

Definition (Formal)

A strategy profile $(s_1^*, s_2^*, .., s_n^*)$ is a **Nash equilibrium** if for every i and every $s_i^{'} \in S_i$ we have $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i', s_{-i}^*)$.

- Think of two players. Denote the Nash equilibrium $\{s_1^*, s_2^*\}$. Nash equilibrium means:

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  - If player 1 plays s_1^*, best player 2 can do is play s_2^*

Nash equilibrium
Best responses

- The *best response* to other player's strategy is the strategy for you that maximizes your payoff given what the others play

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  - If player 2 plays s_2^*, best player 1 can do is play s_1^*

Nash equilibrium
Best responses

- The *best response* to other player's strategy is the strategy for you that maximizes your payoff given what the others play
- The best response to a strategy s_{-i} by the opponents is the set of strategies that maximize your payoffs given that the others plays s_{-i} (maximizes $u_i(s_i', s_{-i})$)

Nash equilibrium
Best responses

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- The best response to a strategy s_{-i} by the opponents is the set of strategies that maximize your payoffs given that the others plays s_{-i} (maximizes $u_i(s'_i, s_{-i})$)
- Nash equilibrium as we defined it is a fixed point of best responses

Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

Nash equilibrium
Coming back to Prisoner's Dilemma

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BR1(P2 plays « Denounce »)=

Nash equilibrium
Coming back to Prisoner's Dilemma

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BR1(P2 plays « Denounce »)={Denounce};

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Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
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BR1(P2 plays « Denounce »)={Denounce};
BR1(P2 plays « Stays Silent »)={Denounce};

Similarly:
BR2(P1 plays « Denounce »)={Denounce};
BR2(P1 plays « Stays Silent »)={Denounce}.

Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

The unique Nash equilibrium is:

Nash equilibrium
Coming back to Prisoner's Dilemma

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10) N	(-1,-25)
	Stays Silent	(-25,-1)	(-3,-3)

The unique Nash equilibrium is: {Denounce, Denounce}.

Nash equilibrium
Back to the beach location game

- Is (0,1) a Nash equilibrium (i.e., both position themselves at the extremes of the beach)?
- Is (1/4,3/4) a Nash equilibrium?

Nash equilibrium
Back to the beach location game

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- ...

Nash equilibrium
Fashion pricing

- You are working for Armani

Nash equilibrium
Fashion pricing

- You are working for Armani
- Main competitor is Ralph Lauren, with shop next door
- It is the end of the season, so unsold clothes are worthless

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- Main competitor is Ralph Lauren, with shop next door
- It is the end of the season, so unsold clothes are worthless
- Should you have sale or keep prices at normal high level?
- RL has similar dilemma..
- If only one shop has sale, that shop attracts some of the other shop's customers and possibly some new customers
- You and RL make independent and simultaneous decisions

Nash equilibrium
Fashion pricing

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- Main competitor is Ralph Lauren, with shop next door
- It is the end of the season, so unsold clothes are worthless
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- If only one shop has sale, that shop attracts some of the other shop's customers and possibly some new customers

Nash equilibrium
Fashion pricing

		RL	
		Sale	No sale
Armani	Sale	40 , 40	50 , 30
	No sale	30 , 70	60 , 60

Nash equilibrium
Fashion pricing

		RL: Sale	RL: No sale
Armani	Sale	40* , 40*	50 , 30
	No sale	30 , 70*	60* , 60

- $BR^{Armani}(Sale) = \{Sale\}$

Nash equilibrium
Fashion pricing

		RL: Sale	RL: No sale
Armani	Sale	40* , 40*	50 , 30
	No sale	30 , 70*	60* , 60

- $BR^{Armani}(Sale) = \{Sale\}$
- $BR^{RL}(Sale) = \{Sale\}$
- There is a unique Nash equilibrium: $\{Sale, Sale\}$

Nash equilibrium
Fashion pricing

		RL: Sale	RL: No sale
Armani	Sale	40* , 40*	50 , 30
	No sale	30 , 70*	60* , 60

- $BR^{Armani}(Sale) = \{Sale\}$
- $BR^{RL}(Sale) = \{Sale\}$

Nash equilibrium
Dominant strategy

- Fashion pricing game is also solvable by iterated deletion of dominated strategy.

Property

If a game is solvable by iterated deletion of dominated strategies, then the solution is a Nash equilibrium.

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Nash equilibrium
Interpretation

- Some justifications of the concept:

Nash equilibrium
Dominant strategy

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If a game is solvable by iterated deletion of dominated strategies, then the solution is a Nash equilibrium.

Property

If all players have a dominant strategy, then the only Nash Equilibrium is one where all players play their dominant strategy.

Nash equilibrium
Interpretation

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Nash equilibrium
Interpretation

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  - Self enforcing agreement: if players communicate and agree initially they will not deviate

Simultaneous games
Outline

Nash equilibrium
Interpretation

- Some justifications of the concept:
  - Introspection: correct conjectures about opponent's play
  - Self enforcing agreement: if players communicate and agree initially they will not deviate
  - Result of learning: situation that arises repeatedly

More strategies

- Up till now only games with 2 players and 2 choices

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- We examine now:

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- We examine now:
  - Games with more choices (but finite number)
  - Games with continuous strategy space

More strategies

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- We examine now:
  - Games with more choices (but finite number)

More strategies: 3x3

		COLUMN		
		Left	Middle	Right
ROW	Up	0, 1	9, 0	2, 3
	Straight	5, 9	7, 3	1, 7
	Down	7, 5	10, 10	3, 5

- Iterated strict dominance:

More strategies: 3x3

		COLUMN		
		Left	Middle	Right
ROW	Up	0, 1	9, 0	2, 3
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	Down	7, 5	10, 10	3, 5

- Iterated strict dominance:

1. "Down" is a strictly dominant strategy: eliminate "Up" and "Straight"

More strategies: 3x3

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- Iterated strict dominance:

1. "Down" is a strictly dominant strategy: eliminate "Up" and "Straight"
2. Once "Up" and "Straight" are eliminated, "Middle" is a dominant strategy: eliminate "Left" and "Right".

- Iterated strict dominance leads to outcome (Down,Middle)

More strategies: 3x3

		COLUMN		
		Left	Middle	Right
ROW	Up	0, 1	9, 0	2, 3
	Straight	5, 9	7, 3	1, 7
	Down	7, 5	10, 10	3, 5

- Iterated strict dominance:

1. "Down" is a strictly dominant strategy: eliminate "Up" and "Straight"
2. Once "Up" and "Straight" are eliminated, "Middle" is a dominant strategy: eliminate "Left" and "Right".

More strategies: 4x3

		COLUMN		
		West	Center	East
ROW	North	2, 3	8, 2	7, 4
	Up	3, 0	4, 5	6, 4
	Down	10, 4	6, 1	3, 9
	South	4, 5	2, 3	5, 2

- No strategy can be eliminated (as long as we restrict to pure dominance).

More strategies: 4x3

		COLUMN		
		West	Center	East
ROW	North	2, 3	8, 2	7, 4
	Up	3, 0	4, 5	6, 4
	Down	10, 4	6, 1	3, 9
	South	4, 5	2, 3	5, 2

- No strategy can be eliminated (as long as we restrict to pure dominance).
- We shall see later on how to solve this game (use of mixed strategies).

More strategies: infinite number
Competition in an oligopoly

- Two firms i and j compete in quantity they produce (called Cournot competition).
- We consider here a situation where they make their choice simultaneously

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- Strategy of player i is quantity q_i

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Competition in an oligopoly

- Two firms i and j compete in quantity they produce (called Cournot competition).
- We consider here a situation where they make their choice simultaneously
- Strategy of player i is quantity q_i
- Given the choice of quantities produced (q_i, q_j), there is a resulting price that emerges in the market: what we call a demand function

More strategies: infinite number
Competition in an oligopoly: Objective of firms

- Each unit of good is of course costly to produce

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- Strategy of player i is quantity q_i
- Given the choice of quantities produced (q_i, q_j), there is a resulting price that emerges in the market: what we call a demand function
- In this case we consider a very simple demand function: price on the market is given by $P = 1 - q_i - q_j$

More strategies: infinite number
Competition in an oligopoly: Objective of firms

- Each unit of good is of course costly to produce
- Here we assume that each unit costs c to produce so that the total cost of production for firm i is given by $C(q_i) = cq_i$

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- If player i knows what player j does, choice is easy, it just maximizes profits, i.e. price$\times$quantity - cost:

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Competition in an oligopoly: Nash equilibrium as solution

- In practice you don't know for sure what the other one will do

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- If player i knows what player j does, choice is easy, it just maximizes profits, i.e. price$\times$quantity - cost:
  - In other words, firm i, if firm j produces q_j, chooses q_i to maximize

$$Pq_i - C(q_i) = (1 - q_i - q_j)q_i - cq_i$$

More strategies: infinite number
Competition in an oligopoly: Nash equilibrium as solution

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- Depends on belief of what the others will do

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Competition in an oligopoly: Best responses

- To determine the Nash equilibrium, consider firm i. It takes the quantity of firm j as given and maximizes her own profits by choosing optimally q_i.

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Competition in an oligopoly: Nash equilibrium as solution

- In practice you don't know for sure what the other one will do
- Depends on belief of what the others will do
- No obvious choice: i.e no dominant strategy
- So we look for the Nash Equilibrium

More strategies: infinite number
Competition in an oligopoly: Best responses

- To determine the Nash equilibrium, consider firm i. It takes the quantity of firm j as given and maximizes her own profits by choosing optimally q_i.
- Problem facing player i, given that opponent produces q_j is to maximize

$$\Pi(q_i) = q_i[1 - (q_i + q_j) - c] = -q_i^2 + q_i(1 - q_j - c)$$

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$$\Pi(q_i) = q_i[1-(q_i+q_j)-c] = -q_i^2 + q_i(1-q_j-c)$$
- Reminder: to find a maximum, equalize the derivative to zero
$$\begin{aligned} \Pi'(q_i) &= 0 \\ -2q_i + (1-q_j-c) &= 0 \end{aligned}$$

More strategies: infinite number
Competition in an oligopoly: Nash equilibrium

- A Nash equilibrium is a pair (q_i, q_j) such that q_i is a best response to q_j while q_j is itself a best response to q_i.

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$$\begin{aligned} \Pi'(q_i) &= 0 \\ -2q_i + (1-q_j-c) &= 0 \end{aligned}$$
- So best response is
$$BR_i(q_j) = \frac{1-c}{2} - \frac{q_j}{2}$$

More strategies: infinite number
Competition in an oligopoly: Nash equilibrium

- A Nash equilibrium is a pair (q_i, q_j) such that q_i is a best response to q_j while q_j is itself a best response to q_i.
  - $q_i = BR(q_j) = \frac{1-c}{2} - \frac{q_j}{2}$ and $q_j = BR(q_i) = \frac{1-c}{2} - \frac{q_i}{2}$

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  - Replace and get:

$$q_i = \frac{1-c}{2} - \frac{1}{2}\left[\frac{1-c}{2} - \frac{q_i}{2}\right]$$

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  - check yourself that the unique solution is

$$q_i = q_j = (1-c)/3$$

More strategies: infinite number
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  - Replace and get:

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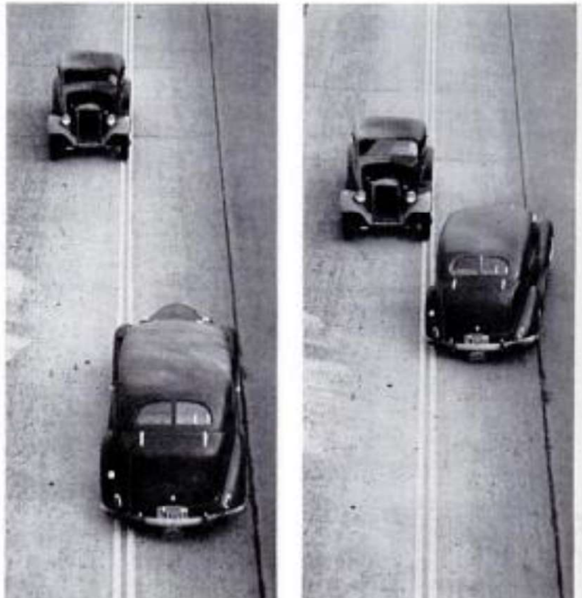
Solution

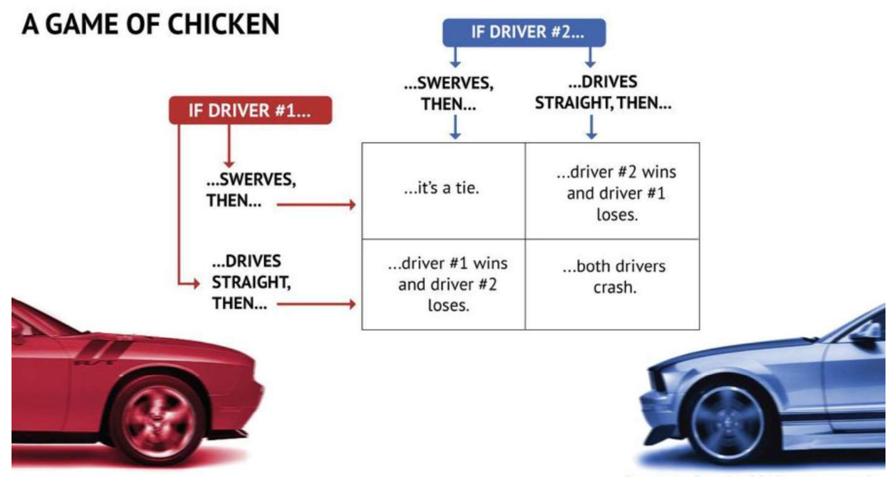
The unique Nash equilibrium is for each firm to choose quantity $q = \frac{(1-c)}{3}$.

Simultaneous games
Outline

Multiple equilibria



Multiple equilibria



Multiple equilibria

There may be multiple Nash equilibria.

Example, the game of chicken (aka hawk-dove) .

		Player 2	
		Straight	Swerve
Player 1	Straight	(Crash, Crash)	(Win, Lose)
	Swerve	(Lose, Win)	(Tie, Tie)

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	Swerve	(Lose, Win)	(Tie, Tie)

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Nash equilibria:

Multiple equilibria

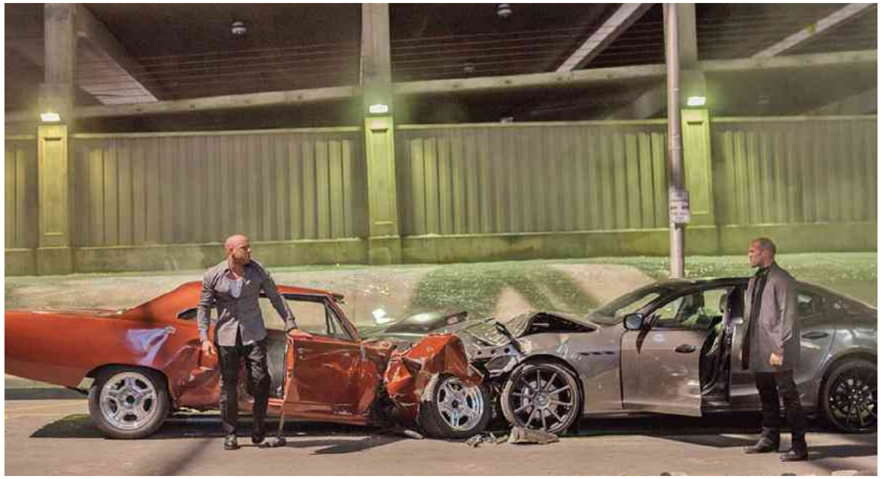
There may be multiple Nash equilibria.

Example, the game of chicken (aka hawk-dove) .

		Player 2	
		Straight	Swerve
Player 1	Straight	(Crash, Crash)	(Win, Lose) N
	Swerve	(Lose, Win) N	(Tie, Tie)

BR1(P2 plays « Straight »)={Swerve};
BR1(P2 plays « Swerve »)={Straight};
Nash equilibria: {(Straight, Swerve); (Swerve, Straight)}

Multiple equilibria



Multiple equilibria

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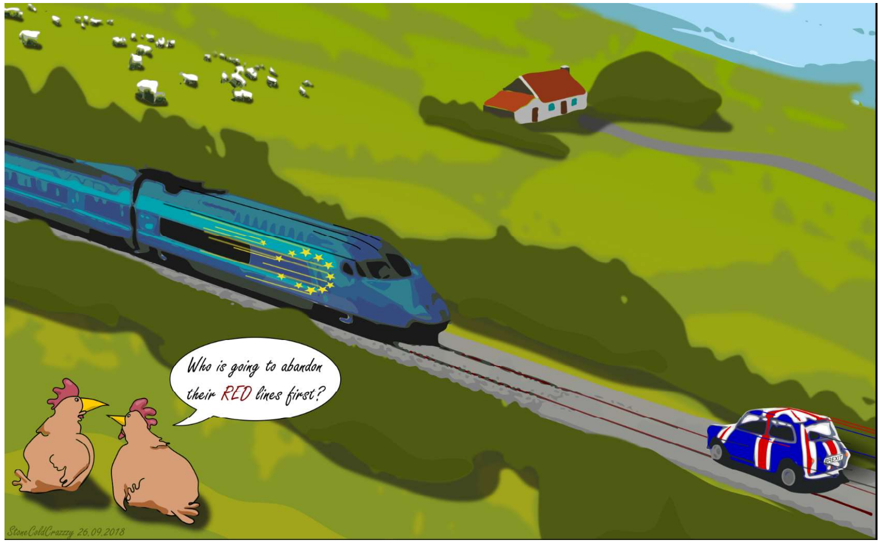

Multiple equilibria
Prisoner's dilemma: playing with your cousin

		Player 2	
		Denounce	Stays Silent
Player 1	Denounce	(-10,-10)	(-6,-25)
	Stays Silent	(-25,-6)	(-3,-3)

- What is the set of Nash equilibrium?

Simultaneous games
Outline

Focal Point

- How to select one equilibrium from multiple equilibria?

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Focal Point

- How to select one equilibrium from multiple equilibria?
- Usually, the selection proceeds from social norms.
- On which side of the road to drive?
  - Dominant strategy: on the side used by other drivers.
- Which side to choose? No side is better than other.
  - UK, Australia, Japan: left-side.

Focal Point

- Choosing a date.
- Choosing a place to meet next week in Paris.

Focal Point

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Focal Point

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- Sunspot can make people moving from one equilibrium to another.

Focal Point
Bank run from depositors

Bank run is a move from one equilibrium to another.

Self-fulfilling prophecy.

Northern Rock bank run on September 2007.
People queuing outside a branch in London to withdraw their savings due to fallout from the subprime crisis.

Focal Point

- Choosing a date.
- Choosing a place to meet next week in Paris.
- The weather can modify the rdv location.
- Sunspot can make people moving from one equilibrium to another.
- Extrinsic fluctuations can cause financial crises.

Focal Point
Bank run from depositors

On March 21, 2013, people queue at an ATM outside a closed Laiki Bank branch in capital Nicosia, Cyprus.

"There are rumours that Laiki Bank (the Greek name for the Popular Bank) will never open again. I want to take out as much as I can," said a depositor.
"It's all about cash now. Only a gambler will take cheques in this situation," said a depositor.

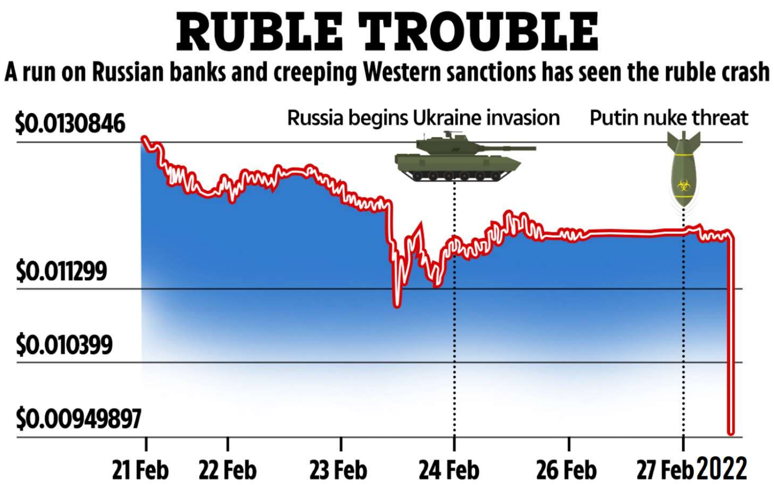
Focal Point
Bank run from depositors

Long lines at Russia's ATMs as citizens rush to withdraw cash amid escalating EU sanctions on 27 Feb 2022

Focal Point
Bank run from depositors

Des policiers gardent l'entrée d'une agence de la banque russe Sberbank devant laquelle des clients font la queue pour retirer leurs avoirs, le 25 février 2022 à Prague (Michal Cizek / AFP)

Focal Point
Bank run from depositors



Focal Point
Bank run from depositors

People wait outside the Silicon Valley Bank headquarters in Santa Clara, California, to withdraw funds after the federal government intervened upon the collapse of the bank. Photograph: Brittany Hosea

Focal Point
Bank run from bondholder

Bank run from bondholders

On March 2008, a bank run began on the securities and banking firm *Bear Stearns*. The non deposit-taking bank had financed huge long-term investments by selling short-maturity bonds, making it vulnerable to panic on the part of its bondholders.

Credit officers of rival firms began to say that *Bear Stearns* would not be able to make good on its obligations. Within two days, Bear Stearns's capital base of $17 billion had dwindled to $2 billion in cash. By the next morning, the *Fed* decided to lend *Bear Stearns* money (the first time since the Great Depression that it had lent to a nonbank).

Stocks sank, and that day *JPMorgan Chase* began to buy *Bear Stearns* as part of a government-sponsored bailout.

Simultaneous games
Outline

Experimental evidence: Nash equilibrium
Ensminger (Oxford University Press, 2004): Public good game

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- The total amount collected is then doubled by the experimenter and this amount is redistributed equally among everyone.

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- For example: $N = 4$. If you have 10, you give 4 and the others give 20 in total. Total is 24, and each gets 0.5 of that. So you will get $10 - 4 + 0.5 * 24 = 18$.

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    - E.g., cinemas, parks, satellite television, fresh air, national defense
- Need for public provision because these goods will tend to be privately under provided

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	Excludable	Non-excludable
Rivalrous	**Private goods** food, clothing, cars, personal electronics	**Common goods (Common-pool resources)** fish stocks, timber, coal
Non-rivalrous	**Club goods** cinemas, private parks, satellite television	**Public goods** free-to-air television, air, national defense

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- Can choose to keep the amount or contribute part or the whole of it to a public good
- Amount contributed doubled by experimenter and divided among the 4 players: so got back 50 percent of the total

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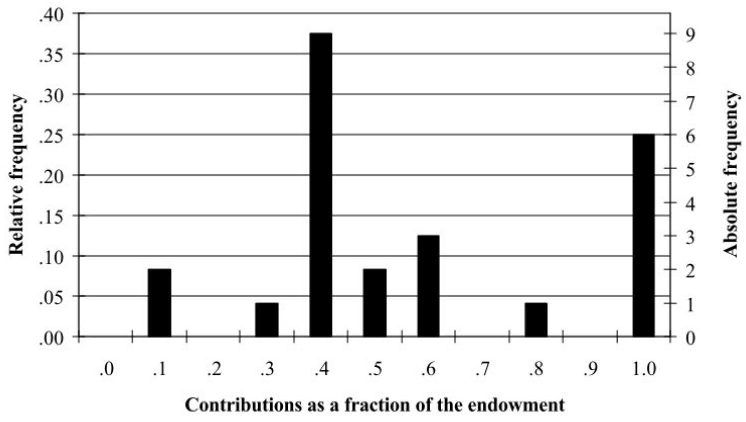
Distribution of offers in the 4-person public goods game (N=24, endowment=50 Kenyan shillings with doubling of contributions by experimenter)



- Results: on average contributions were 60 percent of endowments

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- Players give more than in the NE (where contributions should be zero)
- Example player who gives 20 out of his 50 in a group where other three give 75 total, gets a payoff of:

$$50 - 20 + 0.5 * 95 = 77.5$$

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- What explains this?

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- Can solve for Nash equilibrium

Mixed strategies

Inland Revenue

Tax Payer		Audit	Not audit
	Declare all income	3 , 1	3 , 2
	Lie on Income	0 , 4	5 , 0

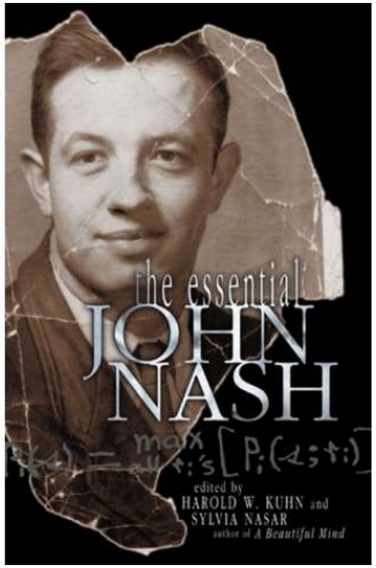
Figure: Tax game

- Does this game have any (pure) strategy equilibrium?

Mixed strategies

Theorem (Nash, 1950):

Every finite game has a strategy equilibrium.



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- In previous classes, all the games we saw had a Nash equilibrium in what is called “pure strategy": i.e where all players play one action for sure
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- In this game, if you know what the other player is going to choose, the strategy that makes you better off makes him worse off
- No equilibrium in pure strategies
- Other example: penalty kicks (most sports in fact)
- Intuitively the only outcome is an outcome where the other player does not know for sure what you are going to play: players randomize

Mixed strategies

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A **mixed strategy** is a strategy where the player randomizes over the set of actions. It is a probability distribution that assigns to each action (or pure strategy) a likelihood of being selected.

- A strategy is defined by the probability you place on each action
- It is as if you were giving these probabilities to a machine that picked accordingly and told you what strategy you should play

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- Example a strategy in tax game for tax authority could be: "audit" with probability 0.4 and "not audit" with probability 0.6

Mixed strategies
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- Payoff for a player is a weighted average of payoff of each action where the weight is the probability: called expected payoff
- Suppose for instance that the tax payer plays a mixed strategy: “declare" with probability 0.2 and “lie” with 0.8.
- Then the payoff of the tax authority if it plays “audit” is:

$$0.2 \times 1 + 0.8 \times 4 = 3.4$$

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- To determine an equilibrium in mixed strategies, we will always use the following essential property:

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Nash equilibrium

- Definition of Nash Equilibrium remains the same: combination of strategies such that if other players play their Nash equilibrium strategies, you also want to play your Nash equilibrium strategy
- Remember strategy is defined for a mixed strategy by a combination of probabilities
- So the probabilities are not any probabilities: they are defined at the equilibrium

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Property (Indifference)

In equilibrium, the players are indifferent (i.e get the same payoff) from all the strategies they play with positive probability.

Mixed strategies

Example

What is the Nash equilibrium of the Tax payer game?

	L	R
U	$(3,1)$	$(3,2)$
D	$(0,4)$	$(5,0)$

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- For example, if the tax payer in equilibrium plays “declare” with probability 0.2 and “lie” with 0.8, then his payoff if he played “declare” for sure and his payoff if he played “lie” for sure should be equal.

Mixed strategies

Example

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	L	R
U	$(3,1)$	$(3,2)$
D	$(0,4)$	$(5,0)$

Solution

Nash equilibrium is such that:

Player 1 plays U with probability $4/5$ and D with probability $1/5$

Player 2 plays L with probability $2/5$ and R with probability $3/5$

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- Fix player 1 at his Nash equilibrium strategy: plays U with probability $4/5$ and D with probability $1/5$. Is player 2 indifferent between L and R:
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$$3 \times q + 3 \times (1-q) = 0 \times q + 5 \times (1-q) \Rightarrow q = \frac{2}{5}$$

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- Player 2 has to be indifferent between L and R, so:

$$1 \times p + 4 \times (1-p) = 2 \times p + 0 \times (1-p) \Rightarrow p = \frac{4}{5}$$

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Simultaneous games
Outline

Mixed strategies

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- Players are indifferent, but the probabilities with which they randomize are very well defined: they leave the other players indifferent

Chiappori et al. (2002): testing mixed strategies

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Chiappori et al. (2002): testing mixed strategies

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Chiappori et al. (2002): testing mixed strategies

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Chiappori et al. (2002): testing mixed strategies

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- Chiappori, Levitt and Groseclose (AER, 2002) using empirical evidence from the French and Italian first-leagues containing 459 penalty kicks over a period of 3 years (1997-2000).

Chiappori et al. (2002): testing mixed strategies

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Chiappori et al. (2002): testing mixed strategies

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- The structure of this game is such that there is no pure-strategy equilibrium.

Chiappori et al. (2002): testing mixed strategies

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  - Do the probabilities with which each player randomize leave the other player indifferent?
- If the indifference property holds, the kicker's scoring probability should be the same whether he kicks L, C or R, and the goalkeeper's probability of averting a goal should the same whether he dives L, C or R.
  - If the players were not indifferent, then it would pay them to adjust their probabilities towards more frequent selection of the strategy with the higher scoring probability (in the case of the kicker) or the strategy with the higher probability of averting a goal (in the case of the goalkeeper).

Jérôme MATHIS (LEDa - Univ. Paris-Dauphin Game Theory Chap.1 Simultaneous games 126 / 133

Chiappori et al. (2002): testing mixed strategies

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Chiappori et al. (2002): testing mixed strategies

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    - For simplicity, for shots involving left-footed kickers, the direction will be reversed so that shooting left correspond to the “natural side” for all kickers.
  - Probability of scoring in the middle is lower than on the sides if goalie does not go the correct way

Chiappori et al. (2002): testing mixed strategies

- Indeed, we have:

TABLE 1—OBSERVED SCORING PROBABILITIES, BY FOOT AND SIDE

	Goalie	
Kicker	Correct side	Middle or wrong side
Natural side ("left")	63.6 percent	94.4 percent
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- The scoring probability when the goalie is mistaken varies between 89 percent and 95 percent (depending on the kicking foot and the side of the kick), whereas it ranges between 43 percent and 64 percent when the goalkeeper makes the correct choice.
- Also, the scoring probability is always higher on the kicker's natural side.

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(iii) *The goalkeeper selects L more often than the right-footed kicker;*

(iv) *The kicker selects C more often than the goalkeeper.*

Chiappori et al. (2002): testing mixed strategies

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  - In the case of (ii), if the goalkeeper selects L and R with equal probability, the right-footed kicker would not be indifferent between L and R, because he would score more often by selecting L (kicking on his stronger side).
  - Selecting C is highly damaging for the kicker if the goalkeeper also selects C. For the kicker to be indifferent between C and either L or R, in accordance with (iv), the goalkeeper must only select C very rarely.

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Chiappori et al. (2002): testing mixed strategies

TABLE 3—OBSERVED MATRIX OF SHOTS TAKEN

	Kicker			
Goalie	Left	Middle	Right	Total
Left	117	48	95	260
Middle	4	3	4	11
Right	85	28	75	188
Total	206	79	174	459

- Predictions (i) & (ii): the kicker and the goalie are both more likely to go L than R.

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  - This prediction is confirmed: in the data, 260 jumps are made to the (kicker's) left, and only 188 to the right.

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  - The same pattern holds for the kicker, although in a less spectacular way (206 against 174).

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- Prediction (iii): The goalkeeper selects L more often than the right-footed kicker.
  - The result emerges very clearly in the data: goalies play "left" 260 times (56.6 percent of kicks), compared to 206 (44.9 percent) instances for kickers.

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- Prediction (iv): The kicker selects C more often than the goalkeeper.
  - The result emerges very clearly in the data: kickers play “center” 79 times in the sample, compared to only 11 times for goalies.