Derivative Instruments (Produits dérivés) - Solution to the Exam

Université Paris Dauphine-PSL - Master 1 I.E.F. (272)

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Answers can be formulated in English or French.

Solution 1 (1 pt) The three year forward price is $F = (S_0 - I)e^{rT}$, with $S_0 = 30€$, $r = 10\%$, $T = 3$, and I denotes the present value of the income, which is $I = 2e^{-0.1\times 1} + 2e^{-0.1\times 2} \simeq 3.45€$. It is then $F = (30 - 3.45)e^{0.1\times 3} \simeq 35.84€$.

Solution 2 (2 pts) The producer of the commodity takes a short futures position. The gain on the futures is $F_1 - F_2 = 1015 - 981$ or \$34. The effective price realized is therefore $S_2 + F_1 - F_2 = 980 + 34$ or \$1014. This can also be calculated as the March 1 futures price ($= F_1 = 1015$) plus the November 1 basis ($S_2 - F_2 = -1$).

Solution 3 (1 pt) The bond pays 1.5€ in 6, 12, 18, and 24 months, and 102.5€ in 30 months. The cash price is

$$1.5(e^{-0.03\times 0.5} + e^{-0.032\times 1.0} + e^{-0.033\times 1.5} + e^{-0.036\times 2.0}) + 101.5e^{-0.037\times 2.5} \simeq 98.28$$

Solution 4 (1 pt) The bond pays 1.5€ in 6, 12, 18, 24, and 30 months, and 101.5€ in 36 months. The bond yield is the value of y that solves

$$1.5e^{-0.5y} + 1.5e^{-1.0y} + 1.5e^{-1.5y} + 1.5e^{-2.0y} + 1.5e^{-2.5y} + 101.5e^{-3y} \simeq 102$$

Solution 5 (1 pt) The cheapest-to-deliver bond is the one for which Quoted bond price - (Most recent settlement price x Conversion factor) is least. Here, the value of such a formula is reported in the fourth column of the following table.

Bond	Price	Conversion factor	
1	96.5	0.963	4.052
2	109	1.089	4.456
3	121	1.224	3.496

The cheapest-to-deliver bond is then bond 3.

Solution 6 (1 pt) The contract gives one the obligation to sell for $F_0 = 40$ € when a forward price negotiated today would give one the obligation to sell for $F_1 = 42$ €. The value of the contract to the party with a short position is $f = -(F_1 - F_0)e^{rT} = -(40-42)e^{-0.08\times 3/12} \simeq -1.96$ €.

Solution 7 (1 pt) Let ρ denotes the correlation between the futures price and the commodity price, and σ_S (resp. σ_F) denotes the standard deviation of monthly changes in the price of commodity A (resp. in a futures price for a contract on commodity B). The optimal hedge ratio is $h^* = \rho\frac{\sigma_S}{\sigma_F} = 0.86\frac{2.2}{2.7} \simeq 0.7$.

Solution 8 (1 pt) Put-call parity is $c + Ke^{-rT} = p + S_0 - D$. In this case $K = 50$ €, $S_0 = 51$ €, $r = 0.06$, $T = 1$, and $c = 6$ €. The present value of the dividend, D, is $1 \times e^{-0.06\times 0.5} = 0.97$. It follows that $p = 6 + 50e^{-0.06\times 1} - (51 - 0.97) = 3.06$ €.

Solution 9 (1 pt) a), c) and d) all generate a higher price (premium) on American call options. The answer is b) as a more liquid underlying asset does not generate a higher price.

Solution 10 (3 pts) The swap involves exchanging the sterling interest of 12×0.072 or £0.864 million for the euro interest of $10 \times 0.06 = 0.6$ million € in 1 month, 13 months, and 25 months. The principal amounts are also exchanged at the end of the life of the swap. The continuously compounded interest rates are :

$$ln(1.05) \simeq 4.88\% \text{ per annum in sterlings; and } ln(1.04) \simeq 3.92\% \text{ per annum in euros}$$

The 1-month, 13-month and 25-month forward exchange rates are :

$$F_{1/12} = 1.16e^{(0.0392-0.0488)\times 1/12} \simeq 1.159;$$

$$F_{13/12} = 1.16e^{(0.0392-0.0488)\times 13/12} \simeq 1.148; \text{ and}$$

$$F_{25/12} = 1.16e^{(0.0392-0.0488)\times 25/12} \simeq 1.137.$$

The values of the three forward contracts corresponding to the exchange of interest for the party paying euros are therefore

$$(0.864 \times 1.159 - 0.6)e^{-0.03922\times 1/12} \simeq 0.4 \text{ million€};$$

$$(0.864 \times 1.148 - 0.6)e^{-0.03922\times 13/12} \simeq 0.376 \text{ million€; and}$$

$$(0.864 \times 1.137 - 0.6)e^{-0.03922\times 25/12} \simeq 0.352 \text{ million€}.$$

The value of the forward contract corresponding to the exchange of principals is

$$(12 \times 1.137 - 10)e^{-0.03922\times 25/12} \simeq 3.358 \text{ million€}.$$

The total euros value of the swap to the party paying euros is then

$$0.4 + 0.376 + 0.352 + 3.358 \simeq 4.486 \text{ million €}.$$

Solution 11 (2 pts) The value of the sterling bond underlying the swap is

$$0.864e^{(-0.0488)\times 1/12} + 0.864e^{(-0.0488)\times 13/12} + 12.864e^{(-0.0488)\times 25/12} \simeq £13.300 \text{ million.}$$

The value of the euro bond underlying the swap is

$$0.6e^{(-0.0392)\times 1/12} + 0.6e^{(-0.0392)\times 13/12} + 10.6e^{(-0.0392)\times 25/12} \simeq 10.942 \text{ million €.}$$

The value of the swap to the party paying sterling is therefore

$$\frac{10.942}{1.16} - 13.300 = -£3.867 \text{ million.}$$

This is consistent with the value found in the previous exercise as

$$(-1) \times (-£3.867 \text{ million}) \times 1.16 = 4.486 \text{ million €.}$$

Solution 12 (3 pts) a) The table that depicts the variation of the trader's profit per share as a function of the asset price at maturity with respect to the strikes writes as :

S_T	Long Put	Short Call	Long stock	π
$S_T \leq K_1$	$K_1 - S_T - P_0$	C_0	$S_T - S_0$	$K_1 + C_0 - P_0 - S_0$
$K_1 < S_T \leq K_2$	$-P_0$	C_0	$S_T - S_0$	$C_0 - P_0 + S_T - S_0$
$K_2 < S_T$	$-P_0$	$C_0 + K_2 - S_T$	$S_T - S_0$	$C_0 - P_0 + K_2 - S_0$

which, with the corresponding numbers ($S_0 = 21$, $K_1 = 16$, $P_0 = 1$, $K_2 = 25$, and $C_0 = 2.5$) gives :

S_T	Long Put	Short Call	Long stock	π
$S_T \leq 16$	$15 - S_T$	2.5	$S_T - 21$	-3.5
$16 < S_T \leq 25$	-1	2.5	$S_T - 21$	$S_T - 19.5$
$25 < S_T$	-1	$27.5 - S_T$	$S_T - 21$	5.5

b) The corresponding diagram is

Figure1

The maximal profit is then $5.5 \times 500 = 2750$€, the maximal loss is $3.5 \times 500 = 1750$€, and the breakeven point is 19.5 €.

Solution 13 (2 pts) We have

$$S_0 - K \leq C_0 - P_0 \leq S_0 - Ke^{-rT}$$

which is equivalent to

$$C_0 - S_0 + Ke^{-rT} \leq P_0 \leq C_0 - S_0 + K$$

with $S_0 = 13$, $K = 14$, $C = 2,5$, $r = 4\%$, and $T = 1/12$. So

$$3.45 \leq P_0 \leq 3.5.$$

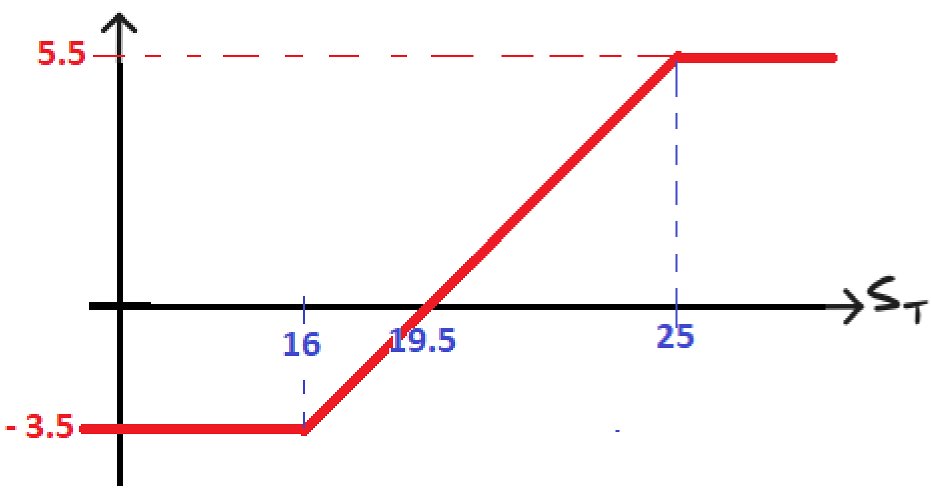

FIGURE 1 – Diagram of the Corridor