

Derivative Instruments (Produits dérivés) - Exam

Université Paris Dauphine-PSL - Master 1 I.E.F. (272)

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Answers can be formulated in English or French.

Exercise 1 (1 pt) The spot price of an investment asset is 30€ and the risk-free rate for all maturities is 10% with continuous compounding. The asset provides an income of 2€ at the end of the first year and at the end of the second year. What is the three-year forward price?

Exercise 2 (2 pts) On March 1 the price of a commodity is \$1,000 and the December futures price is \$1,015. On November 1 the price is \$980 and the December futures price is \$981. A producer of the commodity entered into a December futures contracts on March 1 to hedge the sale of the commodity on November 1. It closed out its position on November 1. What is the effective price (after taking account of hedging) received by the company for the commodity?

Exercise 3 (1 pt) Suppose that 6-month, 12-month, 18-month, 24-month, and 30-month zero rates are 3%, 3.2%, 3.3%, 3.6%, and 3.7% per annum with continuous compounding respectively. Estimate the cash price of a bond with a face value of 100€ that will mature in 30 months and pays a coupon of 3% per annum semiannually.

Exercise 4 (1 pt) A three-year bond provides a coupon of 3% semiannually and has a cash price of 102€. What equation solves the bond's yield?

Exercise 5 (1 pt) Trader A holds a short position in the futures market. He has three bonds with different quoted prices and conversion factors that are eligible for delivery and are given by the following table. The settlement price of the bond is 96 €.

Bond	Price	Conversion factor
1	96,5	0,963
2	109	1,089
3	121	1,224

What is the cheapest-to-deliver bond?

Exercise 6 (1 pt) A short forward contract that was negotiated some time ago will expire in three months and has a delivery price of 40 €. The current forward price for three-month forward contract is 42 €. The three month risk-free interest rate (with continuous compounding) is 8%. What is the value of the short forward contract?

Exercise 7 (1 pt) Suppose that the standard deviation of monthly changes in the price of commodity A is \$2.2. The standard deviation of monthly changes in a futures price for a contract on commodity B (which is similar to commodity A) is \$2.7. The correlation between the futures price and the commodity price is 0.86. What hedge ratio should be used when hedging a one month exposure to the price of commodity A?

Exercise 8 (1 pt) The price of a European call option on a stock with a strike price of 50 € is 6€. The stock price is 51 €, the continuously compounded risk-free rate (all maturities) is 6% and the time to maturity is one year. A dividend of 1 € is expected in six months. What is the price of a one-year European put option on the stock with a strike price of 50 €?

Exercise 9 (1 pt) Which of the following does not generate a higher price (premium) on American call options?

- a. A lower strike price.
- b. A more liquid underlying asset.
- c. A greater term to maturity.
- d. A more volatile price for the underlying asset.

Exercise 10 (3 pts) A currency swap has a remaining life of 25 months. It involves exchanging interest at 6% on € 10 million for interest at 7.2% on £12 million once a year. The term structure of interest rates in both the United Kingdom and France is currently flat, and if the swap were negotiated today the interest rates exchanged would be 4% in euros and 5% in sterling. All interest rates are quoted with annual compounding. The current exchange rate (euro per pound sterling) is 1.16.

What is the euros value of the swap to the party paying euros?

Use the method viewing the swap as a portfolio of forward contracts.

Exercise 11 (2 pts) What is the sterling value of the swap described in the previous exercise to the party paying sterling?

Use the method viewing the swap as a portfolio of bonds.

Exercise 12 (3 pts) A *collar* is an options strategy consisting of buying an out-of-the-money put option while simultaneously writing an out-of-the-money call option with the same expiry date and the same number of contracts. It is typically used by an investor who is already long the underlying asset and wants to temporarily hedge his position due to the increase in the overall market's volatility.

Assume an investor purchases 500 shares of stock X at a price of 21 € per share. He takes a long position in 5 put options (one option contract is 100 shares) with a strike price of 16 € and a premium of 1.00 € and writes 5 call options with a strike price of 25 € with a premium of 2.5 €.

a) Write a table that depicts the variation of the trader's profit per share as a function of the asset price at maturity with respect to the strikes.

b) Draw a diagram showing the variation of the trader's profit per share with the asset price at maturity. What is the maximal profit? What is the maximal loss? What is the breakeven point (i.e., the value of the share that gives zero profit)?

Exercise 13 (2 pts) The price of an American call on a non-dividend-paying stock is 2.5 €. The stock price is 13 €, the strike price is 14 €, and the expiration date is in one month. The risk-free interest rate is 4%.

Derive upper and lower bounds for the price of an American put on the same stock with the same strike price and expiration date.