Master PEI : Game Theory in Banking, Finance, and the International Arena

Final Exam, Sciences Po, November 2022

Jérôme MATHIS

Duration : 110 mn. No document, no calculator allowed.

The financial trilemma

The financial trilemma states that financial stability, financial integration and national financial policies are jointly incompatible. Any two of the three objectives can be combined but not all three ; one has to give.

In the following, we develop a game-theoretic framework to underpin the trilemma. In our model, an ailing financially integrated bank must be recapitalized by national financial policies. The strategic interaction between the countries involved to rescue the bank may lead them to bailout the bank conditionally on its financial integration (i.e., its level of activity conducted abroad).

The underlying strategic interaction is depicted as a simultaneous game played by the countries (or national governments) in which an ailing bank conducts its activities. Each country $i \in N := \{1, ..., n\}$, $n \geq 2$, chooses whether to participate (P) or not ($\neg P$) to the bailout of the bank. We denote country i's strategic space as $S_i := \{P, \neg P\}$. The bank is refunded if at least one country participate (i.e., if at least one country $j \in N$ plays $s_j = P$) ; and is closed otherwise (i.e., when $s := (s_1, ..., s_n) = (\neg P, ..., \neg P)$).

The social benefits of recapitalization is denoted by B. Its cost is denoted as C. We assume $B > C > 0$. The social (international) benefits can be decomposed into countries (national) benefits according to a profile (n-tuple) of weights $(\alpha_i)_{i \in N} \in [0,1]^n$, where the weights are normalized such that they sum up to 1.

When the bank is closed, the payoff of all countries is normalized to zero. When the bank is rescued, all countries enjoy their shares of benefits while the cost of the rescue is equally split among the participating countries. Formally, given an action profile $s \in \{P, \neg P\}^n$, country i's payoffs writes as :

$$g_i(s) = \begin{cases} \alpha_i B & \text{if} \quad s_i = \neg P \quad \text{and} \quad \text{there is at least one } j \in N_{-i} \text{ such that } s_j = P \\ \alpha_i B - \frac{C}{m} & \text{if} \quad s_i = P \quad \text{and} \quad \text{the number of } j \in N \text{ such that } s_j = P \text{ is } m \geq 1 \\ 0 & \text{if} \quad s = (\neg P, ..., \neg P) \end{cases}$$

where N_{-i} denotes the set of countries other than country i.

We restrict our analysis to pure strategies. In the following, each question is worth one point.

Part A (10 pts). Two countries.

Assume $N := \{1, 2\}$.

A.1) Assume [H1] : $\alpha_i B - C < 0 < \alpha_i B - \frac{C}{2}$ for any $i \in N$.

- **A.1.a)** What is the set of Nash equilibrium ?
- **A.1.b)** What is the set of Pareto efficient outcomes ?
- **A.1.c)** Draw the corresponding payoff matrix and report your previous answers using arrows and the symbols (N) and (P) to indicate the outcomes that are Nash equilibrium and/or Pareto efficient.

— **A.1.d)** What kind of game is it ?

— **A.1.e)** Would the previous results change in a case of sequential interaction ? Why ?

A.2) Let H (resp. F) denote the bank's home (foreign) country. The home country is supposed to have the highest national benefits (i.e., $\alpha_H > \alpha_F$). The higher the domestic share α_H, the less financially integrated is the bank. Let H (resp. F) be country 1 (resp. 2). Assume [H2] : $\alpha_H B - C > 0 > \alpha_F B - C$.

— **A.2.a)** What is the set of Nash equilibrium ?

— **A.2.b)** What is the set of Pareto efficient outcomes ?

— **A.2.c)** Draw the corresponding payoff matrix and report your previous answers using arrows and the symbols (N) and (P) to indicate the outcomes that are Nash equilibrium and/or Pareto efficient.

A.3) Assume $C = \frac{2}{3}B$. Let country 1 (resp. 2) still denotes the bank's home (foreign) country, with $\alpha_H > \alpha_F$.

— **A.3.a)** How do the hypothesis [H1] and [H2] rewrite ? How does this translates to share values α_H and α_F ?

— **A.3.b)** Characterize the set of Nash equilibrium as a function of α_H on $(\frac{1}{2}; \frac{2}{3}) \cup (\frac{2}{3}; 1]$.

Part B (8 pts). More than two countries.

Assume $N = \{1, 2, ..., n\}$, with $n \geq 3$.

B.1) Let $i \in N$, and $s_{-i} = (\neg P, ..., \neg P) \in \{P, \neg P\}^{n-1}$. Under which condition is $g_i((s_i = \neg P, s_{-i}))$ strictly larger than $g_i((s_i = P, s_{-i}))$?

B.2) Let $i \in N$, and assume $s_{-i} \in \{P, \neg P\}^{n-1}$ is such that there are $k \geq 1$ contributors among the counterparts of country i. Which payoff is the largest between $g_i((s_i = \neg P, s_{-i}))$ and $g_i((s_i = P, s_{-i}))$?

B.3) Assume $\alpha_i B < C$. Does country i have any strictly dominant strategy ? If yes, which one ?

B.4) Assume [H3] : $\alpha_i B - C < 0$ for any $i \in N$. Characterize the set of Nash equilibrium.

B.5) Assume [H4] : $\alpha_i B - C < 0 < \alpha_i B - \frac{C}{n}$ for any $i \in N$. Is there any efficient equilibrium (i.e., any Pareto-efficient Nash equilibrium outcome) ?

B.6) Let $i \in N$ and $s_{-i} \in \{P, \neg P\}^{n-1}$. Characterize country i's best-reponse correspondence in pure strategies, denoted as $s_i^*(s_{-i})$. (Hint. Consider the three following cases : i) $s_{-i} \neq (\neg P, \neg P, ..., \neg P)$ or $\alpha_i B - C < 0$; ii) $s_{-i} = (\neg P, \neg P, ..., \neg P)$ and $\alpha_i B - C > 0$; and iii) $s_{-i} = (\neg P, \neg P, ..., \neg P)$ and $\alpha_i B - C = 0$.)

B.7) Give a necessary and sufficient condition for the bank to be possibly rescued at equilibrium.

B.8) Let H denote the bank's home country. The home country is supposed to have the highest national benefits (i.e., $\alpha_H > \alpha_j$ for all $j \in N_{-H}$). How to interpret the previous answer ? Conclude.

Questions (4 pts) (2 bonus points)

Are the following statements correct ? If not, give a counter-example.

Q1. (2 pts) The manner in which people discount future payoffs is the same for everyone.

Q2. (2 pts) The manner in which an individual discounts future payoffs is the same between all periods.