Master PEI : Game Theory in Banking, Finance, and the International Arena

Solution to the Final Exam, Sciences Po, November 2022

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Duration : 110 mn. No document, no calculator allowed.

The financial trilemma

Part A (10 pts). Two countries.

Assume $N := \{1, 2\}$.

A.1)

A.1.a) (1 pt) From [H1] we have $\alpha_i B - C < 0$, so $\neg P$ is country i's strictly dominant strategy (i.e., $\neg P \succ_i P$). There is then a unique Nash equilibrium. It consists for the two countries not to participate to the bailout : $Nash = \{(\neg P, \neg P)\}$.

A.1.b) (1 pt) The set of Pareto efficient outcomes writes as : $P = \{(P, \neg P); (\neg P, P); (P, P)\}$. Indeed,

- the outcome $(\neg P, P)$ (resp. $(P, \neg P)$) is the only outcome that provides country 1 (resp. 2) with his maximal payoff;
- From [H1] we have $\alpha_i B - \frac{C}{2} > 0$, so the outcome $(\neg P, \neg P)$ is Pareto dominated by the outcome (P, P); and
- (P, P) by which the cost of the bailout is shared among the two countries is neither Pareto dominated by $(P, \neg P)$ or $(\neg P, P)$ where one of the two countries has to finance the entire bailout solely.

A.1.c) (1 pt) Draw the corresponding payoff matrix and report your previous answers using arrows and the symbols (N) and (P) to indicate the outcomes that are Nash equilibrium and/or Pareto efficient.

The corresponding payoff matrix writes as

$1\backslash 2$	P	$\neg P$
P	$\alpha_1 B - \frac{C}{2}; \alpha_2 B - \frac{C}{2}$ (P)	$\alpha_1 B - C; \alpha_2 B$ (P)
$\neg P$	$\alpha_1 B; \alpha_2 B - C$ (P)	$0; 0$ (N)

A.1.d) (1 pt) This game is a prisoner's dilemma. Indeed, the jointly preferred outcome (P, P) requires both countries to play their strictly dominated strategy P.

A.1.e) (1 pt) No, in a case of sequential interaction the previous results would not change because each country would play his strictly dominant strategy anyway.

A.2)

A.2.a) (1 pt) From [H2] we have $0 > \alpha_F B - C$, so $\neg P$ is country F's strictly dominant strategy (i.e., $\neg P \succ_F P$). From [H2] we have $\alpha_H B - C > 0$, so country H's best response to $\neg P$ is P. There is then a unique Nash equilibrium. It consists for the home country to bailout the entire financial institution solely : $Nash = \{(P, \neg P)\}$.

A.2.b) (1 pt) The set of Pareto efficient outcomes writes as : $P = \{(P, \neg P); (\neg P, P); (P, P)\}$. Indeed,

— the outcome $(\neg P, P)$ (resp. $(P, \neg P)$) is the only outcome that provides country H (resp. F) with his maximal payoff ;

— From [H2] we have $\alpha_H B - C > 0$, so the outcome $(\neg P, \neg P)$ is Pareto dominated by the outcome $(P, \neg P)$; and

— (P, P) by which the cost of the bailout is shared among the two countries is neither Pareto dominated by $(P, \neg P)$ or $(\neg P, P)$ where one of the two countries would have to finance the entire bailout solely.

A.2.c) (1 pt) The corresponding payoff matrix writes as (with $1 = H$ and $2 = F$) :

$1\backslash 2$	P	$\neg P$
P	$\alpha_1 B - \frac{C}{2}; \alpha_2 B - \frac{C}{2}$	$\alpha_1 B - C; \alpha_2 B$
$\neg P$	$\alpha_1 B; \alpha_2 B - C$	$0; 0$

A.3)

A.3.a) (1 pt) For any $i \in \{H, F\}$, the hypothesis [H1] rewrites as $(\alpha_i - \frac{2}{3})B < 0 < (\alpha_i - \frac{2}{3})B$, which is equivalent to

$$\frac{1}{3} < \alpha_F < \alpha_H < \frac{2}{3}$$

The hypothesis [H2] rewrites as $(\alpha_H - \frac{2}{3})B > 0 > (\alpha_F - \frac{1}{3})B$, which is equivalent to

$$\alpha_F < \frac{2}{3} < \alpha_H.$$

A.3.b) (1 pt) By assumption, $\alpha_H > \alpha_F$ and $\alpha_H + \alpha_F = 1$, so $\alpha_H > \frac{1}{2}$. From **A.1)**, we know that under the hypothesis [H1], there is a unique Nash equilibrium which consists for the two countries not to participate to the bailout : $(\neg P, \neg P)$. From **A.2)**, we know that under the hypothesis [H2], there is a unique Nash equilibrium which consists for the home country to bailout the entire financial institution solely : $(P, \neg P)$. So, the set of Nash equilibrium as a function of α_H writes as :

$$\begin{cases} (\neg P, \neg P) & \text{if} \quad \alpha_H \in (\frac{1}{2}; \frac{2}{3}) \\ (P, \neg P) & \text{if} \quad \text{ii)}\ \alpha_H > \frac{2}{3} \end{cases}$$

Part B (8 pts). More than two countries.

Assume n countries, with $n \geq 3$.

B.1) (1 pt) The payoffs are $g_i((s_i = \neg P, s_{-i})) = 0$ and $g_i((s_i = P, s_{-i})) = \alpha_i B - C$, so the condition writes as $\alpha_i B - C < 0$.

B.2) (1 pt) We have $g_i((s_i = \neg P, s_{-i})) = \alpha_i B > \alpha_i B - \frac{C}{k+1} = g_i((s_i = P, s_{-i}))$.

B.3) (1 pt) Yes, from **B.1)** and **B.2)** country i has a strictly dominant strategy which consists in not participating to the bailout (i.e., $\neg P \succ_i P$).

B.4) (1 pt) From **B.3)** there is a unique Nash equilibrium. It consists for all countries not to participate to the bailout : $Nash = \{(\neg P, \neg P, ..., \neg P)\}$.

B.5) (1 pt) No. From [H4], we have $0 > \alpha_i B - C$ for any $i \in N$, so the hypothesis [H3] holds. From **B.4)** the set of Nash equilibrium is then the singleton $\{(\neg P, \neg P, ..., \neg P)\}$. From [H4], we also have $\alpha_i B - \frac{C}{n} > 0$, so $g_i(P, P, ..., P) > 0 = g_i(\neg P, \neg P, ..., \neg P)$. Hence, the unique Nash equilibrium $(\neg P, \neg P, ..., \neg P)$ is Pareto-dominated by $(P, P, ..., P)$, so it is not efficient.

B.6) (1 pt) Following the hint, first, consider case i). When $s_{-i} \neq (\neg P, \neg P, ..., \neg P)$, there is at least one contributor among the counterparts of country i. From **B.2)** $\neg P$ is country i's best-response. When $\alpha_i B - C < 0$, from **B.3)** $\neg P$ is country i's strictly dominant strategy. In both situations, $\neg P$ is country i's best-response.

Second, consider case ii). From $\alpha_i B - C > 0$ and $s_{-i} = (\neg P, \neg P, ..., \neg P)$, we have $g_i((s_i = \neg P, s_{-i})) = 0 < \alpha_i B - C = g_i((s_i = P, s_{-i}))$. So, P is country i's best-response.

Third, consider case iii). From $\alpha_i B - C = 0$ and $s_{-i} = (\neg P, \neg P, ..., \neg P)$, we have $g_i((s_i = \neg P, s_{-i})) = 0 = \alpha_i B - C = g_i((s_i = P, s_{-i}))$. So, both P and $\neg P$ are country i's best-response.

Therefore, country i's best-reponse correspondance in pure strategies writes as :

$$s_i^*(s_{-i}) = \begin{cases} \{\neg P\} & \text{if} \quad \text{i) } s_{-i} \neq (\neg P, \neg P, ..., \neg P) \text{ or } \alpha_i B - C < 0 \\ \{P\} & \text{if} \quad \text{ii) } s_{-i} = (\neg P, \neg P, ..., \neg P) \text{ and } \alpha_i B - C > 0 \\ \{P, \neg P\} & \text{if} \quad \text{iii) } s_{-i} = (\neg P, \neg P, ..., \neg P) \text{ and } \alpha_i B - C = 0 \end{cases}$$

B.7) (1 pt) The bank is possibly rescued at equilibrium if and only if there is at least one country $i \in N$, and a profile of action of country i's counterparts s_{-i}, for which $P \in s_i^*(s_{-i})$. From **B.6)**, the condition writes as $s_{-i} = (\neg P, \neg P, ..., \neg P)$ and $\alpha_i B - C \geq 0$.

B.8) (1 pt) Clearly, there is a country i satisfying the condition $\alpha_i B - C \geq 0$ if and only if $\alpha_H B - C \geq 0$. This rewrites as $\alpha_H \geq \frac{C}{B}$ and has the interpretation that the bank is not too financially integrated. So, from **B.7)** the bank is possibly rescued at equilibrium if and only if the home country is ready to refinance the entire institution solely because the national benefit of the bailout is high enough. We then obtain the financial trilemma in the sense that in the context of strategic national financial policies, the financial stability (by which ailing financial institutions are rescued) requires low financial integration (i.e., $1 - \alpha_H < \frac{C}{B}$) .

Questions (4 pts) (2 bonus points)

Are the following statements correct ? If not, give a counter-example.

Q1. (2 pts) The statement that the manner in which people discount future payoffs is the same for everyone is false. For instance, in chapter 3, we saw that according to the paper of Harrison, Lau and Williams (AER 2002), poor (resp. less educated) people have a lower discount factor than rich (resp. more educated) people, with the interpretation that they are less patient.

Q2. (2 pts) The statement that the manner in which an individual discounts future payoffs is the same among all periods is false. For instance, in chapter 3, we saw that in a study conducted by Read and van Leeuwen (1998), most involved people had a strong preference for the immediate present. Namely, in response to the question : "If choosing today would you choose fruit or chocolate for next week ?" 74 % chose fruit ; while to the question : "For today, what do you choose ?" 70 % chose chocolate.