Game Theory in Banking, Finance, and the International Arena

Master PEI - Autumn 2022

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Chap.3 Repeated games

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1 / 89

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about 10%.

Introduction

Israel.

Game Theory

In 1973, during the Yom Kippur War, the OAPEC

(Organization of Arab Petroleum Exporting Countries)

voted an oil embargo against countries supporting

In 1975, OPEC colluded to raise the price of crude oil

Chap.3 Repeated games

3 / 89

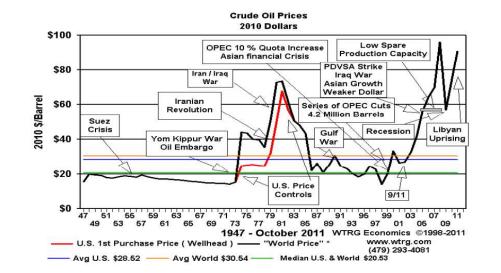
Repeated games Outline

- Introduction
- Motivation
- Discounting
- Finite repetition of games
- Infinitely repeated games
- Application: collusion
- Discounting and time inconsistency
- 8 References

Introduction

Issues of Cooperation: OPEC Cartel

Issues of Cooperation: OPEC Cartel



Issues of Cooperation: OPEC Cartel

By the time of the end of the 1st Gulf War in 1988, Iraq was virtually bankrupt, with most of its debt owed to Saudi Arabia and Kuwait.

Saddam Hussein asked the OPEC to raise the price of crude oil in order to help Iraq to reduce his debt.

In 1990, infuriated by Kuwait who seems to exceed its OPEC quotas and driving down the price of oil, Saddam Hussein chooses to take military action.

Why do some countries not respect the agreement?

Introduction
Issues of Cooperation: OPEC Cartel

Assume there are only two countries: Iraq and Iran.

And two production levels:

The total output on the world market will

Suppose the price will be

Chap.3 Repeated games 5/89

Introduction

Issues of Cooperation: OPEC Cartel

Because the strategic interaction of the OPEC's members comes to prisoner's dilemma.

Assume there are only two countries: Iraq and Iran.

And two production levels: 2 or 4 million barrels a day.

The total output on the world market will be 4, 6 or 8 million barrels.

Suppose the price will be \$25, \$15, or \$10 per barrel.

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Introduction

Issues of Cooperation: OPEC Cartel

Extraction costs are _____

		Iraq's out	tput
		2 million	4 million
Iran's output	2 million	(46,42)	(26,44)
nan s output	4 million	(52,22)	(32,24)

Issues of Cooperation: OPEC Cartel

Extraction costs are \$2 per barrel in Iran and \$4 per barrel in Iraq.

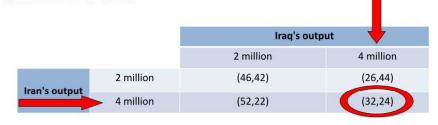
		Iraq's outp	ut
		2 million	4 million
Iran's output	2 million	(46,42)	(26,44)
Iran's output	4 million	(52,22)	(32,24)

Introduction

Issues of Cooperation: OPEC Cartel

Does any country has a dominant strategy?

Yes, both! To produce at the higher of the two available levels.



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Chap.3 Repeated games

9 / 89

Introduction

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Issues of Cooperation: OPEC Cartel

Cooperation would have gotten them resp.

Introduction

Issues of Cooperation: OPEC Cartel

Does any country has a dominant strategy?

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Iran's output	2 million	(46,42)	(26,44)
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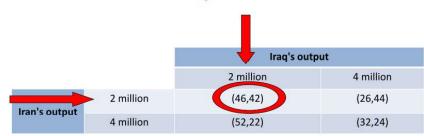
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		2 million	4 million
luanda antonia	2 million	(46,42)	(26,44)
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Issues of Cooperation: OPEC Cartel

Cooperation would have gotten them resp.

\$14 and \$18 million a day more.



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Introduction

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By deviating Iran would obtain

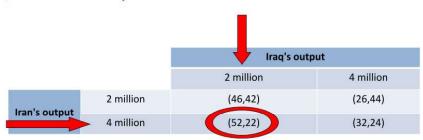
•

		Iraq's ou	tput
		2 million	4 million
	2 million	(46,42)	(26,44)
Iran's output	4 million	(52,22)	(32,24)

Introduction

Issues of Cooperation: OPEC Cartel

By deviating Iran would obtain \$6 million a day more.



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4E / 00

Introduction

Issues of Cooperation: OPEC Cartel

If it takes Iraq a month to detect Iran's cheating and respond, the month's extra profit to Iran is

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That is, just _____.

Both countries can enforce the cooperation by playing the following strategy:

- _____; and

·----·

Issues of Cooperation: OPEC Cartel

If it takes Iraq a month to detect Iran's cheating and respond, the month's extra profit to Iran is \$180 million.

That is, just 13 days of cooperation.

Both countries can enforce the cooperation by playing the following strategy:

- cooperate as long cooperation occurred; and
- if a player cheats, then the other player does not cooperate the next month.

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Chap.3 Repeated games

17 / 89

Introduction

Issues of Cooperation: OPEC Cartel

Such punishment makes cheating too costly.

Why did some countries, as Kuwait, cheat?

The problem is that OPEC is a cartel of more than two members.

Punishment is costly to implement.

In particular when two countries of the cartel are at war (Iran-Irak, 1980-88).

Introduction

Issues of Cooperation: OPEC Cartel

Such punishment makes cheating too costly.

Why did some countries, as Kuwait, cheat?

The problem is that OPEC is a cartel of more than two members.

Punishment is costly to implement. So, any collective punishment is it-self subjected to deviation.

In particular when two countries of the cartel are at war (Iran-Irak, 1980-88).

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Chap.3 Repeated games

10 / 0

Repeated games Outline

Motivation

3 Discounting

4 Finite repetition of games

5 Infinitely repeated games

6 Application: collusion

Discounting and time inconsistency

8 References

Motivation

- In most situations, players do not play only once
- For instance in the fashion game, Armani and Ralph Lauren repeat the sale game every year
- Can radically change the nature of prisoner's dilemma

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21 / 89

Motivation Some terminology

- The game that is repeated is called the stage game
- Repeated games are a special kind of extensive form games
- Strategy is full plan: what you do in every situation depending on what the other players did in previous repetitions
- We still focus on subgame perfect equilibria
- Key distinction is whether the game is repeated a *finite* number of times or an *infinite* number of times.

Repeated games Outline

- 1 Introduction
- 2 Motivation
- 3 Discounting
- 4 Finite repetition of games
- 5 Infinitely repeated game
- 6 Application: collusion
- Discounting and time inconsistency
- 8 Reference

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22 / 0

Discounting

- The payoff of future repetition is discounted
- The idea is just that the future does not have the same value as the present
- ullet Discount factor usually denoted δ
- Payoff is calculated in the following way: suppose repeat a game three times, get 2 in periods 1 and 2 and 1 in period 3, payoff is then

$$2 + \delta 2 + \delta^2 1$$
.

- For a firm, natural to have $\delta = 1/(1+r)$ where r is the interest rate.
- For an individual could be higher.

Discounting Present value

• When a firm evaluates getting a payment of a 1000 € a year from now, it needs to calculate a present value: from the perspective of today, what is it worth?

$$PV = 1000 \frac{1}{1+r}$$

where r is the interest rate

- the idea is the following: if I get the present value today, I can put it in the bank and get the interest rate: a year from now I will have PV + r * PV = (1 + r)PV = 1000, so both are equivalent
- If interest rate is 3 percent, getting $1000 \in$ in a year or the $PV = 1000 \frac{1}{1+0.03} = 970$ today is the same.

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25 / 8

Discounting Present value later years

• What if it is two years from now?

$$PV = 1000 \left(\frac{1}{1+r}\right)^2$$

• If the firm puts PV in the bank for two years, gets

$$(1+r)^2 PV$$

• More generally, the present value of getting a 1000 euros *n* years from now is:

$$PV = \left(\frac{1}{1+r}\right)^n$$
.

Discounting Discount rates

- Do individuals evaluate the future like firms?
- Extensive evidence that they don't: they have a preference for the present that goes far beyond the effect of the interest rate.

Definition

Discount rate ρ is such that an individual is indifferent between getting a amount X a year from now or getting $X \frac{1}{1+\rho}$ today.

We define the **discount factor** $\delta = \frac{1}{1+\rho}$.

- So the more patient you are, the higher is your discount factor but the lower is your discount rate.
- These amounts can vary among individual.

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27 / 89

Discounting Discount rates

• If my discount rate is ρ and I get 10 every year, the present value (i.e equivalent amount today) is:

$$10 + 10 \frac{1}{1+\rho} + 10 \left(\frac{1}{1+\rho}\right)^2 + ... + 10 \left(\frac{1}{1+\rho}\right)^n$$

• Equivalent formulation we have seen is with discount factor: if my discount factor is δ and I get 10 every year, the present value (i.e equivalent amount today) is:

$$10 + 10\delta + 10(\delta)^2 + ... + 10(\delta)^n$$

Discounting

Experimental evidence: Harrison, Lau and Williams (AER 2002)

- They use survey questions with real monetary rewards to elicit individual discount rates
- Experiments carried out across Denmark, using a nationally representative sample of 268 people between 19 and 75 years of age
- The basic question used to elicit individual discount rates is extremely simple:
 - ▶ Do you prefer \$100 today or \$100 + x tomorrow, where x is some positive amount?
 - ▶ If the subject prefers the \$100 today then we can infer that the *discount rate* is higher than *x* percent per day;
 - otherwise, we can infer that it is x percent per day or less.
- Can measure like this at what x they are indifferent
- They consider four possible time horizons: 6 months, 12 months, 24 months, and 36 months.

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29 / 8

Discounting

Experimental evidence: Harrison, Lau and Williams (AER 2002)

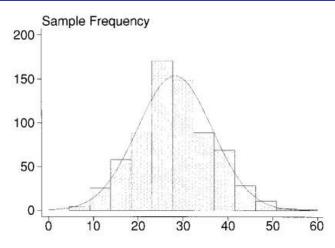


FIGURE 1. ESTIMATED DISCOUNT RATES FOR THE DANISH POPULATION

Discounting

Experimental evidence: Harrison, Lau and Williams (AER 2002)

Table 3-Average Flicted Discount Rates Stratified by Major Demographic Demographic 90-percent confidence interval Estimate Standard error ALL 28.1464 0.53537 27.26233 29 03048 696 Male 28.06626 26,80692 Female 28.22121 26.95507 29.48735 360 146 199 158 193 Young Middle (30–40) Middle (41–50) 28.71521 28.35924 25.05474 0.9551633 27 13791 30.2925 29.79722 26.81503 30 02767 1.256172 27 95331 32 10203 171 280 126 119 32.92452 1.014352 31.24948 34.59955 30.08146 22.68201 22.51315 0.676202 0.752037 1.251744 28.96482 21.44014 20.4461 31.19809 23.92387 24.5802 Unskilled 31.42633 25.73349 30.20636 24.59586 Skilled 0.6889163 26.87113 28.41732 621 75 Not a student 27.48244 26.54756 33,64402 Less educated 30.9838 0.547016 30.0805 31.88711 21.85479 20.58996

- Huge variability among subjects
- The discount rates for men and women appear to be identical
- Discount rates appear to decline with age, at least after middle age.
 - ▶ But retired individuals have higher discount rates
- Poor have much higher discount rates, as one would expect.

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Chap.3 Repeated games

31 / 8

Discounting

Experimental evidence: Harrison, Lau and Williams (AER 2002)

Demographic characteristic	Estimate	Standard error	90-percent con	fidence interval	Observations
ALL	28.1464	0.53537	27.26233	29.03048	696
Male	28.06626	0.76262	26.80692	29.3256	336
Female	28.22121	0.7667374	26.95507	29.48735	360
Young	28.71521	0.9551633	27.13791	30.2925	146
Middle (30-40)	28.35924	0.8708021	26,92125	29,79722	199
Middle (41–50)	25.05474	1.065985	23.29444	26.81503	158
Old	30.02767	1.256172	27.95331	32.10203	193
Poor	32.92452	1.014352	31.24948	34.59955	171
Lower middle	30,08146	0.676202	28,96482	31.19809	280
Upper middle	22.68201	0.7520371	21.44014	23.92387	126
Rich	22.51315	1.251744	20,4461	24.5802	119
Unskilled	31.42633	0.7387784	30,20636	32,6463	295
Skilled	25.73349	0.6889163	24.59586	26.87113	401
Not a student	27.48244	0.5661343	26.54756	28.41732	621
Student	33.64402	1.291917	31.51063	35.7774	75
Less educated	30.9838	0.547016	30.0805	31.88711	506
More educated	20.58996	0.7659382	19.32514	21.85479	190

- There is a large difference between the discount rates of skilled and unskilled individuals (as well as for more and less educated), with those that have skills having a significantly lower discount rate.
 - ► Those with longer investments in education are also those with substantially lower discount rates.
- Students have a higher discount rate than nonstudents.

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Discounting

Experimental evidence: Ruffle and Wilson (JEBO 2019)

Distribution of reasons for getting a tattoo.

Motive	Hidden	Visible
Expression of Individuality	119 (46.7%)	30 (44.1%)
Like the Way the Tattoo Looks	119 (46.7%)	20 (29.4%)
Statement of Personal Identity	90 (35.3%)	20 (29.4%)
Remember Particular Time	77 (30.2%)	17 (25.0%)
Memorialize Loved One	51 (20.0%)	11 (16.2%)
Impulsive Decision	36 (14.1%)	11 (16.2%)
Create Certain Image of Me	25 (9.8%)	8 (11.8%)
Other	9 (3.5%)	7 (10.3%)
Most Friends are Tattooed	9 (3.5%)	1 (1.5%)
Belong to Group who are Tattooed	2 (0.8%)	3 (4.4%)
Political/Environmental Statement	2 (0.8%)	2 (2.9%)

Notes: Number of subjects (percentages) that selected each motive for getting their hidden tattoos and again separately for their visible tattoos. Percentages sum to greater than 100% because they could select more than one motive for each tattoo category.

- Is there any relationship between time preferences and tattoos?
- Apparently Yes. Tattooed individuals, especially those with visible tattoos, would be more short-sighted than nontattooed individuals.

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Chap.3 Repeated games

33 / 8

Discounting Other experimental evidence

- A lot of studies: they find different results, but in general discount rates are high (for sure never below 10 percent on average)
- Warner and Pleeter (AER, 2001) discount rates estimated from a choice given to military personnel of a voluntary separation package where you had to choose between initial lump sum and an annuity.
 - ► Find discount rates close to 17%!
 - * Much higher than the interest rate.
 - * Saved taxpayers \$1.7 billion in separation costs.
 - ▶ Proportion of population choosing the lump sum:
 - ★ Officers: 50%; Non-officers: 90%!

Repeated games Outline

- 1 Introduction
- 2 Motivation
- 3 Discounting
- 4 Finite repetition of games
- 5 Infinitely repeated games
- 6 Application: collusion
- Discounting and time inconsistency
- References

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Finite repetition of games Back to Fashion

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		Sale	No sale
	Sale	40 , 40	50,30
Armani	No sale	30,70	60,60

Finite repetition of games Back to Fashion

- Suppose the game is repeated 5 times (you know in 5 years Armani is retiring)
- Solve the game by backwards induction

Result

The unique SPNE is such that (sale, sale) is the outcome in every period.

 In general if stage game has a unique NE, then unique SPNE in a finite repetition is the repetition of NE in each stage game

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Chap.3 Repeated games

37 / 89

Finite repetition of games Back to Fashion

Proof.

Solve by backwards induction

Period 5: last period, it is as if you were playing the game once. Only NE is that both play Sale

Period 4: what happens in this period does not affect the outcome of period 5.

So as if it was the last period and you were playing the game once. Only NE is that both play Sale

Previous periods: exactly the same reasoning

Finite repetition of games

Repetition of game with multiple equilibria

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What if you have multiple equilibria in the stage game?

Example

A B

(4,4) (0,0) (0,5)

B (0,0) (1,1) (0,0)

C (5,0) (0,0) (3,3)

Solution

Stage game has two Nash Equilibria: (B, B) and (C, C)

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Chap.3 Repeated games

30 / 80

Finite repetition of games Equilibria in this Example

- Suppose the game is repeated twice
- A lot of equilibria:
 - ▶ both players play B in period 1 and B in period 2 regardless of what happened in period 1
 - ▶ both players play C in period 1 and C in period 2 regardless of what happened in period 1
 - ▶ both players play B in period 1 and C in period 2 regardless of what happened in period 1
- More interesting question: can play A in first period be part of an equilibrium?

Finite repetition of games Equilibria in this Example

- Consider the following strategy (same for both players):
 - In first period: Play A
 - ► In second period: If first period outcome is (A,A), play C, otherwise play B.
- So in second period, no point in deviating: both (B,B) and (C,C) are Nash equilibria
- In first period
 - If follow strategy get $4 + \delta 3$
 - ▶ In best deviation get $5 + \delta 1$

Result

If $\delta > 1/2$, this is an equilibrium and (A,A) is played in the first period.

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Chap.3 Repeated games

41 / 89

Finite repetition of games Equilibria in this Example

- Key idea is that there are two equilibria: one is "better" than the other
- Worst NE is used as a punishment in the final periods if one of the players deviates from the NE strategy
- Same logic is going to apply when we repeat the game an infinite number of times

Finite repetition of games Conclusion

- Repeat exactly the same game over and over again
- Discount the payoff from the next repetition at rate δ
- Important: to show that an equilibrium candidate is indeed a subgame perfect equilibrium, need to show it is a best response in all subgames (i.e need to consider any possible history in the game)

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Chap.3 Repeated games

13 / 8

Finite repetition of games Conclusion

• When the game is repeated a finite number of times, we have a number of results:

Result

If the stage game has a unique NE, then the SPNE of the repeated game can only be the repetition of this Nash Equilibrium

This first result can be shown by backwards induction.

Result

If the stage game has several NE, then the outcome of the repeated game can be different.

Result

If one of the equilibria is clearly better, it can be used as a reward and the other as a punishment.

Repeated games Outline

- 1 Introduction
- Motivation
- 3 Discounting
- 4 Finite repetition of games
- Infinitely repeated games
- 6 Application: collusion
- Discounting and time inconsistency
- References

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Chap.3 Repeated games

45 / 8

Infinitely repeated games

 Game repeated infinite number of times so need formula on infinite sums:

$$1+\delta+\delta^2+\delta^3+\ldots=\frac{1}{1-\delta}.$$

- Infinite game so cannot use backwards induction: no clear end to the game.
- Look at every history and check there is no profitable deviation

Infinitely repeated games

Consider infinite repetition of the following stage-game

Example

$$\begin{array}{ccc} & C & D \\ C & (3,3) & (-1,4) \\ D & (4,-1) & (1,1) \end{array}$$

 It is an iterated prisoner's dilemma where players have to choose whether to cooperate (C) or defect (D)

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Chap.3 Repeated games

47 / 80

Infinitely repeated games Grim trigger strategy

- Reminder: a strategy is what a player plays every period depending on what happened in the past
- "Grim trigger" strategy:
 - Play C in the first period and in every period so long as no one ever played D; and
 - ▶ Play D if either player has ever played D in the past.

Solution (Sustaining cooperation)

If $\delta \geq \frac{1}{3}$, both players playing the grim trigger strategy constitutes a subgame perfect equilibrium.

The outcome is then that (C, C) is played every period.

Infinitely repeated games Grim trigger strategy

- To show this is a SPNE: need to show that it sustains a Nash equilibrium in *every* subgame.
- There are two types of subgames:
 - those in which one player played D in the past; and
 - those in which no player ever played D in the past

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Chap.3 Repeated games

49 / 8

Infinitely repeated games Subgame 2

- First consider subgames where one player at least played D in the past (i.e "deviated")
- Need to check that grim trigger strategy leads to a nash equilibrium in that subgame
- Grim trigger strategy in this subgame says play D
- If the other player plays his grim trigger strategy, I also want to play it ((D,D) is the nash equilibrium of the stage game)

Infinitely repeated games Subgame 1

- Now consider subgames where all players played C in the past (i.e all "cooperated")
- Grim trigger strategy dictates to play C
- The payoffs are:
 - ▶ Play C payoff is

$$3+3\delta+3\delta^2+...$$

Play D payoff is

$$4 + 1\delta + 1\delta^2 + ...$$

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Chap.3 Repeated games

51 / 9

Infinitely repeated games Subgame 1

• Play the grim trigger strategy is indeed an equilibrium if:

$$3 + 3\delta + 3\delta^2 + 3\delta^3 + ... > 4 + 1\delta + 1\delta^2 + 1\delta^3 + ...$$

- In other words, deviation:
 - ▶ Gains you 4 3 = 1 in this period
 - ▶ But then get 1 instead of 3 for the rest of the game
- Condition can be written:

$$(3-1)(\delta + \delta^2 + \delta^3 + ...) \ge 4-3$$

$$\Leftrightarrow \ 2\delta(1+\delta+\delta^2+...) \geq 1 \ \Leftrightarrow \ 2\delta(\frac{1}{1-\delta}) \geq 1 \ \Leftrightarrow \ \delta \geq 1/3.$$

Repeated games Outline

- Application: collusion

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Chap.3 Repeated games

Application: collusion on prices

- In markets where there are a small number of firms, numerous case of agreement on prices
- This is illegal
- Cases are often investigated by antitrust authorities who want to promote competition

Application: collusion on prices Lysine cartel

- The "lysine price-fixing conspiracy" was an organized effort during the mid-1990s to raise the price of the animal feed additive Lysine.
- Cartel including an american firm, two japenese and two Korean
 - Goal was to meet to agree on prices: they were able to raise prices by 70% during the last year of cooperation
- Investigation lead to \$105 million in fines and three year sentence for executive of American firm

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Chap.3 Repeated games

Application: collusion Leniency program

- The lysine cartel was the first successful prosecution of an international cartel by the U.S. Department of Justice in more than 40 years.
 - Since then, the DoJ has discovered and prosecuted scores of international cartels.
- How did the U.S. Department of Justice succeed?
- Cartel was denounced by the manager of the american firm under leniency program
 - ▶ The idea of the leniency program is that the first informant does not get a fine whereas later one get full penalty (in the US)
 - This case inspired film "The informant"

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Chap.3 Repeated games

Chap.3 Repeated games

Application: collusion on prices The game

- Stage game:
- $N \ge 2$ firms choose their price
- Consumers purchase from least expensive firm and split equally across firms in case of ties
- Firms have constant marginal cost c
- Profits are given by revenues minus costs:

$$\pi(p) = pQ(p) - cQ(p) = (p-c)Q(p)$$

• If there was a monopoly (i.e N=1), the monopolist would choose a price p_m to maximize $\pi(p) = (p-c)Q(p)$. We note π_m the monopoly profits

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Game Theory

Chap.3 Repeated games

57 / 89

Application: collusion on prices The game: Nash equilibrium in the stage game

- Unique Nash equilibrium in the stage game is to price at marginal cost
- All firms compete aggressively and make zero profits
- From a social welfare perspective this is optimal
- If the game is infinitely repeated, can have collusion

Application: collusion on prices The game

Result

In the infinite repetition of this game, if $\delta \geq \frac{N-1}{N}$, there is a subgame perfect equilibrium in which at every period the firms all price at the monopoly price.

Proof.

Consider grim-trigger strategies:

- choose monopoly price p_m if in the past repetitions everyone chose monopoly price; and
- choose price equal marginal cost if someone deviated

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Chap.3 Repeated games

E0 / 90

Application: collusion on prices The game

Proof.

Same procedure again to show that this is a Nash equilibrium

Need to consider two types of subgames:

- those in which all firms played the monopoly price in the past
- those in which at least one firm did not

In subgame after someone deviated, other players will price at marginal cost so I should also price at marginal cost

Application: collusion on prices The game

Proof.

In subgames where noone deviated in previous repetition.

If follow equilibrium strategy, share monopoly profits with the N-1 other firms, get:

$$\frac{1}{N}\frac{1}{1-\delta}\pi_m$$

If deviate, best is to choose price just below monopoly price and obtain all profits, but after that get zero. So firm prefers not to deviate if and only if

$$\frac{1}{N}\frac{1}{1-\delta}\pi_m \geq \pi_m$$

which is quivalent to

$$\delta \geq \frac{N-1}{N}.$$

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Game Theory

Chap.3 Repeated games

61/8

Application: collusion on prices Using leniency program

- Leniency program creates prisoner's dilemma: if the others inform, better for me to inform
- Can cooperation in the cartel still be an equilibrium?
 - Yes, but makes it harder (need players to be even more patient)

Application: collusion Keeping an eye on competitors

- Key is to be able to monitor actions of competitors
- Antitrust authorities keep a close eye on trade organizations
- Website that was listing all prices of gas on French highways was temporarily banned

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Chap.3 Repeated games

62.10

Application: collusion Keeping an eye on competitors

GAN Assurances: « Your insurance at an unbeatable price ».

Direct Assurance: « If you find a lower price elsewhere, we rebate double the difference ».

Assume a particular home insurance contract is sold by both companies 300 €.

Application: collusion

Keeping an eye on competitors

Suppose *Gan* cuts its price to 275€.

Without the *Direct Assurance*'s advertisement, *GAN* could expect to attract more customers.

Unfortunately, ______.

Customers are tempted just to buy the *Direct* Assurance contract for 300 € and then claim a 50€ rebate.

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Chap.3 Repeated games

65 / 89

Application: collusion Keeping an eye on competitors

Suppose *Gan* cuts its price to 275€.

Without the *Direct Assurance*'s advertisement, *GAN* could expect to attract more customers.

Unfortunately, the price cut has the reverse effect.

Customers are tempted just to buy the *Direct* Assurance contract for 300 € and then claim a 50€ rebate.

Application: collusion

Keeping an eye on competitors

So *Direct Assurance* cuts his price to 275€.

Direct Assurance has been quickly informed by the Gan's cut.

And, Gan is now worse-off because:

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Chap.3 Repeated games

67 / 0

Application: collusion Keeping an eye on competitors

So *Direct Assurance* cuts his price to 275€.

Direct Assurance has been quickly informed by the Gan's cut.

And, Gan is now worse-off because:

- charging a lower price; but
- without attracting more customers.

Application: collusion De Beers

- De Beers probably most successful cartel ever formed
 - Unlawfully monopolised the supply of diamonds and conspired to fix, raise and control diamond prices.
- Problem of monitoring was solved by controlling completely the sales channel
- Difficult to go through different channel because of guarantee of quality
- De Beers had stockpiles of diamonds that allowed it to punish deviators by flooding the market

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Chap.3 Repeated games

69 / 89

Application: collusion Conclusion

- Collusion is easier when:
 - ▶ there are only a small number of firms
 - * E.g., OPEC cartel
 - when deviations can be easily observed
 - * E.g., Gan vs Direct Assurances
 - when punishment can be easily implemented
 - * E.g., Gan vs Direct Assurances, de Beers
 - when players care about future

Application: Fiscal Amnesty

Incentives to repatriate funds.

Italy. April 2010. Fine of 5%.

In 2009 the Italian tax amnesty yielded €80 billion, while the *Bank of Italy* estimated that Italian citizens held around €500bn in undeclared funds outside the country.

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Chap.3 Repeated games

71 / 90

Application: Fiscal Amnesty



March 2009, G20, Silvio Berlusconi is submitting to Angela Merkel and Nicolas Sarkozy his project for a common fiscal amnesty.

Application: Fiscal Amnesty

This is the third italian fiscal amnesty over the 2000's!

Repeated amnesties break many incentives to truthfully report ones revenues and is criticized as an encouragement to tax evasion.

How to become credible in never repeating the amnesty program?

Robert Barro propose that the government offer a tax amnesty, then _____ those who turn themselves in.

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Chap.3 Repeated games

73 / 89

Application: Fiscal Amnesty

This is the third italian fiscal amnesty over the 2000's!

Repeated amnesties break many incentives to truthfully report ones revenues and is criticized as an encouragement to tax evasion.

How to become credible in never repeating the amnesty program?

Robert Barro propose that the government offer a tax amnesty, then renege on its promise and prosecute those who turn themselves in.

Application: Fiscal Amnesty

This would raise even more revenue than a simple amnesty.

Once the government cheats on its amnesty, who would believe the government were it to try again?

By destroying its credibility, the government can make .

Why the U.S. do not use such a strategy?

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Chap.3 Repeated games

75 / 80

Application: Fiscal Amnesty

This would raise even more revenue than a simple amnesty.

Once the government cheats on its amnesty, who would believe the government were it to try again?

By destroying its credibility, the government can make a credible commitment not to offer an amnesty again.

Why the U.S. do not use such a strategy?

Application: Fiscal Amnesty

Because the government may have the need for a future fiscal amnesty.

Because the government don't want to loose credibility on other topics.

_____.

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Chap.3 Repeated games

77 / 89

Application: Fiscal Amnesty

Because the government may have the need for a future fiscal amnesty.

Because the government don't want to loose credibility on other topics.

The long run cost may exceed the short run benefits.

Discounting and time inconsistency

- The payoff of future repetition is discounted
- The idea is just that the future does not have the same value as the present
- Discount factor usually denoted δ
- For a firm, natural to have $\delta = 1/(1+r)$ where r is the interest rate
- For an individual could be higher

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Chap.3 Repeated games

70 / 90

Discounting and time inconsistency

- In our examples, we always considered case where the discount rate is:
 - 1 same for all players
 - 2 same between all periods (period 0 and 1 but also between period 56 and 57)
- According to Harrison, Lau and Williams (AER 2002) in practice 1 tends not to be true.
- Let's see now if 2 is true.
- Note that we can solve games that consider these complications, just makes notation very heavy.

Discounting and time inconsistency

- If a person prefers 100 today to 110 in 1 year, then time consistency means she should prefer 100 in 5 years to 110 in 6 years and 100 in 10 years to 110 in 11 years
- If you discounting is constant over time then you are time consistent
- What is the evidence?

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Chap.3 Repeated games

81 / 8

Discounting and time inconsistency Experimental evidence: Read and van Leeuwen (1998)

- Systematically observe reversals: People may prefer 110 in 31 days over 100 in 30 days but prefer 100 now over 110 tomorrow
- Read and van Leeuwen (1998): "Predicting Hunger: The Effects of Appetite and Delay on Choice."
 - If choosing today would you choose fruit or chocolate for next week?
 - * 74 percent choose fruit.
 - ► For today what do you choose?
 - * 70 percent choose chocolate.

Discounting and time inconsistency

Experimental evidence: Read and van Leeuwen (1998)

- Evidence seems to indicate:
 - 1 strong preference for immediate present
 - 2 constant after that: two choices in the future are evaluated using the same discount rates
- "Typical patterns":
 - Prefer 100 now over 110 tomorrow
 - ▶ Prefer 110 in 31 days over 100 in 30 days
 - ▶ Prefer 110 in 81 days over 100 in 80 days

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Chap.3 Repeated games

83 / 80

Discounting and time inconsistency A different view

• In model we used up till now, if I get 10 every year, the present value is:

$$10 + 10\delta + 10(\delta)^2 + ... + 10(\delta)^n + ...$$

In practice seems to be more of the form:

$$10 + \beta \left[10\delta + 10(\delta)^2 + ... + 10(\delta)^n + ... \right]$$

where β is the preference for the present

Discounting and time inconsistency Consequences

Important consequences:

- Justifies behavior saying: I continue smoking today (because preference for immediate pleasure), but I will stop tomorrow (because my discount rate tomorrow is going to be smaller).
- However when tomorrow arrives you evaluate the choice as you did today.
- Can distinguish the case where you are aware of this time inconsistency (sophisticated) or you are naive.
- Other example: procrastinating on a costly task you need to do.

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Chap.3 Repeated games

85 / 89

Discounting and time inconsistency

Experimental evidence: Della Vigna and Malmendier (AER, 2006)

- Della Vigna and Malmendier (AER, 2006): "Paying not to go to the gym"
- Data on type of membership and day to day attendance of 7752 health club members over three years
- Potential members can choose between three contracts:
 - pay per visit option
 - monthly contract, flat-rate
 - annual contract, flat-rate

Discounting and time inconsistency

Experimental evidence: Della Vigna and Malmendier (AER, 2006)

- They found that the customers consistently chose contracts that cost them almost twice as much as they needed to spend.
- With pay per visit option, price per visit is between \$10 and \$12
- On average users who choose flat-rate contract (monthly or annual) predicted 9.5 monthly visits but went to gym just 5 times a month
 - in the monthly contract: they pay a price per average attendance of over \$17;
 - ▶ in the annual contract: they pay a price per average attendance of over \$15.
- Share of those who pay less than \$10 is 20 percent in the monthly contract and 24% in yearly
- Decreasing average attendance over time in monthly contract.

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Chap.3 Repeated games

87 / 89

Discounting and time inconsistency

Experimental evidence: Della Vigna and Malmendier (AER, 2006)

- On average more than 2 full months elapse between last attendance and contract termination with associated payments \$187. At least 4 months for 20% of users.
- Explanation: situation with immediate effort costs and delayed health benefits.
- Model with present bias can explain these findings
 - ► For naive agents: they tend to overestimate the self control in the future, so they overestimate their future attendance
 - ► For sophisticated agents, the contract serves as a commitment device

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Chap.3 Repeated games 89 / 89

