Game Theory in Banking, Finance, and the International Arena

Master PEI - Autumn 2022

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Chap.2 Sequential games

Sequential games
Outline

Introduction

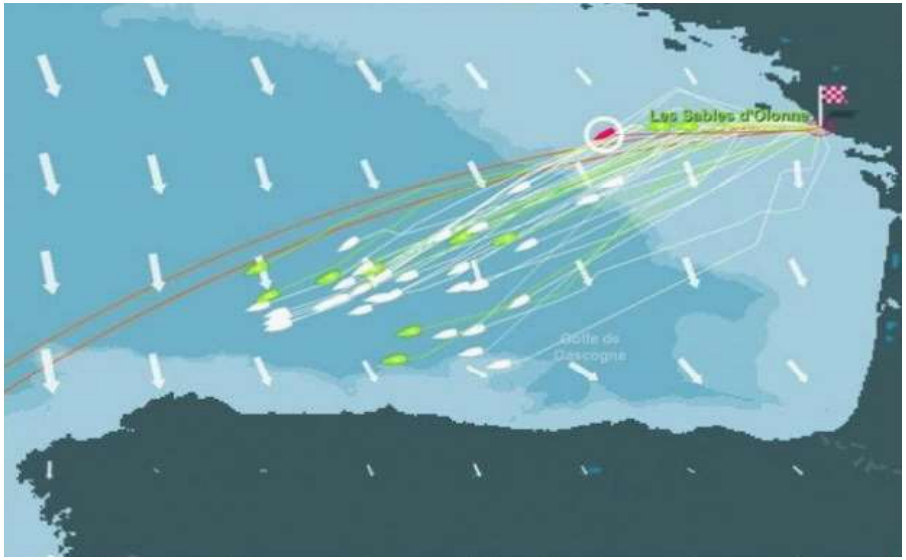
- Up till now we have seen games where players play simultaneously
- In most situations, players do not move simultaneously
- We now move to games where they play in turn and can observe what happened before
- Called sequential games.
- One player usually moves first and the other can observe the move

Introduction
To lead or not to lead

- Widespread strategy: « follow the leader ».
  - Why ?
  - If the leader's position is due to his ability then... he is the person others want to follow.
- Counter-example: Vendée Globe (round-the-world single-handed yacht race)

Introduction
To lead or not to lead

Introduction
To lead or not to lead



Introduction
To lead or not to lead

- Counter-example: Vendée Globe 1st imitates 2nd!
  - The leader imitates the follower even when the follower is clearly pursuing a poor strategy. Because in sailboat racing close doesn't count: only winning matters.

Introduction
To lead or not to lead: Stock-market analysts and economic forecasters

- Leading forecasters have an incentive to follow the pack and produce predictions similar to everyone else's.
  - This way people are unlikely to change their perception of these forecasters' abilities.
- Newcomers take the risky strategies:
  - They tend to predict boom or doom.
  - Usually they are wrong and are never heard of again.
  - But once they are proven to be correct they move to the ranks of the famous.
- Example: Ranking on the stock pickers sell-side analysts
  - Best on the Street Survey (Wall Street Journal);
  - StarMine (Thomson Reuters); and
  - The All-America Research Team (Institutional Investor).

Introduction
To lead or not to lead: Stock-market analysts and economic forecasters


BEST ON THE STREET: 2012 ANALYSTS SURVEY

THE JOURNAL REPORT

THE WALL STREET JOURNAL. Thursday, May 10, 2012

What Makes a Great Stock Analyst?

After two decades of our Best on the Street survey, some qualities stand out

BEST ON THE STREET 20 YEARS THE WALL STREET JOURNAL

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Introduction
To lead or not to lead: Stock-market analysts and economic forecasters


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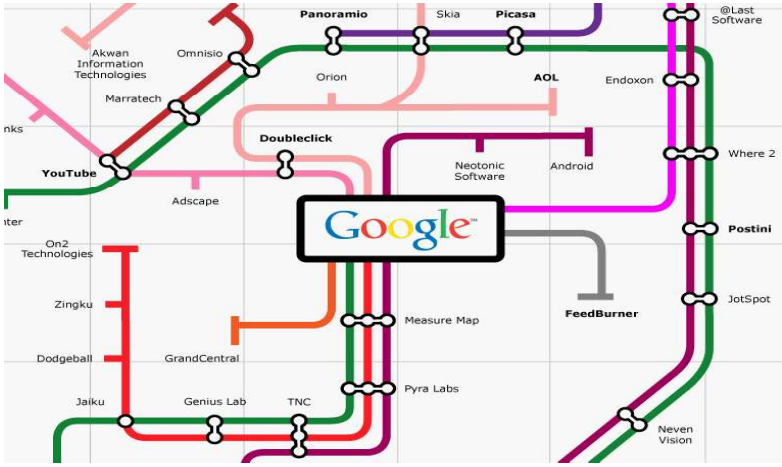
Left to right: C. Stephen Tusa Jr., J.P. Morgan; Stephen Chin, UBS; Jeffrey Zekauskas, J.P. Morgan

Introduction
To lead or not to lead: Industrial and technological competition

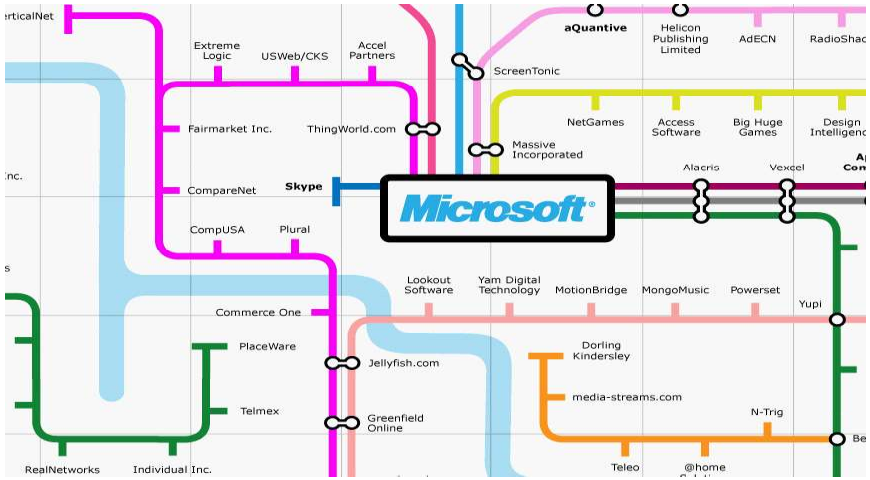
- IBM vs Apple.
  - In 1990's IBM was less known for its innovation than for its ability to bring standardized technology to the mass market.
  - More new ideas had come from Apple and Sun.
- Start-up companies have to take risky innovations to gain market share.
- In Silicon Valley, big firms as Microsoft, Apple, or Google usually:
  - observe start-up companies activity;
  - after a period of natural selection, they choose which survival to acquire.

Introduction
To lead or not to lead: Industrial and technological competition



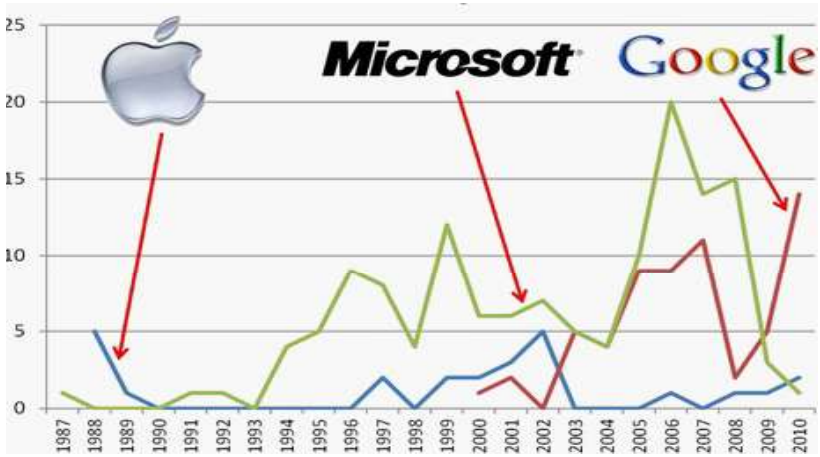
Google acquisitions and investments

Introduction
To lead or not to lead: Industrial and technological competition



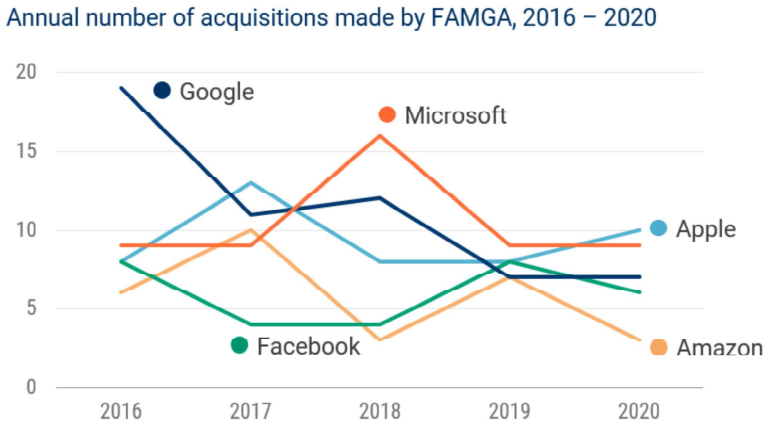
Microsoft acquisitions and investments

Introduction
To lead or not to lead: Industrial and technological competition



Number of acquisitions per year

Introduction
To lead or not to lead: Industrial and technological competition



Introduction
To lead or not to lead: Industrial and technological competition

Google vs Facebook

- One firm may have the leadership in one field and be a follower in another.
- Two firms may compete by... imitating each other leadership activity.
- Google Buzz is a social networking and messaging tool created by Google to compete with Facebook.
- Facebook now integrates Bing as an internet search engine, so members don’t have to leave anymore the site to conduct additional research.

Introduction
To lead or not to lead: Sport vs Business

- There are two ways to move second:
  - imitate as soon as the other has revealed his approach (as in sailboat racing); or
  - wait longer until the success or the failure of the approach is known (as in computer).
- The second strategy is more advantageous in business because, unlike sports,... the competition is usually not winner-take-all.

Sequential games
Outline

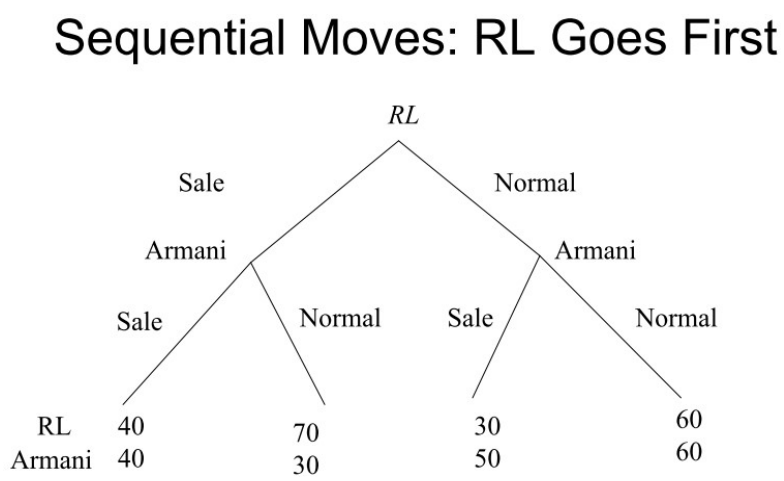
Simultaneous vs sequential move
Back to fashion

		RL: Sale	RL: No sale
Armani: Sale		40 , 40	50 , 30
Armani: No sale		30 , 70	60 , 60

Simultaneous vs sequential move
Back to fashion

- Situation represented by a tree
- At each node of the tree, one player makes a decision
- Each branch of the tree represents a different decision

Simultaneous vs sequential move
Back to fashion



Sequential games
Outline

Solution concept

- Need to be more careful in definition of strategies!!!
- A strategy is a full contingent plan: states what you will do at every node in the game where you play
- Define what you do even at points of the game unlikely to be reached.
- In the previous case:
  - Strategy for RL is sale or normal
  - Strategy for Armani is more complicated: it states what Armani plays
    - If RL plays Sale
    - If RL plays Normal

Solution concept

- Nash equilibria defined as it was before:

Definition (Informal)

A **Nash equilibrium** is a strategy profile where each player's strategy is the best response to that of the other.

- In this case the following are Nash equilibria:
  - RL plays Sale / Armani plays Sale if RL plays Sale and plays Sale if RL plays Normal
  - RL plays Normal / Armani plays Sale if RL plays Sale and plays Normal if RL plays Normal

Solution concept

- First equilibrium based on a non credible threat.
  - RL does not play Normal because of the threat that if he does, Armani will play Sale: but threat not credible
- Need equilibrium concept that is stronger.

Solution concept

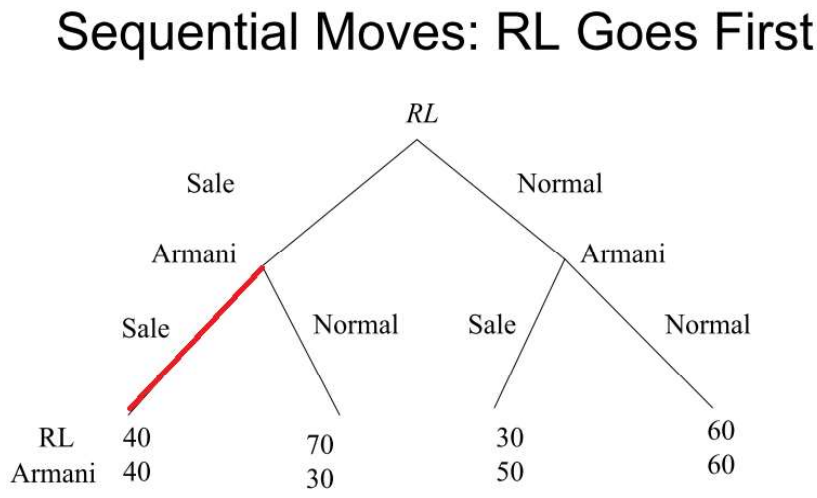
Definition

A **Subgame Perfect Nash Equilibrium** is a combination of strategies, one for each player, that generates a Nash Equilibrium in every subgame.

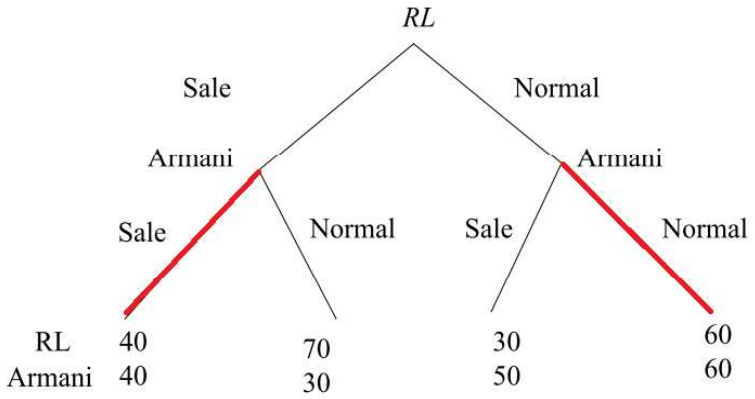
- Subgame: any game starting at some node of the tree
- To find subgame perfect nash equilibrium, use **backwards induction**:
  - Start from the end of the game
  - Determine optimal choice of the player playing last
  - Pull back up the payoffs corresponding to the actions he chose

Sequential games
Outline

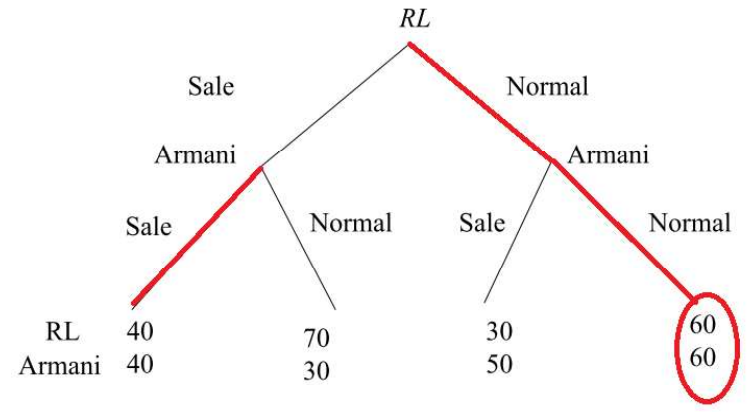
Order of moves
Back to fashion



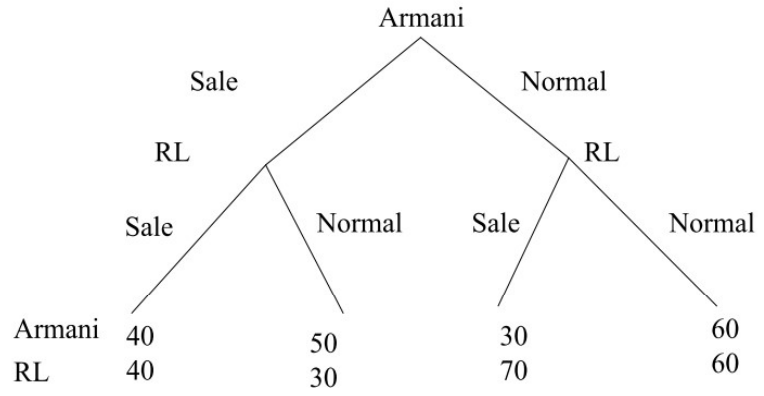
Sequential Moves: RL Goes First



Sequential Moves: RL Goes First



What if Armani goes first



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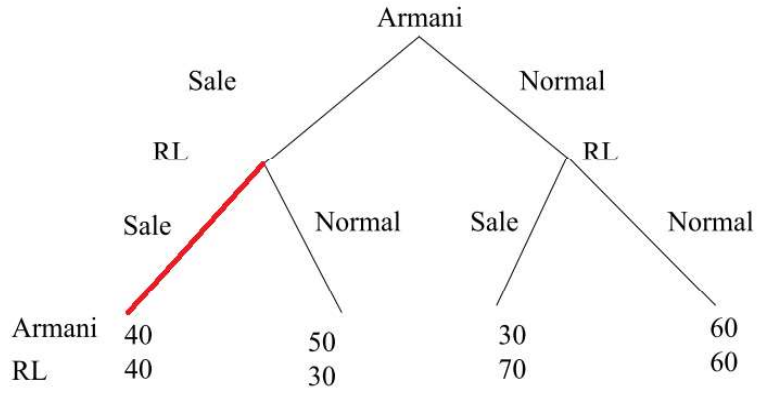

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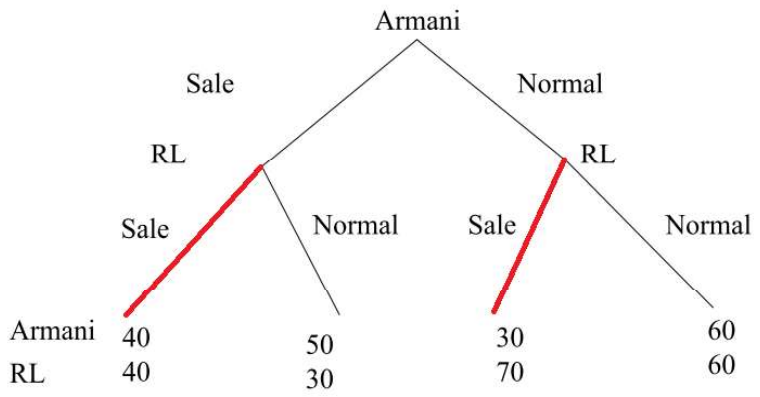
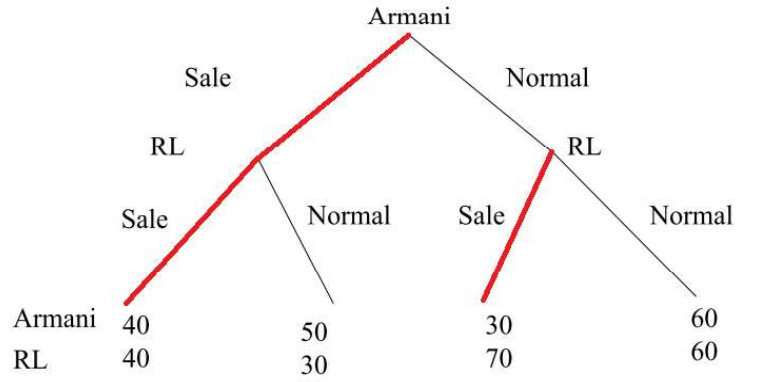


What if Armani goes first



Competition for 3D innovation between U.S. and Japan.

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low		
	High		

Assume that for a similar quality product, U.S. obtains a higher market share. But Japan has lower costs.

Announcing his strategy

Both sides regard a high-effort race as the worst scenario.

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(.,.)	(.,.)
	High	(.,.)	(1,1)

Announcing his strategy

Each side's second worst outcome is to offer a lower quality product than its competitor does.

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(.,.)	(2,.)
	High	(.,2)	(1,1)

Announcing his strategy

The Japanese like best the situation in which they pursue high effort and U.S. follows low effort.

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(.,.)	(2,4)
	High	(.,2)	(1,1)

Announcing his strategy

The second best outcome for Japan is competition of similar low quality products.

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(.,3)	(2,4)
	High	(.,2)	(1,1)

Announcing his strategy

This last situation is the U.S.'s first-best because they would obtain the higher market share…

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(4,3)	(2,4)
	High	(.,2)	(1,1)

Announcing his strategy

… while offering a better quality product would give them a just higher market share, but at high costs of effort.

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(4,3)	(2,4)
	High	(3,2)	(1,1)

Announcing his strategy

U.S. has a dominant strategy:

_ _ .

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(4,3)	(2,4)
	High	(3,2)	(1,1)

Announcing his strategy

U.S. has a dominant strategy:

Low.

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(4,3)	(2,4)
	High	(3,2)	(1,1)

Announcing his strategy

So Japan will choose:

_ _ _ _ _ _ _.

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(4,3)	(2,4)
	High	(3,2)	(1,1)

Announcing his strategy

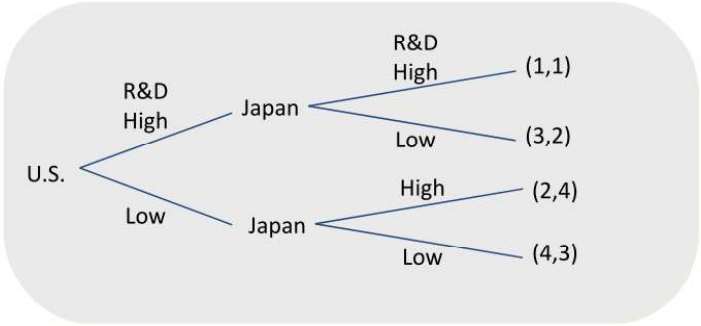
So Japan will choose:

a high level.

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(4,3)	(2,4)
	High	(3,2)	(1,1)

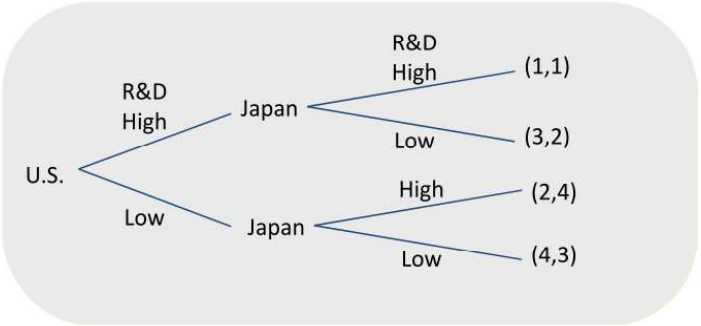
Announcing his strategy

Suppose now the U.S. _ _ _ _ _ _.



Announcing his strategy

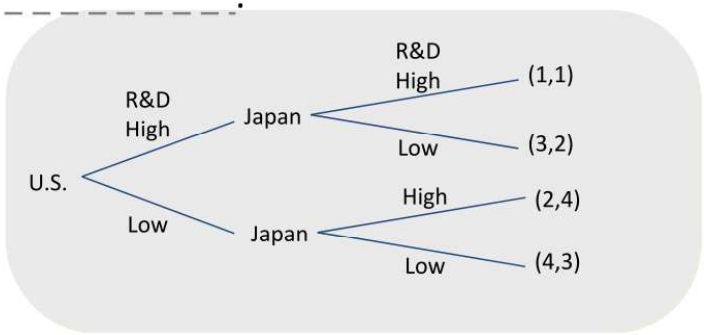
Suppose now the U.S. preempts.



Announcing his strategy

The solution is given by

_ _ _ _ _ _ _ _ _ _ _ _.

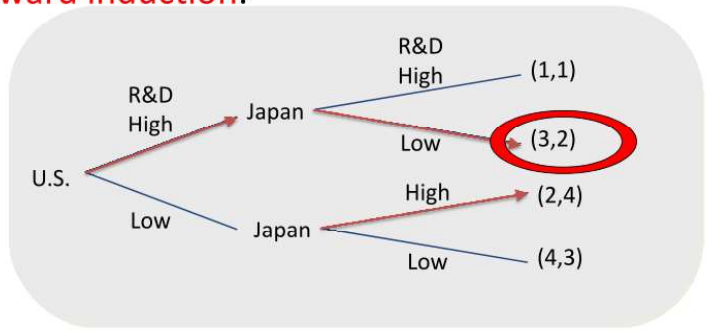


It consists in for U.S. to announce « _ _ _ », and for Japan to respond « _ _ _ ».

Announcing his strategy

The solution is given by

backward induction.



It consists in for U.S. to announce « High », and for Japan to respond « Low ».

Announcing his strategy

Comparing with the simultaneous move…

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(4,3)	(2,4)
	High	(3,2)	(1,1)

Announcing his strategy

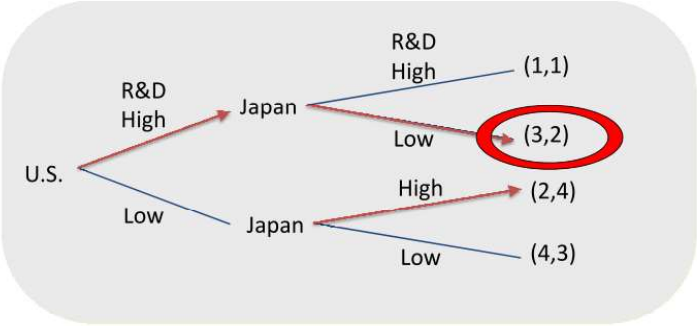
Comparing with the simultaneous move…

		Japanese R&D effort	
		Low	High
U.S. R&D effort	Low	(4,3)	(2,4)
	High	(3,2)	(1,1)

Announcing his strategy

... U.S. increase their payoffs.

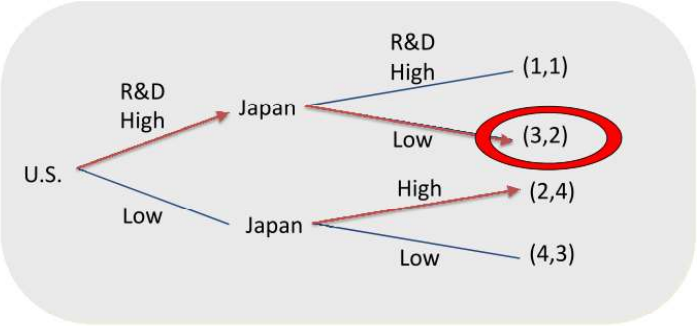
So they have an advantage to _ _ _ _ _ _ _ !



Announcing his strategy

... U.S. increase their payoffs.

So they have an advantage to move at first!



Sequential games
Outline

Bargaining

- The solution of any bargaining depends on:

1. Who gets to make an offer to whom?
2. What happens if the parties fail to reach an agreement?

Bargaining

- By posting a price in retail stores, the sellers make a «take-it-or-leave-it» offer.
- In the case of wage bargaining, a labor union makes a claim and then the company decides whether to accede. If it does not, it may a counteroffer, or wait for the union to adjust its demand.
- An essential feature of negotiations is that « time is money ».
- When negotiation becomes protracted, the pie begins to shrink.
- If failure to reach a wage agreement leads to a labor strike, the firm loses profits and workers lose their wages.

Bargaining

- Charles Dickens's Bleack House (1852) novel illustrates the extreme case.
- At the novel's core is long-running litigation in England's Court of Chancery which has far-reaching consequences for all involved.
- The case revolves around a testator who apparently made several wills.
- The litigation was so prolonged that the entire estate was swallowed up by lawyers' fees.

Bargaining

- Consider the following game:
  - 2 players have to split a pie.
  - Player 1 makes an offer. If player 2 refuses, the pie shrinks to zero.
- What is player 1's optimal strategy?
  - 1 offers the splitting (99%,1%)

Bargaining

- Suppose there is a second round of negotiation:
  - If 2 refuses 1's offer, then the pie shrinks to $\frac{1}{2}$ and 2 can make an counteroffer.
  - If 1 refuses 2's counteroffer then the pie shrinks to zero.
- What are players' optimal strategies?
  - Let us proceed by backward induction: looking ahead and reasoning back

Bargaining

- At the 2nd round, 2 offers the splitting
  - (99%,1%) of the remaining pie (i.e., half of the original pie).
- Considering what will happen during the 2nd round, if 1 wants to incentivize 2 to accept his 1st round offer, he has to offer to 2:
  - at least 49,5% of the whole pie.
- Hence, 1 offers the splitting (50,5%, 49,5%) and 2 agrees.
  - (It would be 50:50 if the pie were continuously divisible.)

Bargaining

- Let us add a 3rd round:
  - at each step, the pie loses 1/3.
- At the 3rd round, 1 offers the splitting:
  - (99%,1%) of the remaining pie (i.e., third of the original pie).
- At the 2nd round, 2 offers at least:
  - 1/3 of the original pie and keeps the remaining, i.e. $(\frac{1}{2})$x(2/3)=1/3.
- At the 1st round, 1 offers the splitting :
  - (2/3,1/3) of the original pie.

Bargaining

- What about the 50:50 splitting?
  - It is the outcome when the number of rounds is even. I.e., when 2 makes the final offer.
- For n steps, we obtain the following splitting:
  - $(50, 50)$ if n is even;
  - $(\frac{n+1}{2n}, \frac{n-1}{2n})$ if n is odd.

Sequential games
Outline

War and Peace

Some years ago, Sudan was ...

War and Peace

... a prey to Libya.

War and Peace

But Libya did not want to draw back troops away from its eastern border with Egypt!

War and Peace

Three enemies created stability.

War and Peace

1967: Six-Day War.

War and Peace

1978: Camp David Accords. Carter bring together Begin and El-Saddate. Peace Nobel prize 1978.

Sequential games
Outline

Complexity

Theorem

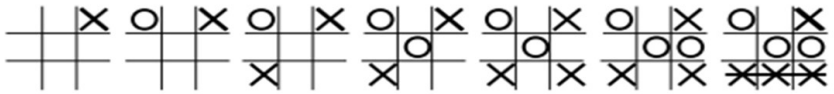
Every sequential game with a finite number of actions possess a best response strategy.

Problem

The number of leaves (or terminal nodes) may be very high!

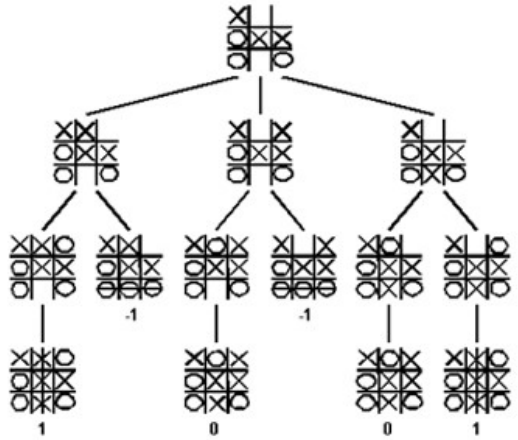
Complexity
Tic-tac-toe

- For instance, even for a game as simple as tic-tac-toe:



- Winning the game requires at least 5 moves. (So there are at least 3 crux)

Complexity
Tic-tac-toe



- The corresponding game tree has a minimum of 9x8x7x6x5=15 120 terminal nodes!

Complexity
Tic-tac-toe

- We can use a symmetric argument.
- There are 9 possibilities for the first move.
- By symmetry, we can reduce it to 3: corner, edge, or middle. And, so on. . .
- Using backward reasoning enables us to compute an algorithm that beat the Backgammon world champion, almost every chess players, . . .

Sequential games
Outline

Promises and threats

Republicans vs Democrats.

A Republican government would like to make Congress votes a budget cut for the sake of a smaller deficit.

The Democrats disapprove because they prefer a Keynesian economic stimulus plan.

Each party has two choices.

Promises and threats

Democrats has to choose between:

- « supporting » the budget proposal, by voting « yes » to some law propositions; and

- « attacking », by voting « no » to all propositions.

		Republicans	
Democrats	support		
	attack		

Promises and threats

Republicans has to choose between :

- « completely supporting » the budget proposal, by voting « yes » to all law propositions; and

- «compromising», by voting « yes » to some, but not all law propositions.

		Republicans	
		support completely	compromise
Democrats	support		
	attack		

Promises and threats

In the case where both parties confront one another by Republicans voting «yes» and Democrats voting «no» to all propositions, the budget cut is blocked.

Opinion polls indicate that people would be infuriated by status quo outcome, and they would criticize Democrats.

		Republicans	
		support completely	compromise
Democrats	support		
	attack	Status quo	

Promises and threats

Democrats would like Republicans to compromise.

		Republicans	
		support completely	compromise
Democrats	support	(2,.)	(3,.)
	attack	(1,.)	(4,.)

Promises and threats

Republicans would like Democrats to support.

		Republicans	
		support completely	compromise
Democrats	support	(.,4)	(.,3)
	attack	(.,2)	(.,1)

Promises and threats

Republicans have a dominant strategy :

		Republicans	
		support completely	compromise
Democrats	support	(2,4)	(3,3)
	attack	(1,2)	(4,1)

Promises and threats

Republicans have a dominant strategy :

to « support completely ».

		Republicans	
		support completely	compromise
Democrats	support	(2,4)	(3,3)
	attack	(1,2)	(4,1)

Promises and threats

Democrats’ best response is:

_ _ _ _ _ _ _ _ _

		Republicans	
		support completely	compromise
Democrats	support	(2,4)	(3,3)
	attack	(1,2)	(4,1)

Promises and threats

Democrats’ best response is:

to « support ».

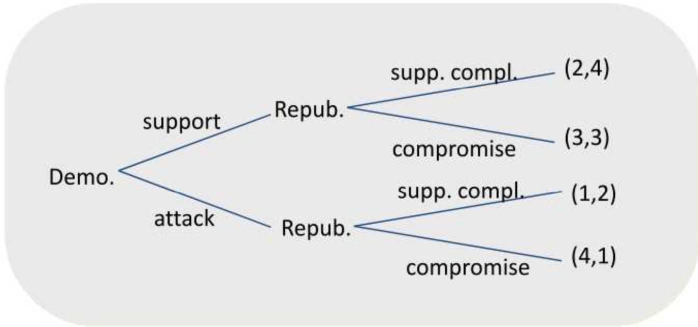
		Republicans	
		support completely	compromise
Democrats	support	(2,4)	(3,3)
	attack	(1,2)	(4,1)

Promises and threats

Would Democrats benefit from announcing their strategy?

Republicans _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ .

So playing at first _

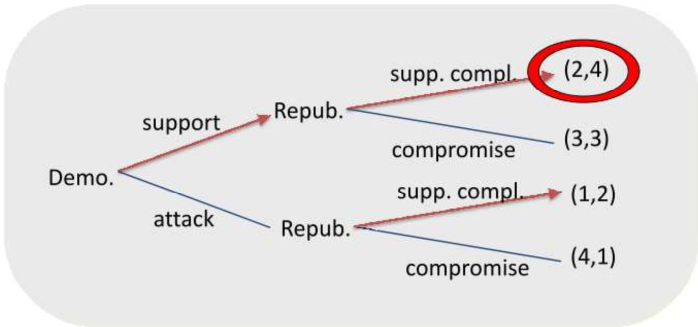


Promises and threats

Would Democrats benefit from announcing their strategy?

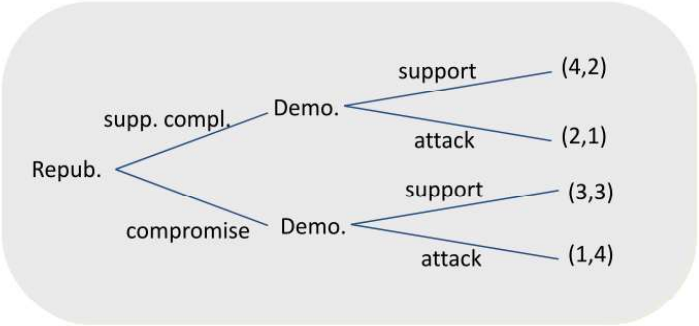
Republicans still have their dominant strategy.

So playing at first does not change anything for Democrats!



Promises and threats

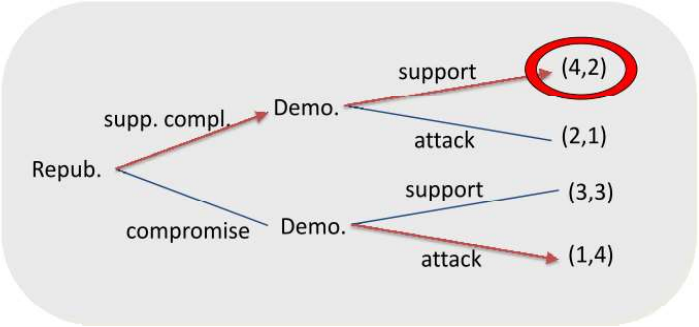
Would Democrats benefit from letting Republicans to play at first, then best-responding?



Promises and threats

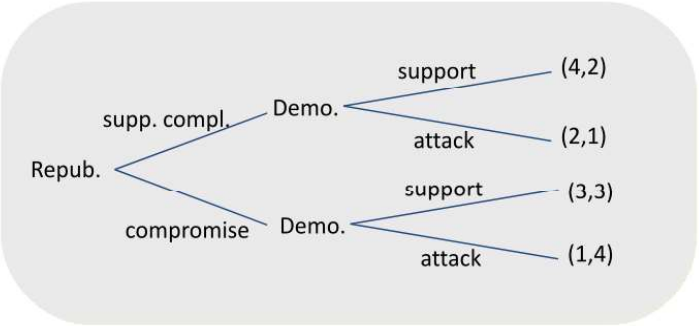
Would Democrats benefit from letting Republicans to play at first, then best-responding?

This still does not change anything!



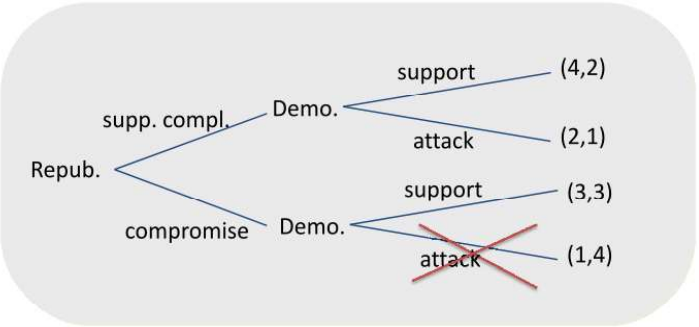
Promises and threats

An other solution would consist in Democrats:

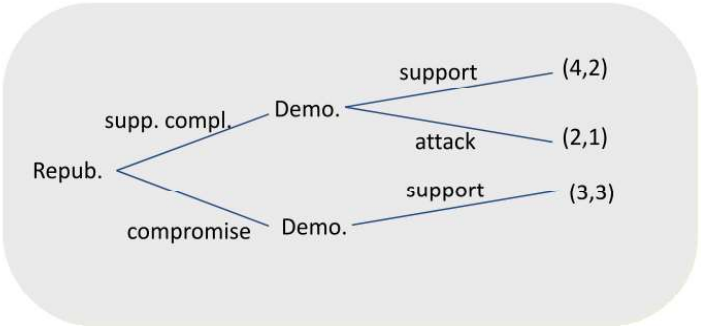


Promises and threats

An other solution would consist in Democrats:
promising to «support» if Republicans «compromise».



Promises and threats



Promises and threats

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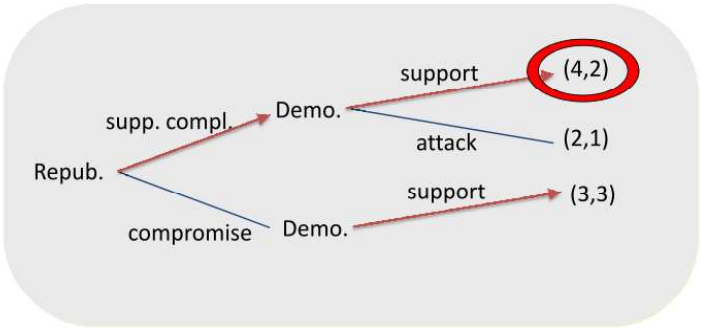
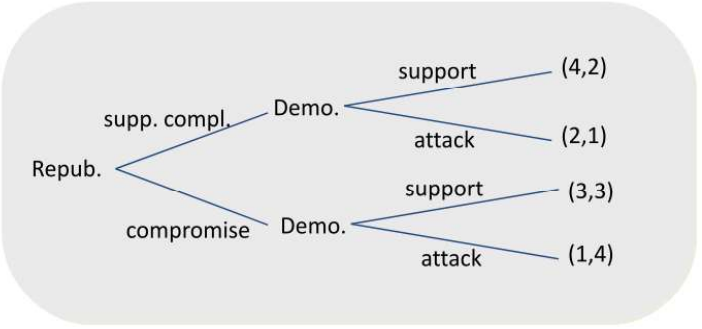


Promises and threats

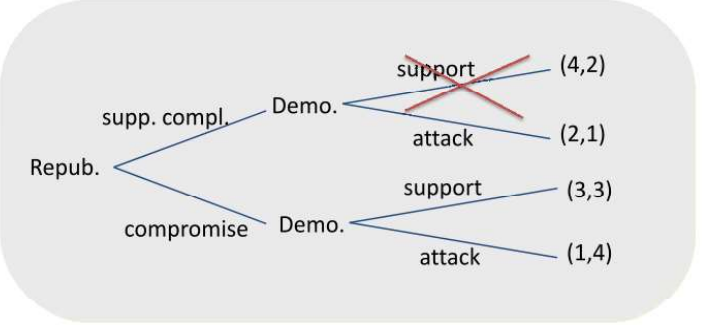
An other solution would consist in Democrats to



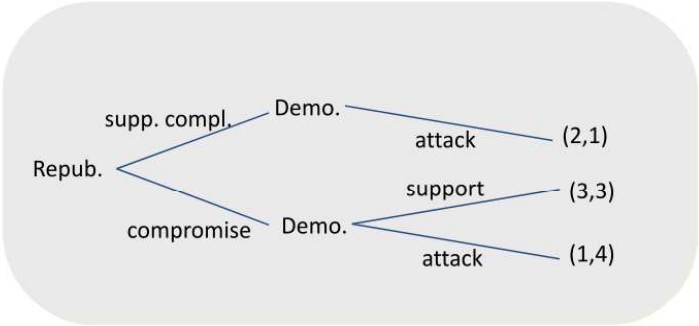
Promises and threats

An other solution would consist in Democrats to

threaten to « attack » if Republicans « supp. compl. »

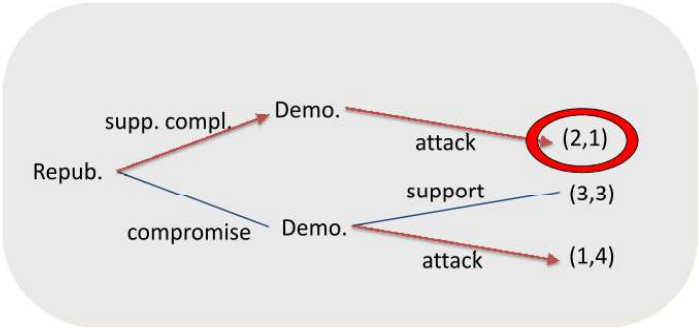


Promises and threats



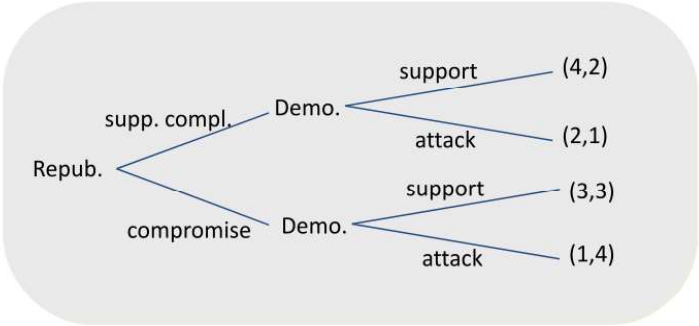
Promises and threats

But Democrats would be worse-off!



Promises and threats

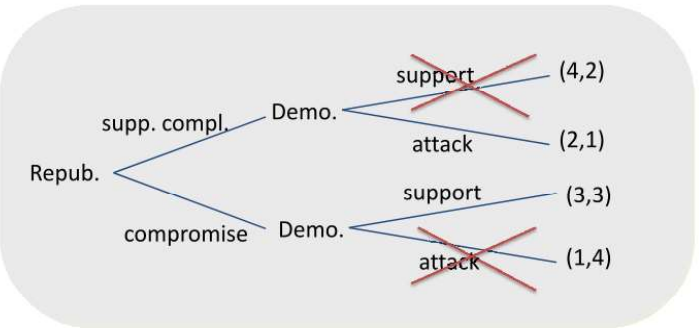
The solution is Democrats:



Promises and threats

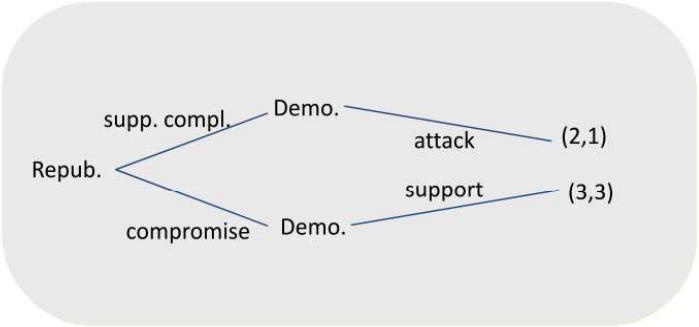
The solution is Democrats:

promising to «support» if Republicans «compromise»; and to threaten to «attack» if Republicans «supp. compl. ».



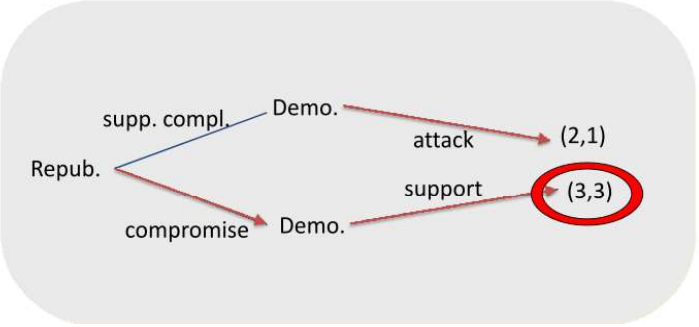
Promises and threats

_ _.



Promises and threats

In this case Republicans do compromise.

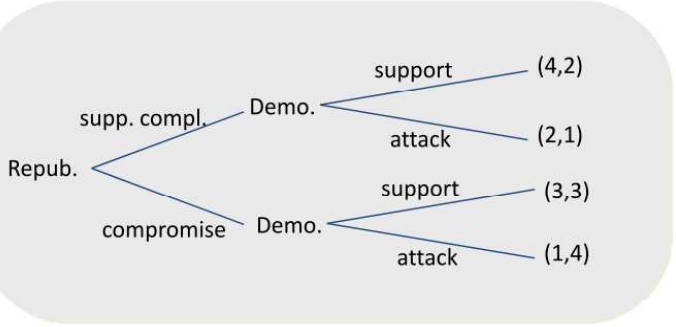


Promises and threats

Such a strategy requires Democrats to be credible.

Because once Republicans have « compromised » …

… they have _ !

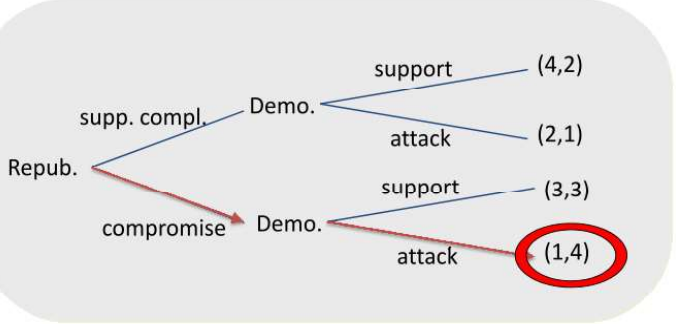


Promises and threats

Such a strategy requires Democrats to be credible.

Because once Republicans have « compromised » …

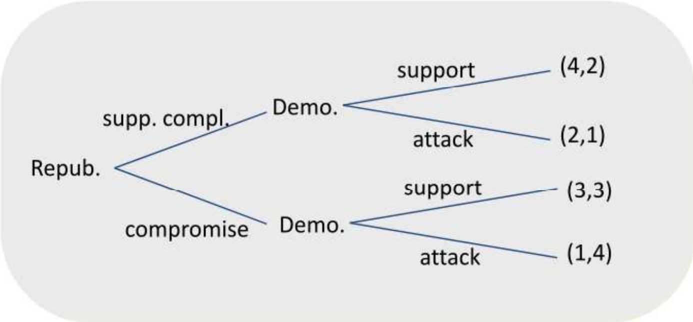
… they have a profitable deviation: to « attack »!



Promises and threats

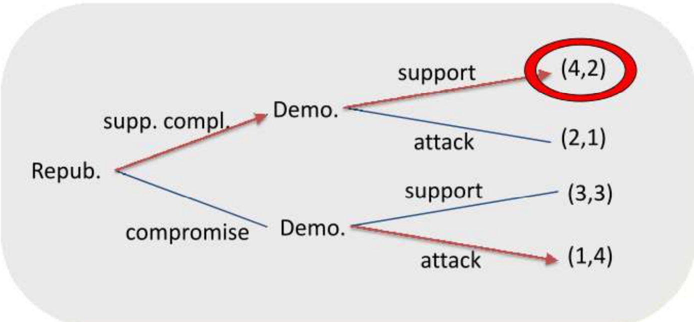
If Republicans anticipate such a deviation,

_ _



Promises and threats

If Republicans anticipate such a deviation,
we are back to the initial situation.



Promises and threats

Interacting repeteadly may confer credibility to _ _ _ _ _ _ _ _ _ _ _ _ _.

If the interaction occurs only once, _

Promises and threats

Interacting repeteadly may confer credibility to promises and threats.

If the interaction occurs only once, it is difficult to credibly commit on a strategy that does not use a best-response at all nodes.

Sequential games
Outline

Burning bridges behind

- In sequential games we saw that the order of moves is essential
- Players can then take actions prior to the game, that will change the outcome
- These actions can:
  - Make threats credible
  - Change directly the payoffs of the game

Burning bridges behind

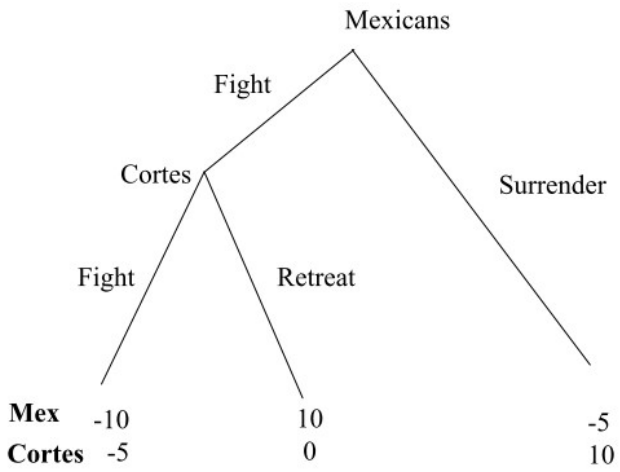
« *Quemar las naves !* »

Hernán Cortés
1485 - 1547

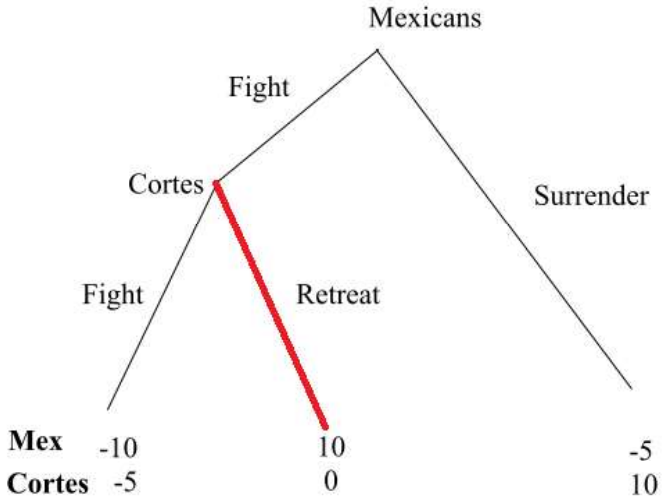
Burning bridges behind

- Most classical examples of these types of moves can be found throughout history
- Famous example is Cortes' conquest of Mexico
- Upon arrival in Cempoalla he gave orders to burn down all ships
- What did this change?

Burning bridges behind



Burning bridges behind



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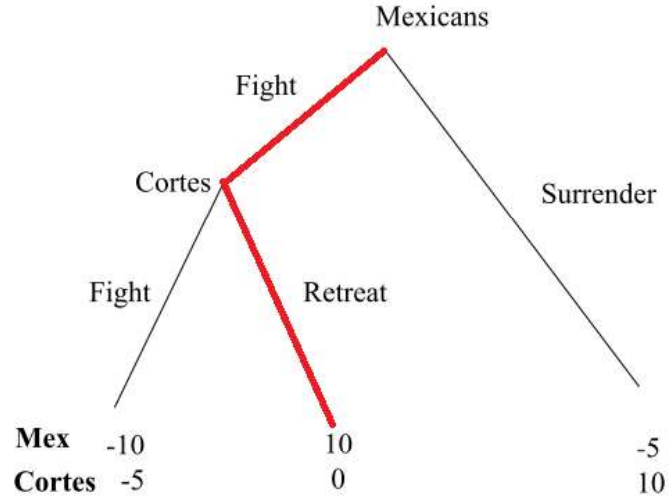
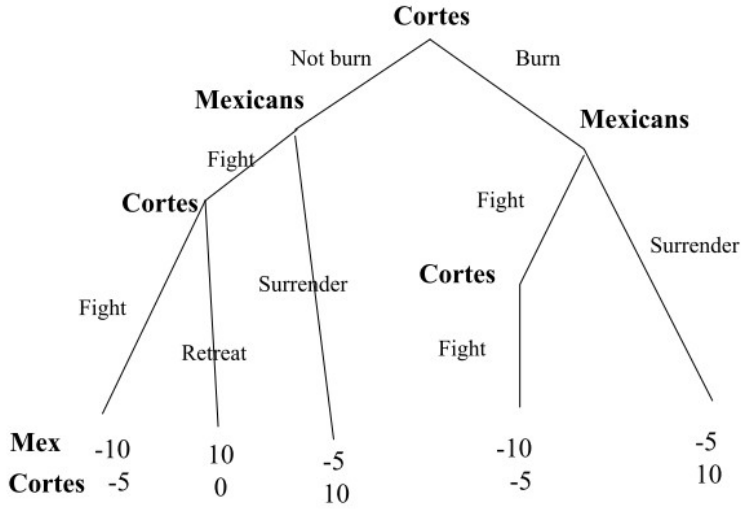
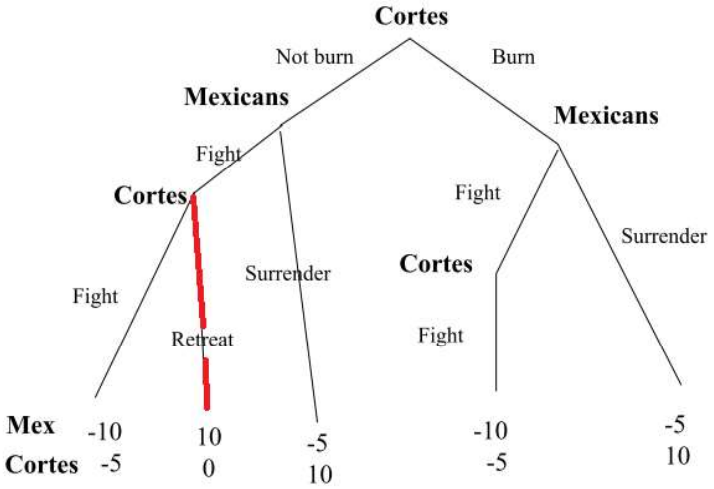


Burning bridges behind



Burning bridges behind



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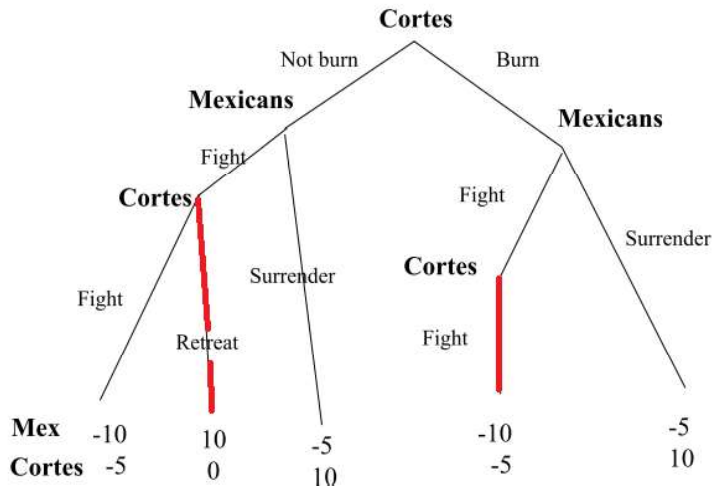

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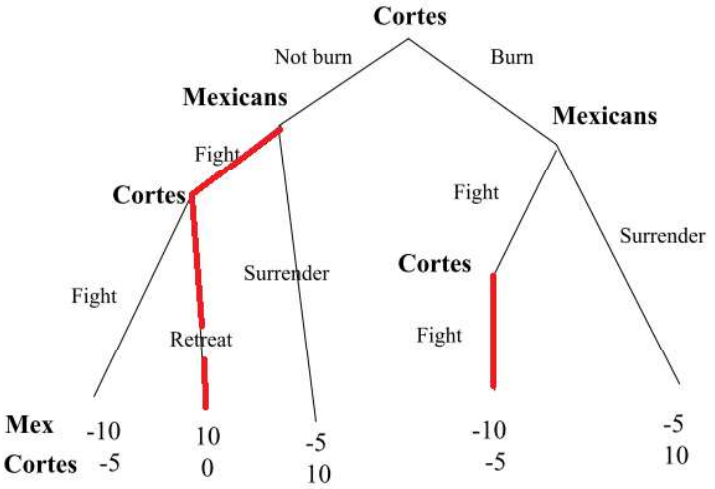
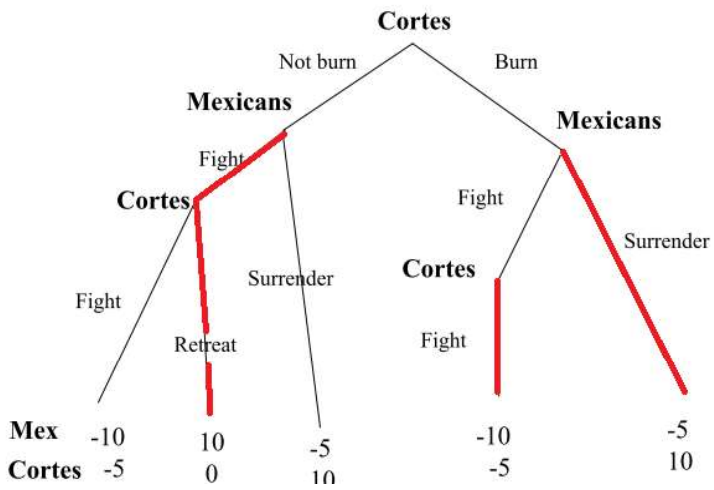
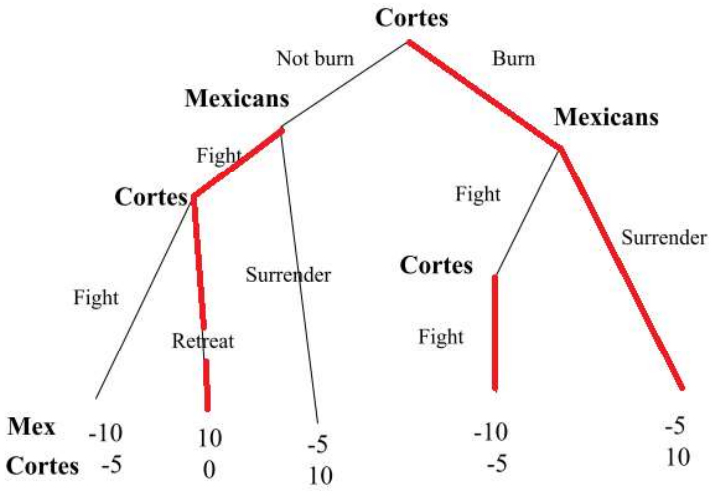


Burning bridges behind



Burning bridges behind



Burning bridges behind
Politics

- Such examples abound in history of warfare
  - Involve burning down ships, bridges (ways to retreat)
- Not limited to warfare
- Campaign promises: are they credible?
  - Credibility is linked to the cost of going back on the promise
  - How can the cost be changed?
  - Delegating policy to independent agencies
  - In particular monetary policy
    - Limiting *stop-and-go* policy
  - Goal is to create *credibility*