Exercises Chapter 3: Repeated games

Exercise 1: Price competition in Duopoly

Two firms are playing an infinitely-repeated prisoner's dilemma pricing game of the following form:

		Firm 2	
		Low	High
Firm 1	Low	(5,5)	(20,0)
	High	(0,20)	(10,10)

The firms simultaneously set prices at regular intervals. In the equilibrium of this game, each firm selects the low price. While the equilibrium results in profits of \$5 for each firm, collusion can potentially result in payoffs of \$10. The firms utilize trigger strategies in order to maintain the collusive outcome. We suppose firms' interest rate (r) is strictly positive.

a) Suppose that both firms adopt the grim trigger strategy. They continue colluding until one of them cheats. Upon one of them defecting, they play the equilibrium strategy for the rest of the game.
What has to be true about the interest rate (r) for collusion to be sustainable?

b) Suppose that both firms adopt a tit-for-tat strategy. They initially collude. In future periods, a firm colludes if its competitor did in the previous period, and elects the lower price if its competitor cheated in the previous period.
What has to be true about the interest rate (r) for collusion to be sustainable?

Exercise 2: Monopolistic competition

Three firms are in monopolistic competition for producing goods that are imperfect substitutes. They choose their prices simultaneously. Consumers' demand for the firm *i*, with *i=1,2, and 3*, writes as $q_i = 100 - 3p_i + \sum_{j\neq i} p_j$, where p_i denotes firm *i*'s price. We assume production costs are zero.

a) Solve the Nash equilibrium of the stage game. What are the associated profits?

b) Find the strategies and profits associated with the "cooperative" solution that would maximize the total profit.

c) Consider now the corresponding infinitely repeated game. Let δ_i denotes firm *i*'s discount factor. Define a trigger strategy that may sustain cooperation at equilibrium.

d) Show that there are values of δ_i (*i*=1,2,3) such that cooperating at every stage sustain a SPNE. Give the corresponding strategies and value for δ_i (*i*=1,2,3).

Exercise 3: Tacit collusion

Consider two firms that compete in price $T + 1$ times. At each date t, $t = 0,1, \dots, T$, the firms choose their prices (p_{1_t}, p_{2_t}) simultaneously. Let $\pi^i(p_{i_t}, p_{j_t})$ be firm i's profit at date t when it charges p_{i_t} and its rival charges p_{j_t}. Consider a firm i's discount factor $\delta_i \in (0,1)$, so that the discounted value of firm i's profits writes as:

$$\sum_{t=0}^{T} \delta_i^t \, \pi^i(p_{i_t}, p_{j_t})$$

At date t, if $p_{i_t} > p_{j_t}$ then firm i makes zero profit; if $p_{i_t} = p_{j_t}$ then $\pi^i(p_{i_t}, p_{j_t}) = \alpha_i \pi(p_{i_t}, p_{j_t})$, where $\pi(p_{i_t}, p_{j_t})$ denotes the aggregate profits at date t, that is shared between the firms according to α_1 and α_2, two positive real numbers satisfying $\alpha_1 + \alpha_2 = 1$. The unit cost of production is c.

B_1) Is any collusion sustainable as a (time-invariant) equilibrium in finite horizon? Why?

B_2) In infinite horizon, give the smaller discount factors $\underline{\delta_1}$ and $\underline{\delta_2}$ such that any pair (δ_1, δ_2) satisfying $\delta_1 \geq \underline{\delta_1}$ and $\delta_2 \geq \underline{\delta_2}$ allows to fully collude at (time-invariant) equilibrium. Start by exhibiting an adapted trigger strategies profile.

B_3) Explain why $\underline{\delta_i}$ is a decreasing function of α_i.