Master PEI : Game Theory in Banking, Finance, and the International Arena

Chapter 3 : Solution to Additional Exercises and Problems

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Problem 1: Bargaining

Let us proceed by backward induction.

At the last period, T, the player who plays at second will accept any (non-negative) offer because he will not have any possibility to make any counter-offer. The player that starts period T will then take the whole cake.

Formally, if T is even then player 1 offers $(x_T,\ 1-x_T)=(1,0)$, and if T is odd then player 2 offers $(x_T,\ 1-x_T)=(0,1)$.

- If $T=0$ then player 1 makes the offer $(x_0,\ 1-x_0)=(1,0)$ and player 2 accepts immediately.
- If $T=1$ then at period 1 player 2 makes the offer $(x_1,\ 1-x_1)=(0,1)$ and player 1 accepts.
  - At period 0, player 2 accepts player 1's offer $(x_0,\ 1-x_0)$ if and only if it satisfies: $1-x_0 \geq \delta_2(1-x_1)=\delta_2$.
  - So player 1 offers $(x_0,\ 1-x_0)=(1-\delta_2,\delta_2)$ and player 2 accepts.
- If $T=2$ then player 1 makes the final offer $(x_2,\ 1-x_2)=(1,0)$ and player 2 accept immediately.
  - At period 1, player 2 offers $(x_1,\ 1-x_1)=(\delta_1,1-\delta_1)$ and player 1 accepts.
  - So at period 0, player 2 accepts player 1's offer $(x_0,\ 1-x_0)$ if and only if it satisfies: $1-x_0 \geq \delta_2(1-\delta_1)$.
  - So player 1 offers $(x_0,\ 1-x_0)=(1-\delta_2(1-\delta_1),\delta_2(1-\delta_1))$ and player 2 accepts.
- If $T=3$ then player 2 makes the final offer $(x_3,\ 1-x_3)=(0,1)$ and player 1 accept immediately.
  - At period 2, player 1 offers $(x_2,\ 1-x_2)=(1-\delta_2,\delta_2)$ and player 2 accepts.
  - At period 1, player 2 offers $(x_1,\ 1-x_1)=(\delta_1(1-\delta_2),1-\delta_1(1-\delta_2))$ and player 1 accepts.
  - So at period 0, player 2 accepts player 1's offer $(x_0,\ 1-x_0)$ if and only if it satisfies: $1-x_0\geq\delta_2(1-\delta_1(1-\delta_2))$.
  - So, player 1 offers $x_0=1-\delta_2(1-\delta_1(1-\delta_2))=(1-\delta_2)(1+\delta_1\delta_2)$ and player 2 accepts.

Overall, for $k<T$, if T is odd, player 2's offer has to satisfy $x_k=\delta_1 x_{k+1}$; while if T is even, player 1's offer has to satisfy $1-x_k=\delta_2(1-x_{k+1})$, that is $x_k=1-\delta_2(1-x_{k+1})$.

For instance, for $T=5$, proceeding by induction we find:

$$x_0=1-\delta_2(1-\delta_1(1-\delta_2)(1+\delta_1\delta_2))=(1-\delta_2)\left(1+\delta_1\delta_2+{\delta_1}^2{\delta_2}^2\right)$$

More generally, for T odd we find:

$$x_0=(1-\delta_2)\left(1+\delta_1\delta_2+{\delta_1}^2{\delta_2}^2+\ldots+{\delta_1}^{\frac{T-1}{2}}{\delta_2}^{\frac{T-1}{2}}\right)$$

Observe that along the equilibrium path the game ends at period 0 with player 2 accepting player 1's offer. Any threat of the subsequent behavior in case of a refusal has to be credible to sustain the describe strategy profile as a SPE. The threats described previously are the only credible threat. For instance at last period T, any final offer that would propose a strictly positive amount to the opponent is not credible.

Problem 2: The OECD's solution to tax multinational enterprises' income

Part A. Tax competition between two countries : implementing a minimum corporate tax

A.1) The corresponding payoff matrix writes as

$A\backslash B$	L	H
L	$LY^A; LY^B$	$L(Y^A+\alpha^B Y^B); H(1-\alpha^B)Y^B$
H	$HY^A(1-\alpha^A); L(Y^B+\alpha^A Y^A)$	$HY^A; HY^B$

A.2) From our initial assumption, we have :

- $\alpha^A < H-L = (H-L)\frac{Y^A}{Y^A}$, so $LY^A < HY^A(1-\alpha^A)$ and $BR^A(L)=\{H\}$;
- $\alpha^B < \frac{H-L}{L}\frac{Y^A}{Y^B}$, so $L(Y^A+\alpha^B Y^B) < HY^A$ and $BR^A(H)=\{H\}$;
- $\alpha^B < H-L = (H-L)\frac{Y^B}{Y^B}$, so $LY^B < H(1-\alpha^B)Y^B$ and $BR^B(L)=\{H\}$;
- $\alpha^A > \frac{H-L}{L}\frac{Y^B}{Y^A}$, so $L(Y^B+\alpha^A Y^A) > HY^B$ and $BR^B(H)=\{L\}$.

So, country A has a strictly dominant strategy of imposing the tax rate H (i.e., $H \succ_A L$) and country B has no dominant strategy.

A.3) From **A.2)**, at equilibrium A plays its strictly dominant strategy H and B best responds by playing L. So, there is a unique Nash equilibrium, which consists in for country A (resp. B) to tax at rate H (resp. L). Formally, the set of Nash equilibrium writes as $\{(H,L)\}$.

From the previous analysis, the outcome (H,H) (resp. (H,L)) maximizes country A's (resp. B's) payoff. The outcomes (L,L) and (L,H) are Pareto-dominated by (H,H). So, the set of Pareto optima is $\{(H,H),(H,L)\}$.

Suppose country A applies a new law according to which domestic companies which are taxed at a lower rate abroad have to pay the difference in tax to country A. We assume that companies have not yet had time to change tax location.

A.4) The corresponding payoff matrix writes as

$A\backslash B$	L	H
L	$LY^A; LY^B$	$L(Y^A+\alpha^B Y^B); H(1-\alpha^B)Y^B$
H	$HY^A(1-\alpha^A)+(H-L)Y^A\alpha^A; L(Y^B+\alpha^A Y^A)$	$HY^A; HY^B$

Country A still has a strictly dominant strategy of imposing the tax rate H, and B still best responds by playing L. So, the set of Nash equilibrium is the same as in the previous answer : $\{(H,L)\}$.

A.5) Although the unique Nash equilibrium is the same as in the previous answer, the payoffs associated to the equilibrium Pareto dominates the previous payoffs since country A has increased his tax revenue by $(H-L)Y^A\alpha^A$ while country B has the same payoff.

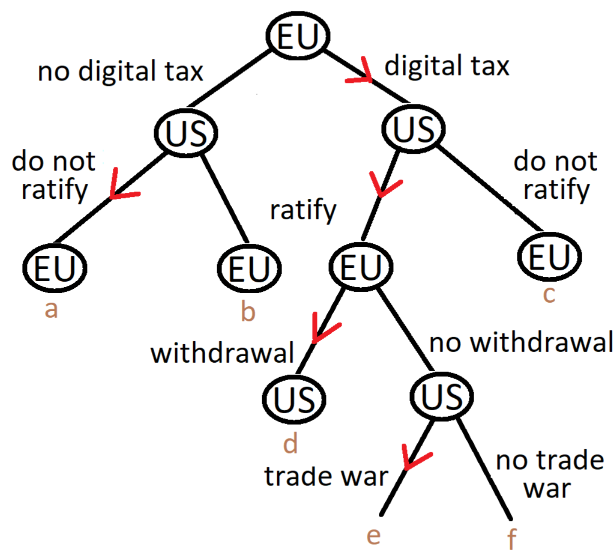
A.6) The corresponding payoff matrix writes as

$A\backslash B$	L	H
L	$LY^A; LY^B$	$L(Y^A+\alpha^B Y^B); H(1-\alpha^B)Y^B$
H	$HY^A; LY^B$	$HY^A; HY^B$

In which case, country B also has a strictly dominant strategy of imposing the tax rate H (i.e., $H \succ_B L$). So, there is a unique Nash equilibrium, which consists in for each country to tax at the high rate : $\{(H,H)\}$.

Part B. Using threat to achieve tax cooperation between the EU and the US

B.1) The corresponding game tree is :



From 1) $d \succ_{EU} a$, and from 2) $d \succ_{EU} f$. From 3) $a \succ_{US} b$, from 4) $d \succ_{US} c$, and from 5) $e \succ_{US} f$. By backward induction, there is a unique subgame perfect Nash equilibrium which sequentially consists in : for the EU to implement the digital tax ; for the US to ratify the tax suggested by the OECD only in case of a EU digital tax ; for the EU to withdraw its digital tax after the US ratification ; and for the US to start a trade war in case of absence of EU withdrawal.

Part C. Minimum tax in repeated interaction

C.1) For any country $i \in \{1,2\}$, $BR^i(\tau^*) = \frac{\tau^*}{2} + \frac{1}{20} = \frac{1}{8} + \frac{1}{20} = \frac{28}{160} = \frac{7}{40} = 17.5\% \neq \tau^*$, so the rate τ^* is not sustainable. The one-shot Nash equilibrium satisfies

$$BR^i(BR^j(\tau_i)) = \tau_i \iff \frac{1}{2}(\frac{\tau_i}{2} + \frac{1}{20}) + \frac{1}{20} = \tau_i \iff \tau_i^N = \frac{1}{10} = 10\%.$$

So, there is a unique Nash equilibrium, which is given by $(\tau_1^N, \tau_2^N) = (\frac{1}{10}, \frac{1}{10})$. This equilibrium is symmetric and we denote $\tau^N = \frac{1}{10}$.

C.2) Grim-trigger strategies prescribe the countries to set the rate τ^* as long as no deviation is observed, and set the static Nash-equilibrium tax rate τ^N forever after a deviation is observed. The optimal deviation of country i from cooperation is given by $BR^i(\tau_j^*) = 17.5\%$. Country i finds it optimal not to deviate at period $\bar{k} \geq 1$ if the following incentive condition holds :

$$\sum_{k=0}^{+\infty} \delta^k g_i(\tau^*, \tau^*) \geq \sum_{k=0}^{\bar{k}-1} \delta^k g_i(\tau^*, \tau^*) + \delta^{\bar{k}} g_i(\tau_i = BR^i(\tau_j^*), \tau_j = \tau^*) + \sum_{k=\bar{k}+1}^{+\infty} \delta^k g_i(\tau^N, \tau^N)$$

which is equivalent to the incentive condition for deviation at period 0 :

$$\sum_{k=0}^{+\infty} \delta^k g_i(\tau^*, \tau^*) \geq g_i(\tau_i = BR^i(\tau_j^*), \tau_j = \tau^*) + \sum_{k=1}^{+\infty} \delta^k g_i(\tau^N, \tau^N)$$

This condition is equivalent to

$$\frac{g_i(\tau^*, \tau^*)}{1-\delta} \geq g_i(\tau_i = BR^i(\tau_j^*), \tau_j = \tau^*) + \frac{\delta g_i(\tau^N, \tau^N)}{1-\delta}$$

$$\iff g_i(\tau^*, \tau^*) \geq g_i(\tau_i = BR^i(\tau_j^*), \tau_j = \tau^*)(1-\delta) + \delta g_i(\tau^N, \tau^N)$$

$$\iff \delta(g_i(\tau_i = BR^i(\tau_j^*), \tau_j = \tau^*) - g_i(\tau^N, \tau^N)) \geq g_i(\tau_i = BR^i(\tau_j^*), \tau_j = \tau^*) - g_i(\tau^*, \tau^*)$$

$$\iff \delta \geq \frac{g_i(\tau_i = BR^i(\tau_j^*), \tau_j = \tau^*) - g_i(\tau^*, \tau^*)}{g_i(\tau_i = BR^i(\tau_j^*), \tau_j = \tau^*) - g_i(\tau^N, \tau^N)} \equiv \bar{\delta}$$

where the last equivalence uses the fact that the denominator is positive. Indeed, since $g_i(\tau_i, \tau_j)$ increases in τ_j, from $\tau_j^* = 25\% > 10\% = \tau_j^N$, we have $g_i(\tau^N, \tau^N) \leq g_i(\tau_i^N, \tau_j^*)$ and, by definition of $BR^i(.)$, the RHS is lower than $g_i(\tau_i = BR^i(\tau_j^*), \tau_j^*)$.

C.3) If a minimum corporate tax rate is set internationally at level $\underline{\tau} \in (\tau^N, \tau^*)$ the static Nash-equilibrium tax rate τ^N can no longer be used as a punishment after a deviation is observed. In particular, at discount factor $\bar{\delta}$, from $g_i(\underline{\tau}, \underline{\tau}) > g_i(\tau^N, \tau^N)$ we have

$$\sum_{k=0}^{+\infty} \bar{\delta}^k g_i(\tau^*, \tau^*) < g_i(\tau_i = BR^i(\tau_j^*), \tau_j = \tau^*) + \sum_{k=1}^{+\infty} \bar{\delta}^k g_i(\underline{\tau}, \underline{\tau})$$

so the incentive condition does not hold and the minimum discount factor $\bar{\delta}$ is not valid anymore. The countries have to be more patient for their tax cooperation to be sustainable.

We can conclude that although the static theory of tax competition implies that a minimum tax cannot be harmful (except, perhaps, at an extremely high level), this is no longer true in dynamic tax competition. Indeed, a lower bound on tax rates restricts the ability of countries to punish deviators, which makes cooperation harder to sustain. For more discussion on this result, see Kiss, Á. (2012). Minimum taxes and repeated tax competition. *International Tax and Public Finance*, 19(5), 641-649.