Master PEI: Game Theory in Banking, Finance and the International Arena

Chapter 2: Solution to Additional Exercises and Problems

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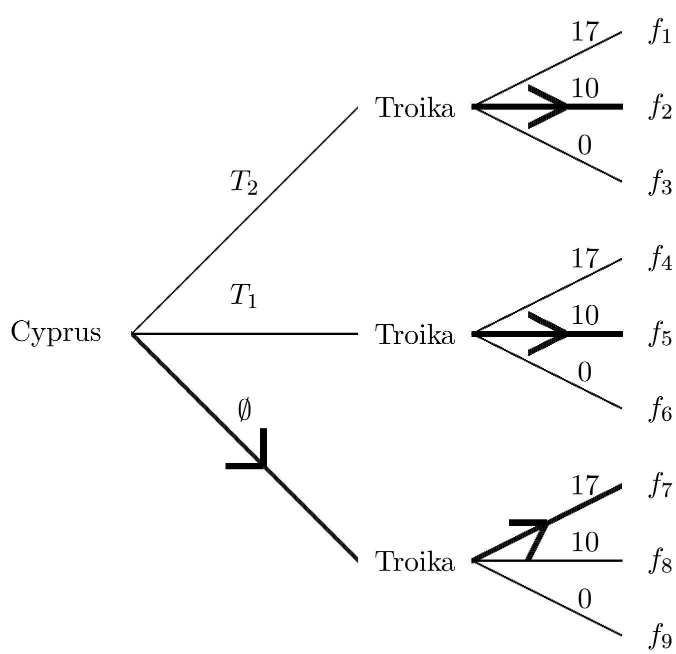
Problem 1: Cyprus: Taxation of bank deposits to avoid a euro exit

1. From (I_4)&(I_5) the three possible Cyprus actions are: not to implement any tax, denoted by $\varnothing$; to adopt the tax T_1; or to adopt the tax T_2.

   From (I_6) the three possible troika's (EU, ECB and IMF) actions are: to not take part in the rescue plan, denoted by 0; to lend 10 billion euros, denoted by 10; or to lend 17 billion euros, denoted by 17.

2. The sequential form game played by Cyprus and the troika where Cyprus moves first depicts as (ignore the array for the moment):



3. From (I_3) Cyprus rescue requires a help estimated at 17 billion euros. The leaves corresponding to a rescue are: $\{f_1, f_2, f_4, f_5, f_7\}$.

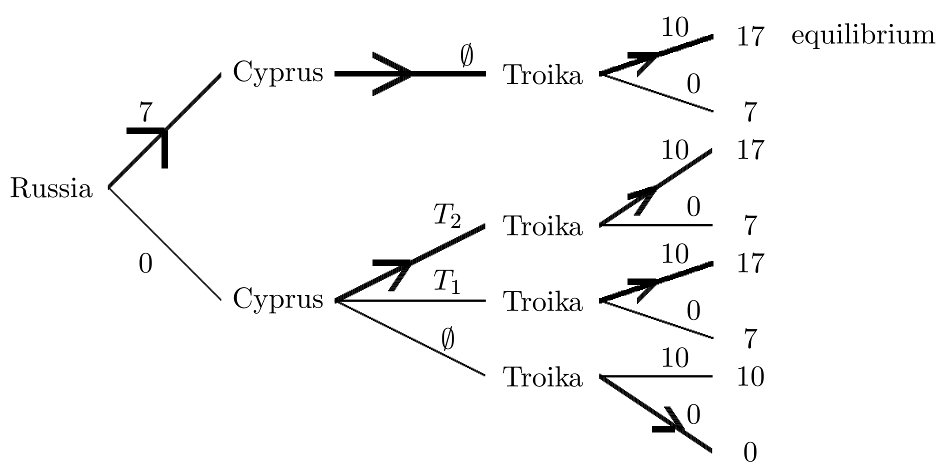
4. From $(I_1)\&(I_2)\&(I_3)$ the troika prefers a Cyprus rescue, but wants to be involved as less as possible. Therefore, troika's preferred leaves when Cyprus plays: T_2 is f_2; T_1 is f_5; and $\varnothing$ is f_7.

5. From (I_5), Cyprus prefers f_2 to f_5. From (I_2) Cyprus prefers f_7 to f_2 and f_5. Finally, among the three preceding leaves, Cyprus prefers f_7.

6. The backward induction outcome is f_7: the troika is the only player involved in the Cyprus rescue.

7. From (I_8) there are only two possible troika's actions: to not take part in the rescue plan, denoted by 0; to lend 10 billion euros, denoted by 10. From (I_7) Russia's actions are: to not take part in the rescue plan, denoted by 0; to lend 7 billion euros, denoted by 7.

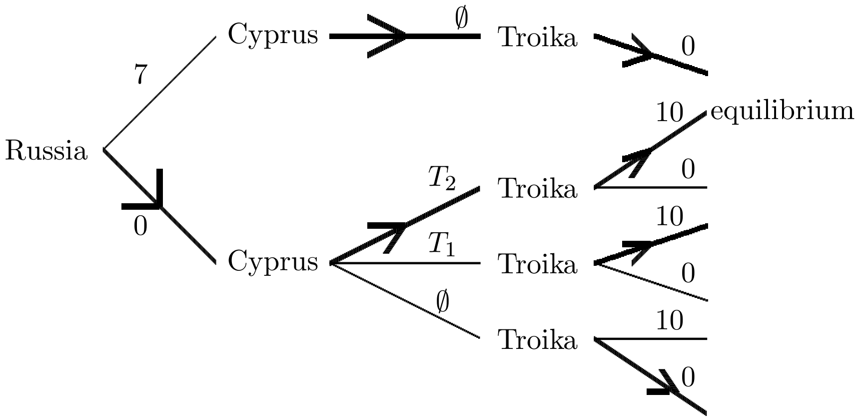
8. The sequential form game played by Russia, Cyprus and the troika where Russia moves first and Cyprus moves second depicts as (ignore the array for the moment):



9. The backward induction outcome consists in Russia lending 7 billion euros and the troika lending 10 billion euros.

10. From (I_2) the troika prefers this equilibrium to the one of question **6** because in both cases Cyprus is rescued but the troika lends a lower amount in the second case.

11. From (I_9) the troika will provide assistance only if Russia does not participate. The new game tree in the sequential order of question **8** depict as (ignore the array for the moment):



12. The backward induction outcome consists in Cyprus implementing the tax T_2 to raise 7 billion euros and the troika lending 10 billion euros.

13. From (I_2) the troika prefers this equilibrium to the one of question **6** because the rescue is performed at a lower lending amount from the troika (10 rather than 17 billion euros). From (I_9) the troika also prefers this equilibrium to the one of question **10** because now there is no more Russia participation.

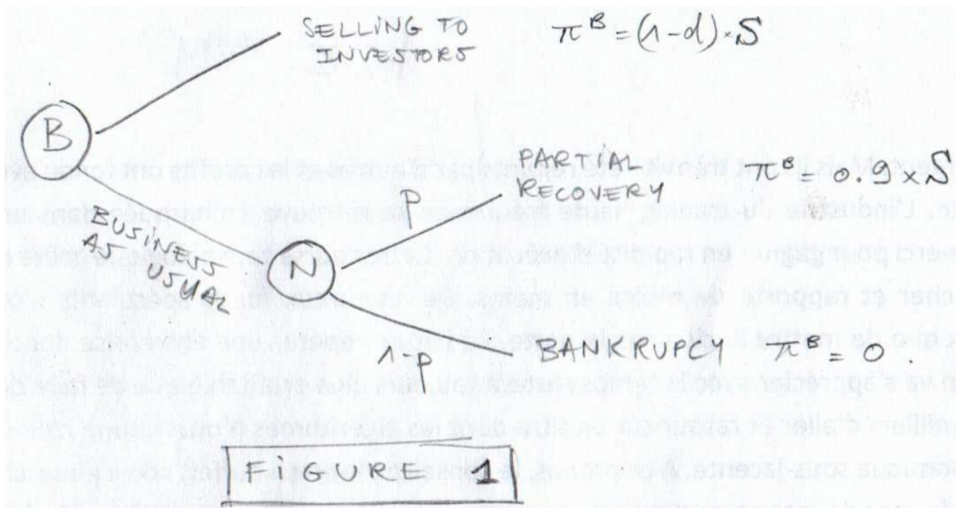
14. By announcing a bailout reduction from 17 to 10 billion euros (I_8) and by providing Cyprus government with incentives to renounce to any Russian assistance (I_9), the troika succeeds to implement his first-best equilibrium outcome.

Problem 2: Italian Banks struggling with a burden of bad debt and loans.

Part A. *B* is the only player and has only two possible actions.

1. The corresponding game tree is

Figure 1



2. When $p = 0.7$ and $d = 25\%$, B's optimal strategy is to sell. Indeed, B's payoff when it sells is

$$(1 - d) \times S = 0.75 \times S$$

while when it chooses to conduct business as usual his expected payoffs is

$$p \times 0.9 \times S + (1 - p) \times 0 = 0.7 \times 0.9 \times S = 0.63 \times S.$$

3. From the two previous equations, B prefers to sell when

$$(1 - d) \times S \geq p \times 0.9 \times S$$

that is when

$$p \leq \frac{1 - d}{0.9} = \bar{p}.$$

Hence, B's optimal strategy is to sell when $p < \bar{p}$ and to conduct business as usual otherwise.

Part B. *B* is not the only player and has more than two possible actions.

1. The corresponding game tree without the payoffs is

Figure 2 (see Figure 3)

2. From (I_2) and (I_3), EU's expected payoff associated to its rejection is

$$-3 \times 0.5 + 0 \times 0.5 = -1.5$$

while from (I_1) EU's payoff associated to its acceptance is -1. Therefore EU's optimal strategy is to accept the Italian government bail-out.

3. From (I_4), G's optimal behavior is to accept to bail-out.

4. From (I_6), following a financial market rejection of its issuance of additional shares, B's expected payoff associated to conduct business as usual is

$$0.4 \times 0.9 \times S + 0.6 \times 0 = 0.36 \times S$$

while from (I_7) its payoff associated to sell is

$$(1 - 0.4) \times S = 0.6 \times S.$$

From **B.2)** and **B.3)**, we know that if B asks the government for a bail-out it will be accepted by both G and EU. From (I_5), B's payoff would then be

$$(1 - 0.3) \times S = 0.7 \times S.$$

Therefore, B's optimal behavior following a financial market rejection of its issuance of additional shares is to ask the government for a bail-out.

5. From **A.2)**, we know that B prefers to sell than to conduct business as usual at the initial node, and that selling provide B with a payoff of $0.75 \times S$. Reasoning backward, B's expected payoff when it chooses to raise capital from the financial market is

$$0.6 \times 0.8 \times S + 0.4 \times 0.7 \times S = 0.76 \times S.$$

Therefore, the whole game has a unique subgame perfect Nash equilibrium. It consists for the Italian bank (B) to try first to raise capital from the financial market. In case of a market rejection, then the bank request the Italian government (G) to bail it out. The Italian government will accept. Despite the European rules, the European union (EU) will accept the Italian government bail-out.

6. The whole game tree with the payoffs and the solution path is

Figure 3

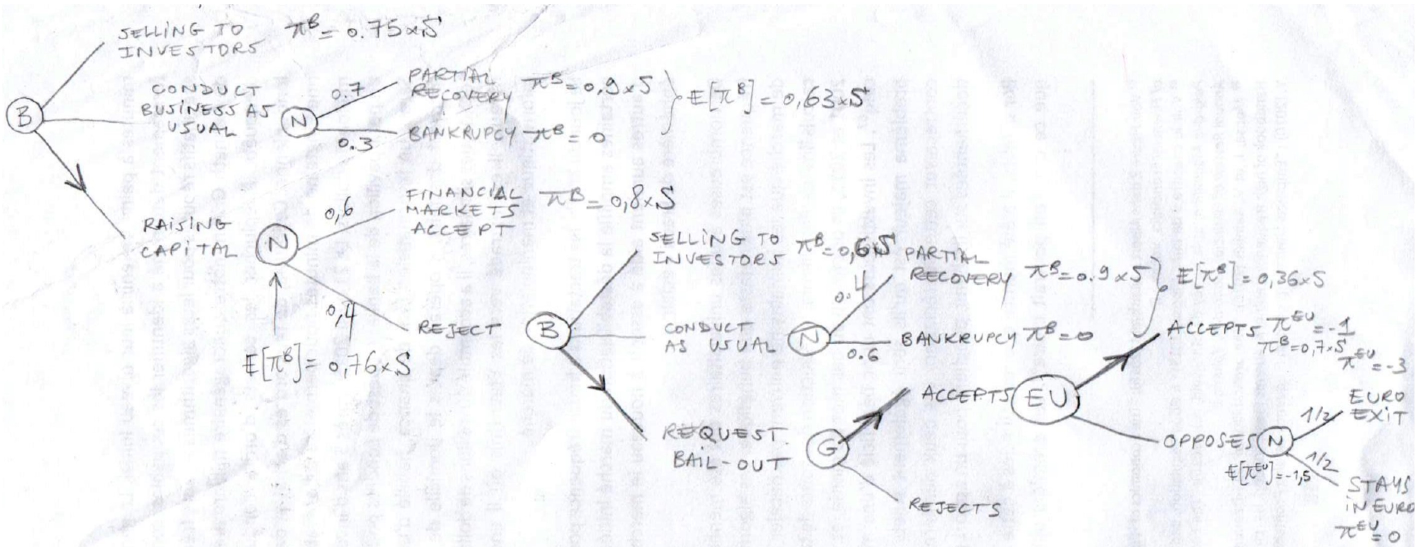


FIGURE 3

Problem 3: Paris and Frankfurt compete to woo Britain's banks post-Brexit.

Part C. City banks.

C1. The game tree where Paris plays first and Frankfurt move second writes as in Figure 1.

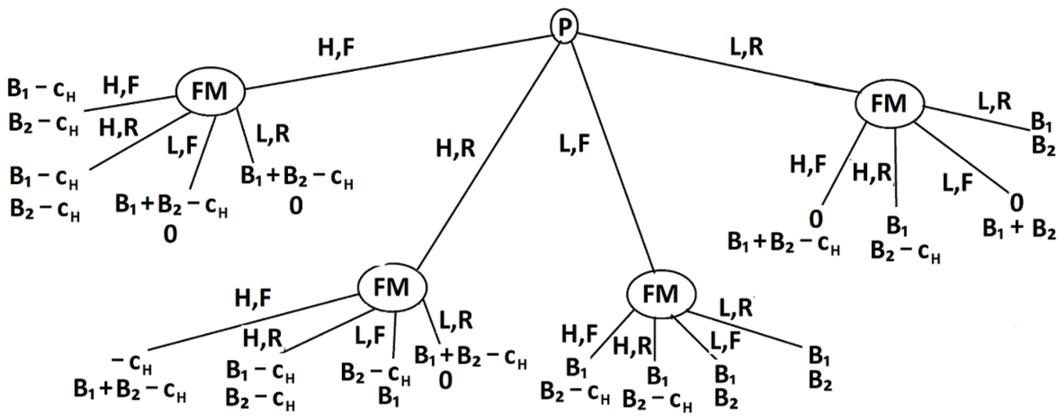


Figure 1: 1

C2. From $B_2 - c_H > 0$ the set of subgame perfect Nash equilibria is easily obtain by backward induction:

Figure 2

It contains two equilibria and writes as $\{((L, F), (L, F); ((L, F), (L, R)))\}$. The payoffs associated with any of these two equilibria is the same: (B_1, B_2). It consists of Paris attracting the first group of banks with a low tax cut and a flexible hiring-and-firing regime (i.e., (L, F)); and for Frankfurt to attract the second group of banks with a low tax cut and any kind of hiring-and-firing regime (i.e., (L, F) or (L, R)).

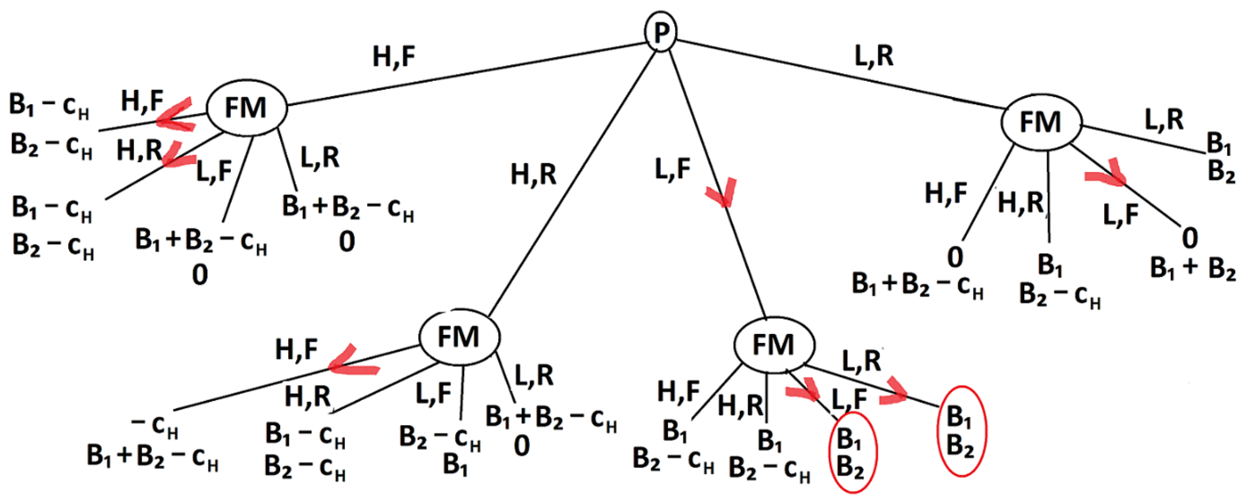


Figure 2: Figure 2