Master PEI: Game Theory in Banking, Finance and the International Arena

Chapter 1: Solution to Additional Exercises and Problems

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Problem 1: Greek debt crisis

1. ECB.

		(E_1) realized	
		Yes	No
ECB's choice	Yes	a	b
	No	c	d

From (I_2) we obtain $a > b$ and $c > d$.
From (I_3) we obtain $a > c$ and $b > d$.

The ECB has then a dominant strategy which consists in playing "Yes", i.e., to allow Greek government bonds as collateral for its repo operations.

2. Private sector banks.

		(E_2)&(E_3)&(E_5) realized	
		Yes	No
Banks' choice	Yes	a	b
	No	c	d

From (I_1) we obtain $a > b$ and $c > d$.
From (I_4) we obtain $c > a$ and $d > b$.

The private banks have then a dominant strategy which consists in playing "No", that is to not participate.

3. IMF.

		(E_2)&(E_3) realized	
		Yes	No
IMF's choice	Yes	a	b
	No	c	d

From (I_1) we obtain $a > b$ and $c > d$.
From (I_5) we obtain $b < c$ because $b > c$ would contradict $c > d$ and so (I_1).
From (I_6) we obtain $a > c$. Hence $a > c > b > d$.

The IMF has then a dominant strategy which consists in playing "Yes", that is to create additional bail-out package for Greece.

4. Germany.

		$(E_3)\&(E_4)$ realized	
		Yes	No
Germany's choice	Yes	a	b
	No	c	d

From (I_1) we obtain $a > b$ and $c > d$.
From (I_7) we obtain $c < b$ because $c > b$ would contradict $c > d$ and so (I_1).
Hence $a > b > c > d$.

Germany has then a dominant strategy which consists in playing "Yes", that is to create additional bail-out package for Greece.

5. France.

		$(E_1)\&(E_3)$ realized	
		Yes	No
France's choice	Yes	a	b
	No	c	d

From $(I_1)\&(I_8)$ we obtain $a > b$ and $c > d$.
From (I_9) we obtain $c < b$ car $c > b$ contredirait $c > d$ and donc $(I_1)\&(I_8)$.
Hence $a > b > c > d$.

France has then a dominant strategy which consists in playing "Yes", that is to create additional bail-out package for Greece.

6. Greece.

		$(E_2)\&(E_5)$ realized	
		Yes	No
Greece's choice	Yes	a	b
	No	c	d

From (I_1) we obtain $a > b$ and $c > d$.
From (I_{10}) we obtain $c < a$ and $b < d$
Hence $a > c > d > b$.

Greece's best response consists then to play "Yes", that is to implement further austerity measures and privatizations, when the events $(E_2)\&(E_5)$ are realized, and otherwise to play "No", that is to not implement it.

7. Rating agencies. The event to consider is the private banks participating in the loss sharing, i.e., (E_4).

		(E_4) realized	
		Yes	No
Rating agencies' choice	Yes	a	b
	No	c	d

From (I_{11}) we obtain $a > c$ and $d > b$.

Rating agencies' best response consists to play "Yes", that is to downgrade rating, when (E_4) is realized, and otherwise to play "No", that is to not downgrade.

8. From previous answers we have:
- 1) the ECB plays "Yes";
- 2) private banks play "No" and the event (E_4) is not realized;
- 3) the IMF plays "Yes" and the event (E_5) is realized;
- 4)&5) Germany and France play "Yes" and the event (E_2) is realized;

- 6) the events $(E_2)\&(E_5)$ being realized, Greece plays “Yes”; and
- 7) the event (E_4) being not realized, the rating agencies play “No”.

Therefore at Nash equilibrium:
- the ECB continues to take Greek collateral for its repo operations;
- the private banks do not participate in the loss sharing;
- the IMF, Germany and France pay additional bail-out funds to Greece;
- Greece takes further austerity measures and privatises state assets; and
- the rating agencies do not downgrade Greek government debt.

Problem 2: COP21 Climate negotiations between asymmetric countries

1. When Q is such that the catastrophic loss only occurs when both countries choose N, the corresponding matrix payoff writes as

		B's choice	
		N	U
A's	N	$(-L; -L)$	$(b; b - c^B)$
choice	U	$(b - c^A; b)$	$(2b - c^A; 2b - c^B)$

From $-L > b - c^B$, $BR^B(N) = \{N\}$. From $b - c^B < 0$, $BR^B(U) = \{N\}$. So N is B's dominant strategy. From $b - c^A > -L$, $BR^B(N) = \{U\}$. So there is a unique pure strategy Nash equilibrium. It consists for country A to be the only country reducing its emissions. The set of pure strategy Nash equilibria is the singleton $\{(U, N)\}$.

From $b - c^A > -L$, and $b > 0 > -L$, the outcome (N, N) is Pareto-dominated by (U, N).
From $b > 0 > b - c^A$, we have $b > 2b - c^A > 0 > b - c^A$ and the outcome (N, U) gives A its (unique) maximal payoff. Hence, the outcome (N, U) is Pareto-efficient.
A similar argument with respect to c^B rather than c^A, establishes that (U, N) gives B its (unique) maximal payoff. Hence, the outcome (U, N) is Pareto-efficient.
Finally, A is better off under (U, U) than under (U, N), while B is better off under (U, U) than under (N, U), so (U, U) is Pareto-efficient as well.
Therefore, the set of Pareto-efficient outcomes is $\{(U, N); (N, U); (U, U)\}$.

2. When Q is such that the catastrophic loss cannot be avoided, the corresponding matrix payoff writes as

		B's choice	
		N	U
A's	N	$(-L; -L)$	$(-L + b; -L + b - c^B)$
choice	U	$(-L + b - c^A; -L + b)$	$(-L + 2b - c^A; -L + 2b - c^B)$

From $b - c^i < 0$, $i \in \{A, B\}$, N is i's dominant strategy. So there is a unique pure strategy Nash equilibrium. It consists for each country to not reduce its emissions. The set of pure strategy Nash equilibria is the singleton $\{(N, N)\}$.

From $2b - c^A > 2b - c^B > 0$, the outcome (N, N) is Pareto-dominated by (U, U).
From $b > 0 > b - c^i$, $i \in \{A, B\}$, we have $-L + b > -L + 2b - c^i > -L + b - c^i$ and the outcome (N, U) gives A (resp. (U, N) gives B) its (unique) maximal payoff. Hence, the outcomes (N, U) and (U, N) are Pareto-efficient.
Since A is better off under (U, U) than under (U, N),while B is better off under (U, U) than under (N, U), so (U, U) is Pareto-efficient as well.
Therefore, the set of Pareto-efficient outcomes is $\{(U, N); (N, U); (U, U)\}$.

3. The previous situation corresponds to a prisoners' dilemma. The unique nash equilibrium (which here involves no cooperation on emissions reduction) is Pareto-dominated by an outcome that requires each player to play a strictly dominated strategy (here to reduce emissions).

4. When Q is such that the catastrophic loss is only avoided when both countries choose U, the corresponding matrix payoff writes as

		B's choice	
		N	U
A's	N	$(-L;-L)$	$(-L+b;-L+b-c^B)$
choice	U	$(-L+b-c^A;-L+b)$	$(2b-c^A;2b-c^B)$

From $0 > b - c^A$, we have $BR^A(N) = \{N\}$. From $b - c^A > -L$, we have $BR^A(U) = \{U\}$. From $0 > -L > b - c^B$, N is B's dominant strategy. So there is a unique pure strategy Nash equilibrium. It consists for each country to not reduce its emissions. The set of pure strategy Nash equilibria is the singleton $\{(N, N)\}$.

From $2b - c^A > 2b - c^B > -L$, the outcome (N, N) is Pareto-dominated by (U, U).
From $b > b - c^A > -L$, the outcome (N, U) is Pareto-dominated by (U, U).
From $b - c^A > -L$ and $b > 0 > b - c^A$, we have $2b - c^A > -L + b > -L > -L + b - c^A$, so the outcome (U, U) gives A its (unique) maximal payoff and is then Pareto-efficient.
From N being B's dominant strategy, $-L + b > -L$, the outcome (U, N) gives B its (unique) maximal payoff and is then Pareto-efficient.
Therefore, the set of Pareto-efficient outcomes is $\{(U, N); (U, U)\}$.

(a) The matrix payoff corresponding to the situation where A gives to country B a transfer of t if and only if B chooses to reduce its emissions writes as

		B's choice	
		N	U
A's	N	$(-L;-L)$	$(-L+b-t;-L+b-c^B+t)$
choice	U	$(-L+b-c^A;-L+b)$	$(2b-c^A-t;2b-c^B+t)$

(b) A transfer t, from country A to country B, that makes reduction emissions B's weakly dominant strategy, has to satisfy that $\{U\} \in BR^B(N)$ and $\{U\} \in BR^B(U)$, that is

$$-L + b - c^B + t \geq -L \qquad \textit{and} \qquad 2b - c^B + t \geq -L + b$$

so

$$t \geq c^B - b \qquad \textit{and} \qquad t \geq -L + c^B - b$$

Hence, the minimal transfer is

$$t^* = c^B - b$$

(c) The matrix payoff corresponding to the minimal transfer t^*

		B's choice	
		N	U
A's	N	$(-L;-L)$	$(-L+2b-c^B;-L)$
choice	U	$(-L+b-c^A;-L+b)$	$(3b-c^A-c^B;b)$

(d) From $0 > b-c^A$, we have $BR^A(N)=\{N\}$. From $b-c^A > -L$, we have $BR^A(U)=\{U\}$. Clearly, we have $BR^B(N)=\{N,U\}$ and $BR^B(U)=\{U\}$. So there are two unique pure strategy Nash equilibria. One in which no country reduce its emissions, the other in which they both reduce their emissions. The set of pure strategy Nash equilibria is $\{(N,N);(U,U)\}$.

Clearly, (U,U) maximizes B's payoff. From $2b-c^B > 0 > b-c^A > -L$ we have $3b-c^A-c^B > -L+2b-c^B > -L > -L+b-c^A$ so (U,U) maximizes A's payoff. The outcome (U,U) is the unique maximum of country A's (resp. B's) payoff. Therefore, the set of Pareto-efficient outcomes is the singleton $\{(U,U)\}$.

(e) (N,N) gives rise to a Nash equilibrium that is Pareto-dominated. A way to suppress it would for country A to transfer country B an amount that is slightly higher than t^*, so that N becomes country B's strictly dominated strategy. In that case, (U,U) would become the unique Nash equilibrium.

Problem 3: Paris and Frankfurt compete to woo Britain's banks post-Brexit.

Part A. Competition between Paris and Frankfurt.

A1. The corresponding matrix payoff writes as

		FM's choice	
		H	L
P's	H	$(\frac{1}{2} - c_H; \frac{1}{2} - c_H)$	$(1 - c_H; 0)$
choice	L	$(0; 1 - c_H)$	$(\frac{1}{2}; \frac{1}{2})$

From $c_H < \frac{1}{2}$ we have $\frac{1}{2} - c_H > 0$ and $1 - c_H > \frac{1}{2}$, so H is a strictly dominant strategy for every city. The set of pure strategy Nash equilibrium is the singleton $\{(H, H)\}$.

The outcome (H, H) is Pareto-dominated by (L, L).
From $c_H < \frac{1}{2}$ we have $1 - c_H > \frac{1}{2}$, so the outcome (H, L) gives P its (unique) maximal payoff. Hence, the outcome (H, L) is Pareto-efficient.
Similarly, the outcome (L, H) gives FM its (unique) maximal payoff. Hence, the outcome (L, H) is Pareto-efficient.
Finally, P is strictly better off under (L, L) than under (L, H), while FM is strictly better off under (L, L) than under (H, L), so (L, L) is Pareto-efficient as well.
Therefore, the set of Pareto-efficient outcomes is $\{(H, L); (L, H); (L, L)\}$.

A2. The corresponding matrix payoff writes as

		FM's choice	
		F	R
P's	F	$(\frac{1}{2}; \frac{1}{2})$	$(1; 0)$
choice	R	$(0; 1)$	$(\frac{1}{2}; \frac{1}{2})$

Clearly, F is a strictly dominant strategy for every city. The set of pure strategy Nash equilibrium is the singleton $\{(F, F)\}$.

The outcome (F, R) gives P its (unique) maximal payoff. Hence, the outcome (F, R) is Pareto-efficient.
Similarly, the outcome (R, F) gives FM its (unique) maximal payoff. Hence, the outcome (R, F) is Pareto-efficient.
P is strictly better off under (R, R) than under (R, F), while FM is strictly better off under (R, R) than under (F, R), so (R, R) is Pareto-efficient as well.
Finally, since the outcome (F, F) provides the same payoff than does (R, R), (F, F) is Pareto-efficient as well.
Therefore, the set of Pareto-efficient outcomes is $\{(F, F); (F, R); (R, F); (R, R)\}$.

A3. The corresponding matrix payoff writes as

		FM's choice			
		H,F	H,R	L,F	L,R
P's choice	H,F	$(\frac{1}{2}-c_H;\frac{1}{2}-c_H)$	$(1-c_H;-c_H)$	$(1-c_H;0)$	$(1-c_H;0)$
	H,R	$(-c_H;1-c_H)$	$(\frac{1}{2}-c_H;\frac{1}{2}-c_H)$	$(1-\alpha-c_H;\alpha)$	$(1-c_H;0)$
	L,F	$(0;1-c_H)$	$(\alpha;1-\alpha-c_H)$	$(\frac{1}{2};\frac{1}{2})$	$(1;0)$
	L,R	$(0;1-c_H)$	$(0;1-c_H)$	$(0;1)$	$(\frac{1}{2};\frac{1}{2})$

From $0<c_H<\frac{1}{2}$ we have $\frac{1}{2}-c_H>0>-c_H$, so $BR^{FM}\,(P \text{ plays } (H,F))=\{(H,F)\}$.
From $0<c_H<\frac{1}{2}$ and $\alpha<1-c_H$ we have $1-c_H>\max\{\frac{1}{2}-c_H;\alpha;0\}$, so $BR^{FM}\,(P \text{ plays } (H,R))=\{(H,F)\}$.
From $c_H<\frac{1}{2}$ and $\alpha>0$ we have $1-c_H>\max\{1-\alpha-c_H;\frac{1}{2};0\}$, so $BR^{FM}\,(P \text{ plays } (L,F))=\{(H,F)\}$.
From $0<c_H$ we have $BR^{FM}\,(P \text{ plays } (L,R))=\{(L,F)\}$.
By symmetry, the correspondence of P's best response are the same. Hence, $\{(H,F),(H,F)\}$ is a Nash equilibrium and it is the unique pure strategy equilibrium.
This Nash equilibrium is Pareto-dominated by the outcome $\{(L,R),(L,R)\}$.

Part B. European harmonization of tax policies.

B1. By choosing one of the two levels of tax cuts (H or L) the European Union transforms the previous game depicted in question A3, into a 2×2 matrix game. If the EU chooses H the game writes as

		FM's choice	
		F	R
P's choice	F	$(\frac{1}{2}-c_H;\frac{1}{2}-c_H)$	$(1-c_H;-c_H)$
	R	$(-c_H;1-c_H)$	$(\frac{1}{2}-c_H;\frac{1}{2}-c_H)$

This game has a unique Nash equilibrium: $\{(F,F)\}$, with a corresponding payoff $(\frac{1}{2}-c_H;\frac{1}{2}-c_H)$.
If the EU chooses L, the game writes as

		FM's choice	
		F	R
P's choice	F	$(\frac{1}{2};\frac{1}{2})$	$(1;0)$
	R	$(0;1)$	$(\frac{1}{2};\frac{1}{2})$

This game has a unique Nash equilibrium: $\{(F,F)\}$, with a corresponding payoff $(\frac{1}{2};\frac{1}{2})$.
From $0<c_H$ the Nash equilibrium of the first game (when H is chosen) is Pareto-dominated by the one of the second game (when L is chosen), so the level of tax cuts that should be selected by the European Commission is the lowest one: L.