# Master PEI: Game Theory in Banking, Finance and the International Arena

## Chapter 1: Additional Exercises and Problems

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## Problem 1: Greek debt crisis

Consider seven players who during 2012 played a central role in the Greek debt crisis.

1. The **European Central Bank (ECB)** who has to choose whether or not to allow Greek government bonds as collateral for its repo operations.

2. The private **banks** who have to choose whether or not to participate in the loss sharing through a potential Greek restructuring.

3-4-5. The **International Monetary Fund (IMF)**, **Germany** and **France** who each has to choose whether or not to create additional bail-out package for Greece.

6. **Greece** who has to choose whether or not to implement further austerity measures and privatizations.

7. The **rating agencies** who have to choose whether or not to downgrade the rating of the Greek government bonds.

We would like to find the Nash equilibrium associated to a simultaneous move from all players. The multidimensional game is difficult to represent in matrix form. Rather than considering the payoff matrix associated to the game, we simplify the analysis by considering each player's choice as a function of one or several event(s) that would modify the most his payoff. Doing so, for each player, we consider the following matrix:

|  |  | Event(s) Realized | |
| --- | --- | :---: | :---: |
|  |  | Yes | No |
| Player's | Yes | $a$ | $b$ |
| choice | No | $c$ | $d$ |

Warning: the matrix only has two columns even if there are several events. The left column states for the situation where all considered events are realized. The right column states for the situation where at least one event does not realize. The list of events is the following:

$(E_1)$: Rating agencies do not downgrade the rating of the Greek government bonds;

$(E_2)$: Germany and France create additional bail-out package;

$(E_3)$: Greece implements further austerity measures and privatizations;

$(E_4)$: The private banks participate in the loss sharing; and

$(E_5)$: The IMF creates additional bail-out package.

| **Player** | mainly cares about the events: |
|:---:|:---:|
| ECB | $(E_1)$ |
| Private banks | $(E_2)\&(E_3)\&(E_5)$ |
| IMF | $(E_2)\&(E_3)$ |
| Germany | $(E_3)\&(E_4)$ |
| France | $(E_1)\&(E_3)$ |
| Greece | $(E_2)\&(E_5)$ |
| Rating agencies | ...to be determined... |

Each player's payoffs ($a$, $b$, $c$, $d$) can be ordered using the following information – labeled from ($I_1$) to ($I_{11}$) – to characterize his best response.

($I_1$) Whatever his own level of participation, each player (except rating agencies who are indifferent) prefers that his counterpart does exert an effort (as Greece implementing further austerity and/or others create additional bail-out).

($I_2$) Willing to prevent any concern regarding the soundness of the financial system, the ECB prefers that no rating downgrade takes place.

($I_3$) The ECB wants to assist Greece to preserve the monetary union.

($I_4$) The private banks prefer not to participate in the loss sharing no matter how other players behave.

($I_5$) IMF ranks the situation where it exerts effort while at least one of the two countries abandons just above the worst of the four cases.

($I_6$) However the IMF considers that the best situation consists in every one exerting an effort.

($I_7$) Germany wants to make it clear that it takes on the responsibility. So it announces that the situation that it ranks just above the worst of the four is the one where it does not exert any effort while private sector and Greek people do.

($I_8$) Contrary to Germany, France does not want the private banks to participate in the loss sharing because French banks hold a huge amount of Greek bonds. A rating downgrade could worsen banks' balance sheet, leading to a credit crunch then slowing down the economy.

($I_9$) France wants to make it clear that it also takes on the responsibility. So it announces that the situation that it ranks just above the worst of the four is the one where it does not exert any effort while there is no downgrade rating and Greek people suffers from further austerity measures.

($I_{10}$) Greek government accepts to inflict unpopular measures with Greek people if and only if every public sector players provide financial assistance to Greece.

($I_{11}$) Rating agencies prefer to behave honestly: downgrading the rating if the private banks are participating in the losses, and not downgrading otherwise.

From questions 1 to 7, for each player give the corresponding matrix and infer from ($I_1$) − ($I_{11}$) the maximum of relationship between the parameters $a$, $b$, $c$, and $d$. Give player's best response as a function of the corresponding realized event(s) (matrix column).

1. ECB.

2. Private banks.

3. IMF.

4. Germany.

5. France.

6. Greece.

7. Rating agencies. What is the event(s) to consider?

8. Give the Nash equilibrium where players move simultaneously.

# Problem 2: COP21 Climate negotiations between asymmetric countries

There are 2 countries $A$ and $B$, each able to reduce emissions by using clean technologies. The cost of reducing emissions is constant but different among countries $A$ and $B$, with $c^A < c^B$. We can think of country $A$ (resp.$B$) being a developed (resp. developing) country. We can also think of countries $A$ and $B$ being similar in terms of development but $A$ having only access to a low-cost "ordinary abatement", $c^A$, while country $B$ having a highcost technology, $c^B$, for removing carbon dioxide from the atmosphere.

Each country has to choose between action $U$, which consists in using its clean technology to reduce emissions, and action $N$ which consists to not reducing its emissions. Each emission reduction gives each country a benefit, $b$, of avoiding "gradual" climate change. This benefit is collective and cumulative: if only one of the two countries chooses $U$, each country earns benefit $b$; if both countries choose $U$, each country earns $2b$. We assume $2b > c^B > c^A > b > 0$.

Countries' choices are made simultaneously. Abatement short of $Q$ results in catastrophic loss of value $L > 0$. We assume $b - c^A > -L > b - c^B$.

1. We assume that $Q$ is such that the catastrophic loss only occurs when both countries choose $N$. Give the corresponding matrix payoff, characterize both the set of pure strategy Nash equilibria, and the set of Pareto-efficient outcomes.

2. We assume that $Q$ is such that the catastrophic loss cannot be avoided. Give the corresponding matrix payoff, characterize both the set of pure strategy Nash equilibria, and the set of Pareto-efficient outcomes.

3. To which situation corresponds the previous one?

4. We assume that $Q$ is such that the catastrophic loss is only avoided when both countries choose $U$. Give the corresponding matrix payoff, characterize both the set of pure strategy Nash equilibria, and the set of Pareto-efficient outcomes.

5. We consider the previous case. Assume that country $A$ has the possibility to subsidize country $B$'s reduction emissions.

   (a) Give the matrix payoff corresponding to the situation where $A$ gives to country $B$ a transfer of $t > 0$ if and only if $B$ chooses to reduce its emissions.

   (b) What is the minimal transfer $t^*$, from country $A$ to country $B$, so that $U$ becomes $B$'s weakly dominant strategy?

   (c) Give the matrix payoff corresponding to the minimal transfer $t^*$.

   (d) Characterize both the set of pure strategy Nash equilibria, and the set of Pareto-efficient outcomes.

   (e) Is there a way to eliminate the Nash equilibrium that is Pareto-dominated?

# Problem 3: Paris and Frankfurt compete to woo Britain's banks post-Brexit.

At the end of 2017, continental competition is heating up to benefit from the Brexit as half a dozen European Union (EU) cities vie to attract London-based banks and financial services companies worried about losing their access to the European single market. Among them, Paris and Frankfurt vie to become the EU's principal financial centre post-Brexit.

The purpose of this exercise is to study the strategic interaction between France, Germany, the European Commission and City banks.

## Part A. Competition between Paris and Frankfurt.

*This subject consists of 3 parts (A, B and C). Parts A and B deal with simultaneous games. As such, they are presented here. Part C concerns sequential games and will be exposed in the problems of chapter 2.*

Consider two players, Paris, denoted as $P$, and Frankfurt am Main, denoted as $FM$, who are engaged in a post-Brexit bid to lure London bankers to their city. Competition takes place through two channels: tax breaks and flexibility of employment laws.

With increasing populist and anti-EU parties in France and Germany, governments are understandably wary of being seen to bend the rules for bankers. Indulging international high-earners is an extremely sensitive subject for the electorate. This imposes a rational limit for governments on both tax reduction and flexible working legislation.

To simplify the analysis, we assume there are only two possible levels of tax cuts: high, denoted as $H$, and low, denoted as $L$. We also assume there are only two possible levels of hiring-and-firing regime: flexible, denoted as $F$, and rigid, denoted as $R$.

To simplify the payoffs, we normalize the mass of banks at unity, so attracting all the banks (resp. zero banks) gives a city a payoff of 1 (resp. 0). We also assume there is a social cost associated to high tax cuts, denoted as $c_H$, with $0 < c_H < \frac{1}{2}$. Such a cost can be seen as the government's loss of revenue due to lower taxes collected from domestic firms.

All City banks prefer higher tax cuts and a more flexible hiring-and-firing regime. The proportion of banks that prefer a flexible hiring-and-firing regime to high tax cuts is denoted as $\alpha \in (0, 1 - c_H)$ (i.e., $P$'s (resp. $FM$'s) payoff when France chooses $(L, F)$ and Germany chooses $(H, R)$ is $\alpha$ (resp. $(1 - \alpha - c_H)$). In case of similar policies, banks split equally between both cities (i.e., $P$'s and $FM$'s payoffs when France and Germany play the same pair $(x, y)$, with $x \in \{H, L\}$ and $y \in \{F, R\}$, is: $\frac{1}{2}$ if $x \neq H$; and $\left(\frac{1}{2} - c_H\right)$ if $x = H$).

A1.  Ignore the employment legislation channel and consider the simultaneous tax competition between $P$ and $FM$. Give the $2 \times 2$ corresponding matrix payoff, characterize both the set of pure strategy Nash equilibria and the set of Pareto-efficient outcomes.

A2.  Ignore the tax breaks channel and consider the simultaneous employment legislation competition between $P$ and $FM$. Give the $2 \times 2$ corresponding matrix payoff, characterize both the set of pure strategy Nash equilibria and the set of Pareto-efficient outcomes.

A3.  Consider the simultaneous tax and employment legislation competition between $P$ and $FM$. Give the $4 \times 4$ corresponding matrix payoff and characterize the set of pure strategy Nash equilibria. Is the Nash equilibrium Pareto efficient?

## Part B. European harmonization of tax policies.

The European banking union would remain incomplete without a harmonization of tax policies. The European Commission suggests all specific taxes on banks to be centralized and merged into a single financial activity tax.

B1. By considering how EU moving first would transform the matrix considered in question **A3**, which of any unique tax cut level $H$ or $L$ should be chosen by the European Union?