Game Theory with Application in Economics and Finance
Magistère BFA 2 - April 2019 - 1^{st} Exam Session

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Answers can be formulated in English or French.
90 mn. No document, no calculator allowed.

Could Trump's trade war drag out indefinitely? (16 pts)

In just few months, the United States (US) President Donald Trump has managed to assault the international trading order by slapping steep tariffs on billions of dollars' worth of goods from China, Canada, Mexico, and the European Union (EU). All these economies are now responding in kind, retaliating with levies on thousands of US products.

Free trade gives people the freedom to buy cheaper or better-made products from anywhere in the world. It helps companies to cut costs, which drives prices down and boosts the global economy. However this also means that companies in wealthier countries are consequently less likely to buy local products and offer low skilled jobs. While free trade created wealth for some, it has made others poorer, particularly in countries like US where the welfare program is weaker than those in most EU countries.

In early March 2018, the US President announced a 25% tariff on all steel imports, and a 10% tariff on aluminum imports. The objective was to boost demand for US steel, lifting profits for domestic steelmakers and driving new hires. But there were two side effects. First, US companies that require raw materials, like car and airplane makers, saw their costs rise, forcing them to translate those higher prices for their customers. Second, such restrictions encouraged the targeted trade partners to strike back. Only a few months later, China retaliated with tariffs on several billion dollars worth of US exports. The EU retaliated too, targeting US goods, from bourbon to motor-bikes.

The purpose of this exercise is to study the strategic trade interaction between Trump's administration and the European Union.

Part A. One round trade war between the US and the EU (5 pts).

Consider two players, Trump's administration, denoted as T, and the European Union, denoted as EU, who are engaged in a trade war. To simplify the analysis, we assume each player can choose between only two possible actions: higher tariffs (protectionism), denoted as H, and lower tariffs (free trade), denoted as L. The players move sequentially. Trump's administration moves first. The EU only responds to US protectionism - so as long as Trump's administration does not increase the level of tariffs, the EU does not play.

We normalize the players' payoffs to zero in the case of no protectionism. Implementing higher tariffs increases own payoffs by one point but decreases opponent's payoffs by two points (e.g., when (T, EU) plays (H, L) the payoffs are worth $(+1, -2)$). Trump's administration also faces some uncertain electorate effect associated with Europe's retaliation. Imposing economic sanctions on the US may either strengthen or weaken Trump's popularity domestically. A strengthening (resp. weakening), denoted as s (resp. w), occurs with probability $p \in (0, 1)$ (resp. $(1-p)$) and benefits for 2 points (resp. costs 2 points) . For instance, when (T, EU) plays (H, H) the payoffs are worth $(+1, -1)$ with probability p and $(-3, -1)$ with probability $(1-p)$.

A1. (**1 pt**) Draw the game tree associated to this strategic (one round) sequential interaction.

A2. (**2 pts**) Give the set of ex-post Pareto efficient outcomes. (Clue: Compare the payoffs situated on the tree leaves.) Give the set of ex-ante Pareto efficient pair of strategic actions. (Clue: Reason in terms of expected payoff.)

A3. (**2 pts**) Depending on the values of the probability p, what are the players' optimal behaviors? Characterize the set of subgame perfect Nash equilibrium in pure strategies as a function of the probability p.

Part B. Two round trade war between the US and the EU (9 pts).

Consider the previous game with one additional round of interaction. The payoffs are not discounted and are perceived in the end of the game. We assume the probability associated with Trump's popularity is evolving with the number of economic sanctions imposed on the US. We denote by p (resp. p') the probability associated with the first (resp. second) lottery Trump's administration is playing. So, in case where some sanctions (resp. no sanctions) were imposed on the US during the first round of the interaction, the probability Trump's administration is facing in the second round is denoted by p' (resp. p).

B1. (**1 pts**) Draw the game tree associated with this strategic (two round) sequential interaction. (Clue: Draw the game tree depicted in A1. Label each leaf with node n_1, n_2, ...Then draw it again at every such leaf. Compute the corresponding payoffs.)

B2. (**2 pts**) Depending on the values of the probabilities p and p', what are the players' optimal behaviors in the second round?

B3. (**3 pts**) We assume $\min\{p, p'\} > \frac{3}{4}$. What are the players' optimal behaviors in the first round? Characterize the set of subgame perfect Nash equilibrium in pure strategies. What are the corresponding payoffs?

B4. (**3 pts**) We assume $p > \frac{3}{4} > p'$. What are the players' optimal behaviors in the first round? Characterize the set of subgame perfect Nash equilibrium in pure strategies. What are the corresponding payoffs?

Part C. (In)Finite number of rounds trade war between the US and the EU (2 pts).

Assume the probability behind the lottery over Trump's popularity is strictly decreasing with the number of sanctions, and goes below $\frac{3}{4}$ at some point.

C1. (**2 pts**) Will Trump's trade war drag out indefinitely? Why?

Repeated game (4 pts)

R1. (**1 pt**) Consider the following simultaneous game:

		Player 2	
		L	R
Player 1	U	$(2;2)$	$(0;3)$
	D	$(1;0)$	$(1;1)$

Characterize the set of Nash equilibrium. By which outcome is this Nash equilibrium Pareto dominated? Is this game a prisoner's dilemma?

R2. (**1 pt**) Assume this one-shot game is repeated T times, with T a finite number, and players discount their payoffs according to the pair of discount factor $(\delta_1, \delta_2) \in (0,1)^2$. What is the set of subgame perfect equilibrium associated to this repeated interaction?

R3. (**2 pts**) Assume now that T is an infinite number. Is the outcome that Pareto dominates the Nash equilibrium of the stage game sustainable at equilibrium of the repeated interaction? If yes, give a strategy sustaining such an equilibrium. What is the condition on the discount factor so that no player has unilateral profitable deviations?