Game Theory with Application in Economics and Finance

Magistère BFA 2 - April 2019

Solution to 1^{st} Exam Session

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90 mn. No document, no calculator allowed.

Could Trump's trade war drag out indefinitely?

Part A. One round trade war between the US and the EU (5 pts).

A1. (**1 pt**) The corresponding game tree draws as follows

Figure 1

A2. (**2 pts**) The set of ex-post Pareto efficient outcomes (obtained by comparing the payoffs situated on the tree leaves) writes as $\{(L,.,.), (H,H,s)\}$, the associated payoffs are $(0,0)$ and $(1,-1)$.

From $p<1$, the set of ex-ante Pareto efficient pair of strategic actions (reasoning in terms of expected payoff) writes as:

- $\{(L,.), (H,L), (H,H)\}$ if $p>\frac{3}{4}$ with associated (expected) payoffs $(0,0)$, $(1,-2)$, and $(4p-3,-1)$;
- $\{(L,.), (H,L)\}$ if $p\leq\frac{3}{4}$ with associated (expected) payoffs $(0,0)$ and $(1,-2)$.

A3. (**2 pts**) We proceed by backward induction.

EU's optimal behavior. In any cases, $H\succ_{EU} L$ because $-1>-2$.

T's optimal behavior. $H\succeq_T L \iff 4p-3\geq 0 \iff p\geq\frac{3}{4}$.

Hence, the set of subgame perfect Nash equilibrium in pure strategies writes as:

$$SPNE=\begin{cases}\{(H,H)\} & \text{if} \quad p>\frac{3}{4}\\ \{(H,H),(L,H)\} & \text{if} \quad p=\frac{3}{4}\\ \{(L,H)\} & \text{if} \quad p<\frac{3}{4}\end{cases}$$

Figure 2

Part B. Two round trade war between the US and the EU (9 pts).

B1. (**1 pts**) The game tree associated with this strategic (two rounds) sequential interaction draws as

Figure 3

B2. (**2 pts**) In the second round, the players' optimal behaviors are as follows.

EU's optimal behavior. In any cases, $H\succ_{EU} L$ because H provides to EU one point more than L in any subgame starting at nodes n_1, n_2, n_3, and n_4.

T's optimal behavior. $H\succeq_T L$ at node n_1 (resp. n_2) $\iff 4p-3\geq 0$ (resp. $4p-2\geq 1$) $\iff p\geq\frac{3}{4}$.

$H\succeq_T L$ at node n_3 (resp. n_4) $\iff 4p'-2\geq 1$ (resp. $4p'-6\geq -3$) $\iff p'\geq\frac{3}{4}$.

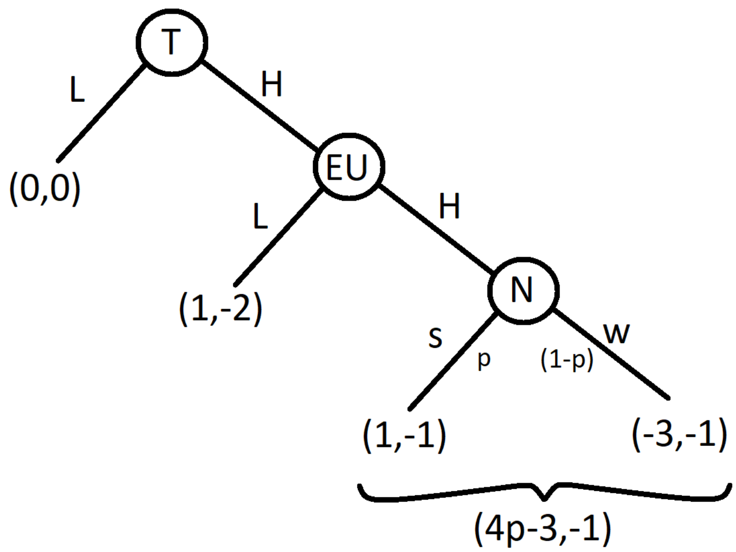


Figure 1: 1

B3. (**3 pts**) Assume $\min\{p, p'\} > \frac{3}{4}$. We already know that EU always play H in the second round. From $p > \frac{3}{4}$ (resp. $p' > \frac{3}{4}$) we know that in the second round T plays H at nodes n_1 and n_2 (resp. at nodes n_3 and n_4).
Hence, in the first round we have $H \succ_{EU} L$ because $-2 > -3$.
Also, T's expected payoff when he plays L is $4p - 3$ and when he plays H is

$$p\,(4p' - 2) + (1 - p)\,(4p' - 6) = 4p + 4p' - 6 = (4p - 3) + (4p' - 3)\,.$$

From $p' > \frac{3}{4}$, we deduce $H \succ_T L$.
Therefore there is a unique subgame perfect Nash equilibrium in pure strategies. It consists for the players to play H at every nodes. The corresponding expected payoffs are: $(4p + 4p' - 6, -2)$.

Figure 4

B4. (**3 pts**) Assume $p > \frac{3}{4} > p'$. We already know that EU always retaliate in the second round by playing H in all cases. From $p > \frac{3}{4}$ (resp. $p' < \frac{3}{4}$) we know that in the second round T plays H at nodes n_1 and n_2 (resp. plays L at nodes n_3 and n_4).
Hence, in the first round we have $H \succ_{EU} L$ because $-1 > -3$.
Also, T's expected payoff when he plays L is $4p - 3$ and when he plays H is $p - 3\,(1 - p) = 4p - 3$. So, $H \sim_T L$.
Therefore there are two subgame perfect Nash equilibria in pure strategies. In both, EU plays H at at every nodes. In one equilibrium, T starts by playing H then plays L in any cases in the second round. In the other equilibrium, T starts by playing L then plays H in any cases in the second round. Both equilibria yield to the same expected payoffs: $(4p - 3, -1)$.

Figure 5

Part C. (In)Finite number of rounds trade war between the US and the EU (2 pts).

C1. (**2 pts**) From Part A and B, we know that EU always choose to retaliate. From Part A we know that when the probability that economic sanctions imposed on the US strengthen Trump's popularity is sufficiently low (lower than $\frac{3}{4}$), Trump's administration prefers not to raise the tariffs. Hence, from the assumption that the probability behind the lottery over Trump's popularity goes below $\frac{3}{4}$ at some point, Trump's trade war will not drag out indefinitely.

Repeated game (4 pts)

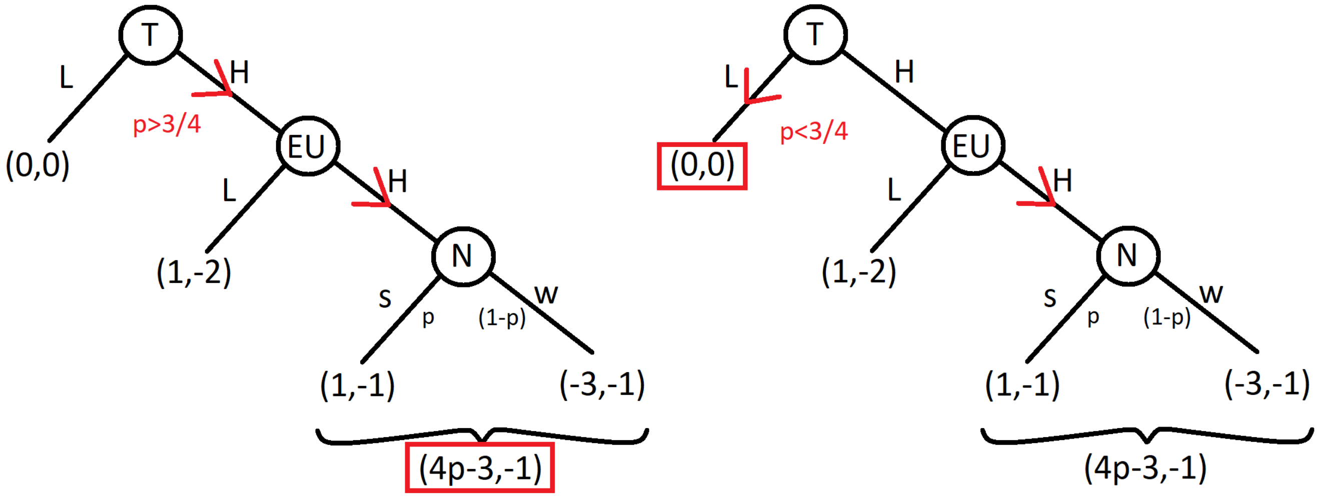


Figure 2: 2

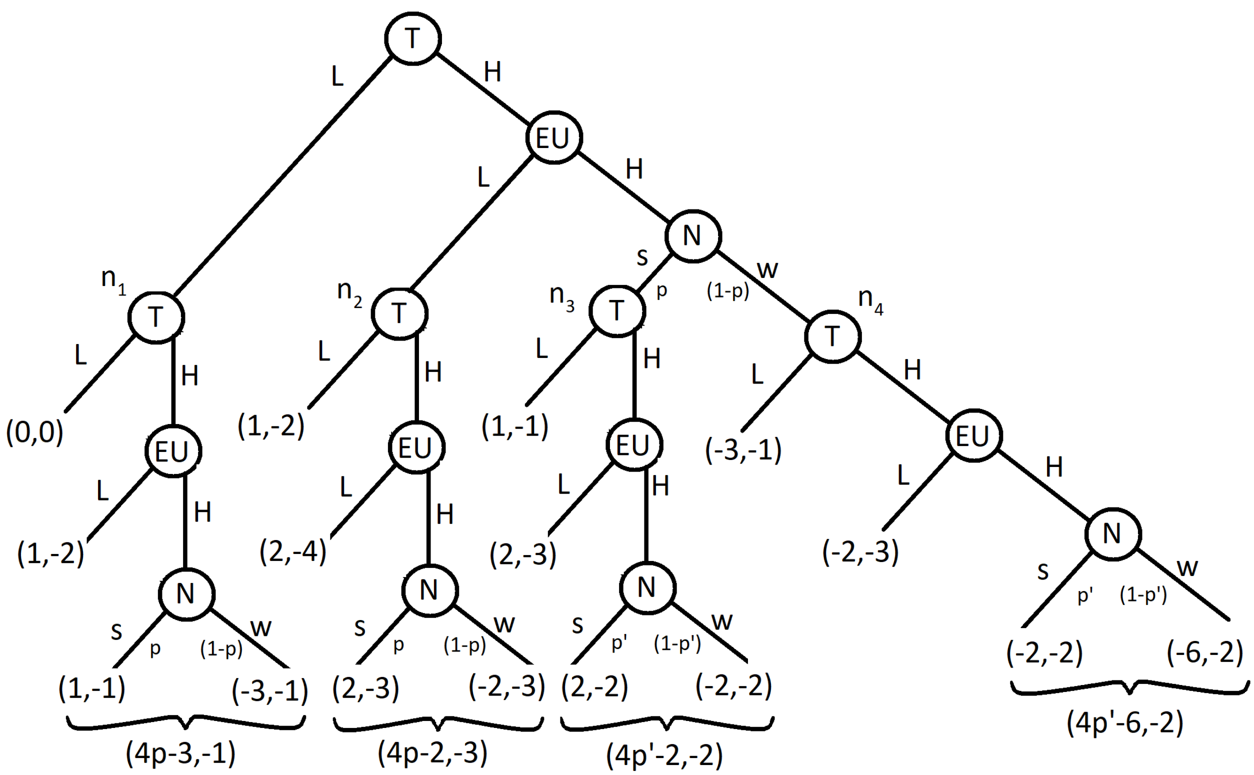
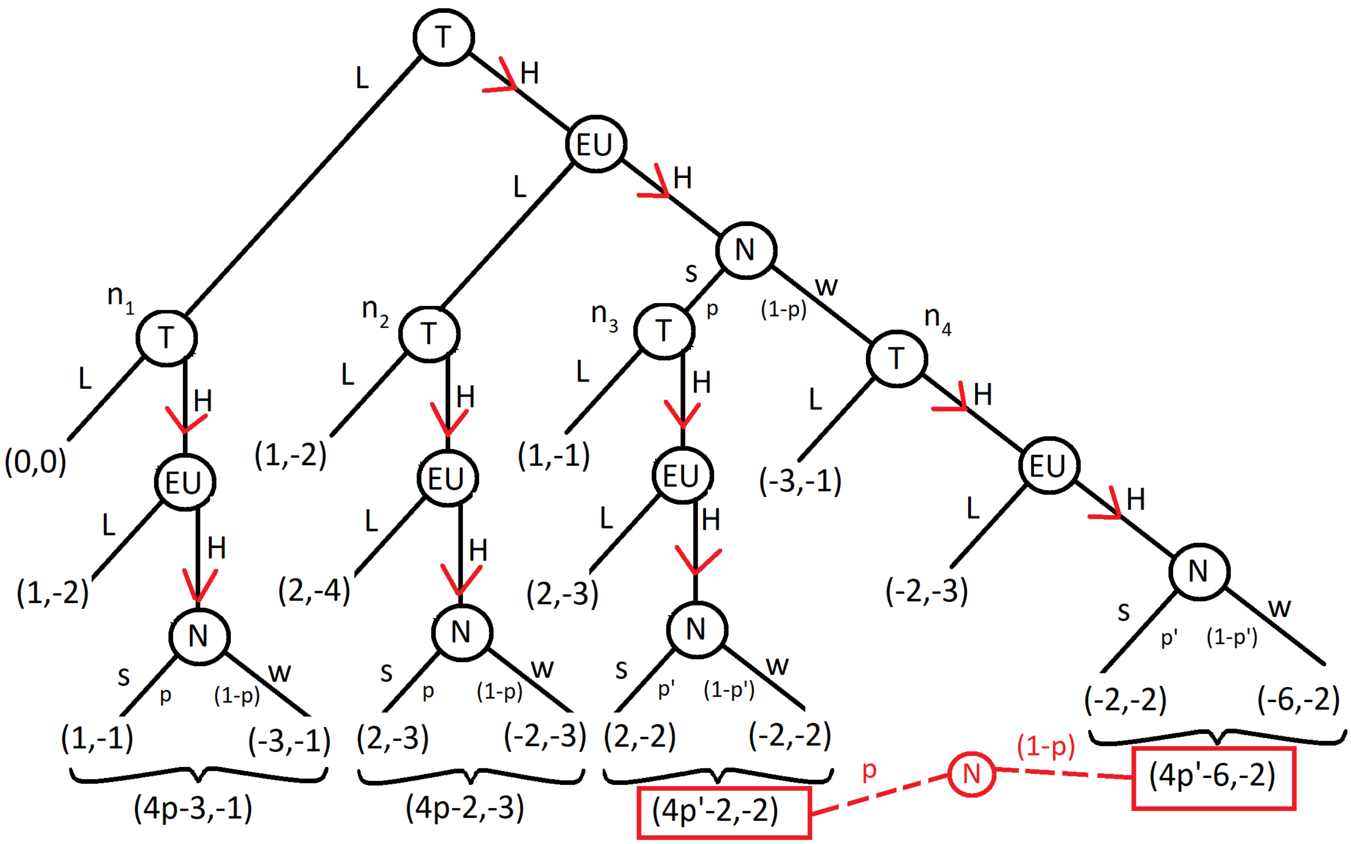


Figure 3: 3


T
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(-3,-1)
(4p-3,-1)
n2
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(1,-2)
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(4p-2,-3)
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p'
(1-p')
(2,-2)
(-2,-2)
(4p'-2,-2)
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(-3,-1)
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(-2,-2)
(-6,-2)
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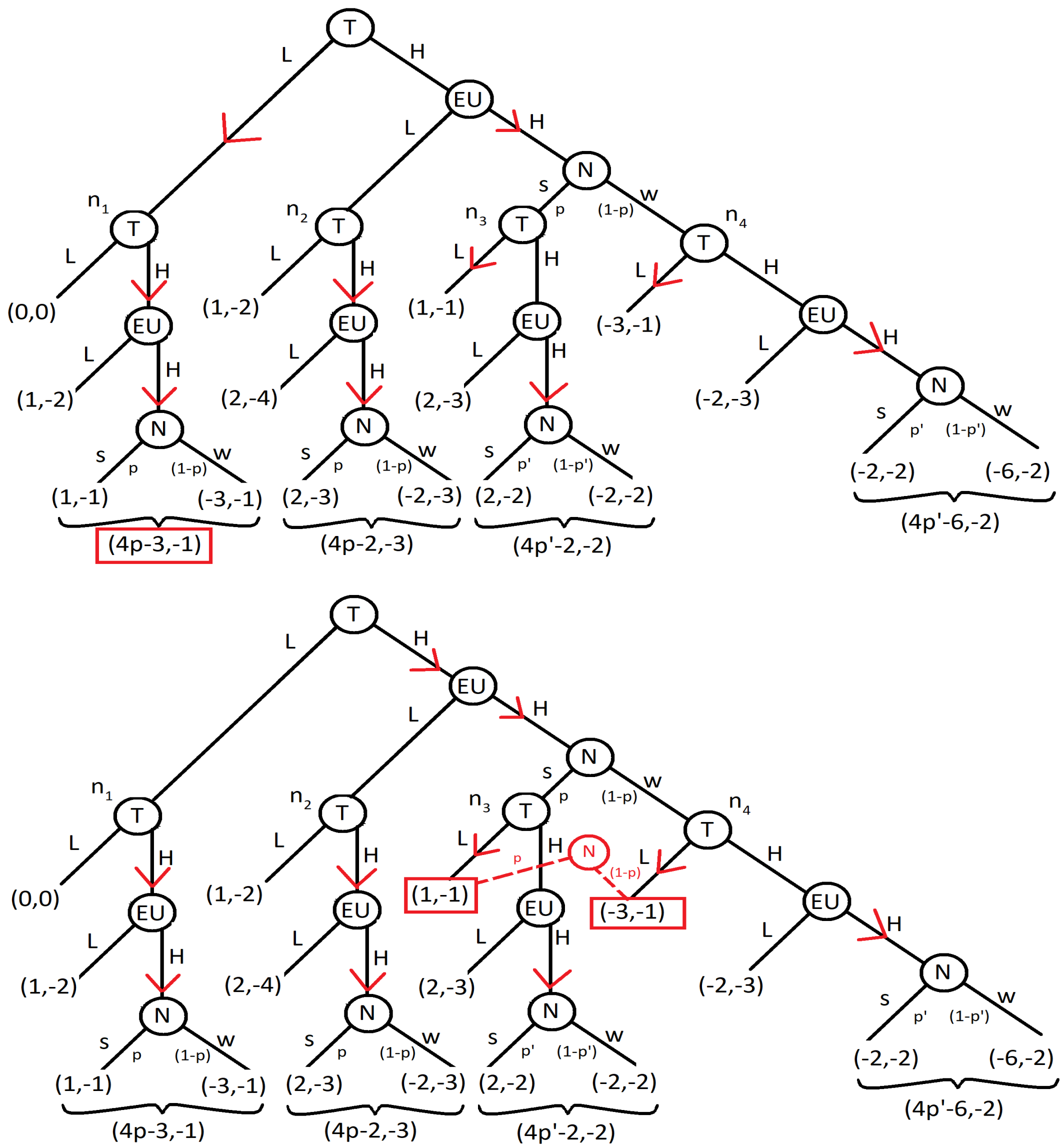
Figure 4: 4



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(4p'-6,-2)

Figure 5: 5

R1. (**1 pt**)

		Player 2	
		L	R
Player	U	$(2;2)$	$(0;3)$
1	D	$(1;0)$	$(1;1)$

Clearly, Player 1 has no dominant strategy so this game is not a prisoner's dilemma. Player 2 has a strictly dominant strategy that consists in playing action R. Player 1's best response to R is D (i.e., $BR^1(R) = \{D\}$). So this game has a unique Nash equilibrium: (D, R). This Nash equilibrium is Pareto dominated by (U, L).

R2. (**1 pt**) Since the stage game has a unique Nash equilibrium, the only subgame perfect equilibrium of the finite repeatition consists in repeating the Nash equilibrium over time. This can be shown by backward induction. At the last period T, player 2 plays his dominant strategy R to which Player 1 best responds with D. At the previous period $T-1$, what is played will not change what happen in the period after. So both players play the stage game Nash equilibrium. And so on...

R3. (**2 pts**) When the interaction is repeated infinitely, the outcome (U, L) that Pareto dominates the Nash equilibrium (D, R) is sustainable at equilibrium of the repeated interaction.
Consider the grim-trigger strategy that consists for player 1 (resp. 2) to starting by playing action U (resp. L) and then repeating this action over time as long as (U, L) has been played in the past, otherwise playing action D (resp. R) for ever.
Player 1 has no unilateral profitable deviations because $BR^1(L) = \{U\}$. So there is no condition on δ_1.
Player 2 has no unilateral profitable deviations if his payoff associated to (U, L):

$$2\sum_{t=1}^{+\infty}\delta_2^t = \frac{2\delta_2}{1-\delta_2} \tag{A}$$

is higher than his most profitable deviation at date k, for any k, which gives as:

$$2\sum_{t=1}^{k-1}\delta_2^t + 3\delta_2^k + 1\sum_{t=k+1}^{+\infty}\delta_2^t \tag{B}$$

$(A - B)$ writes as

$$\begin{aligned}
& 2\sum_{t=1}^{+\infty}\delta_2^t - \left(2\sum_{t=1}^{k-1}\delta_2^t + 3\delta_2^k + 1\sum_{t=k+1}^{+\infty}\delta_2^t\right) \\
= \quad & (2-3)\,\delta_2^k + (2-1)\sum_{t=k+1}^{+\infty}\delta_2^t \\
= \quad & -\delta_2^k + \sum_{t=k+1}^{+\infty}\delta_2^t \simeq -\delta_2^k + \frac{\delta_2^{k+1}}{1-\delta} = \frac{-\delta_2^k + 2\delta_2^{k+1}}{1-\delta}
\end{aligned}$$

which is positive if and only if $-\delta_2^k + 2\delta_2^{k+1} \geq 0$, that is when $\delta_2 \geq \frac{1}{2}$. Hence the pair (δ_1, δ_2) has to belong to the set $(0,1) \times [\frac{1}{2}; 1)$.