

# Derivative Instruments

Paris Dauphine University - Master IEF (272)

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LEDa

Exercises + Solutions Chapter 6

## Exercise (1)

It is January 9. The price of a Treasury bond with a 12% coupon that matures on October 12, in four years, is quoted as 102-07.

What is the cash price?

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*The last coupon has been paid on October 12 of the last year.  
The next coupon will be paid on April 12 of the current year.*

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*Between October 12 (last year) and January 9 (current year), there are*

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*The next coupon will be paid on April 12 of the current year.*

*Between October 12 (last year) and January 9 (current year), there are 89 days.*

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*Between October 12 (last year) and April 12 (current year), there are 182 days.*



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*The cash price of the bond is obtained by adding the accrued interest to the quoted price.*

*The quoted price is  $102\frac{7}{32}$  or*

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*The cash price is therefore  $102.21875 + \frac{89}{182} \times 6 = \$105.15$*

## Exercise (2)

A Eurodollar futures price changes from 96.76 to 96.82.

What is the gain or loss to an investor who is long two contracts?

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## Solution (2)

*The Eurodollar futures price has increased by 6 basis points.*

*The investor makes a gain per contract of  $25 \times 6 = \$150$  or \$300 in total.*

### Exercise (3, Done)

The 350-day LIBOR rate is 3% with continuous compounding and the forward rate calculate from a Eurodollar futures contract that matures in 350 days is 3.2% with continuous compounding.

Estimate the 440-day zero rate.

### Solution (3)

### Exercise (3, Done)

The 350-day LIBOR rate is 3% with continuous compounding and the forward rate calculate from a Eurodollar futures contract that matures in 350 days is 3.2% with continuous compounding.

Estimate the 440-day zero rate.

### Solution (3)

*We have*

$$R_2 =$$

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Estimate the 440-day zero rate.

### Solution (3)

We have

$$R_2 = \frac{F_1 (T_2 - T_1) + R_1 T_1}{T_2}.$$

with  $F_1 =$



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with  $F_1 = 3.2\%$ ,  $T_1 =$

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with  $F_1 = 3.2\%$ ,  $T_1 = 350$ ,  $T_2 = 440$ , and  $R_1 = 3\%$ . So

$$R_2 = \frac{3.2\% \times 90 + 3\% \times 350}{440} = 3.0409\%.$$

## Exercise (4)

It is January 30. You are managing a bond portfolio worth \$6 million. The duration of the portfolio in six months will be 8.2 years. The September Treasury bond futures price is currently 108-15, and the cheapest-to-deliver bond will have a duration of 7.6 years in September.

How should you hedge against changes in interest rates over the next six months?

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## Exercise (5)

Suppose that the Treasury bond futures price is 101-12. Which of the following four bonds is cheapest to deliver?

Bond	Price	Conversion Factor
1	125-05	1.2131
2	142-15	1.3792
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$$\text{Quoted Price} - \text{Futures Price} \times \text{Conversion Factor}$$

*is least.*

*Calculating this factor for each of the 4 bonds we get:*

$$\text{Bond 1: } 125.15625 - 101.375 \times 1.2131 = 2.178$$

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*Bond*

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*Bond 4 is therefore the cheapest to deliver.*

## Exercise (6)

Suppose that the 300-day LIBOR zero rate is 4% and Eurodollar quotes for contracts maturing in 300, 398 and 489 days are 95.83, 95.62, and 95.48.

Calculate 398-day and 489- day LIBOR zero rates.

Assume no difference between forward and futures rates for the purposes of your calculations.

(Hint: The forward rates calculated from the Eurodollar futures are expressed with an actual/360 day count and quarterly compounding. The use of our formula then require these rates to be expressed with continuous compounding and an actual/365 day count)



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$$\frac{365}{90} \ln \left( 1 + \frac{0.0417}{4} \right) = 4.2060\%$$

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*and*



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with  $F_1 = 4.2060\%$ ,  $T_1 = 300$ ,  $T_2 =$

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$$R_2 = \frac{F_1 (T_2 - T_1) + R_1 T_1}{T_2}.$$

with  $F_1 = 4.2060\%$ ,  $T_1 = 300$ ,  $T_2 = 398$ , and  $R_1 =$

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$$\frac{4.2060 \times 98 + 4 \times 300}{398} =$$

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$$\frac{4.2060 \times 98 + 4 \times 300}{398} = 4.0507$$

The 489 day rate is obtained with  $F_1 =$

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with  $F_1 = 4.2060\%$ ,  $T_1 = 300$ ,  $T_2 = 398$ , and  $R_1 = 4\%$ , the 398 day rate is

$$\frac{4.2060 \times 98 + 4 \times 300}{398} = 4.0507$$

The 489 day rate is obtained with  $F_1 = 4.4167\%$ ,  $T_1 = 398$ ,  $T_2 = 489$ , and  $R_1 = 4.0507\%$ :

$$\frac{4.4167 \times 91 + 4.0507 \times 398}{489} =$$

## Solution (6)

and

$$\frac{365}{90} \ln \left( 1 + \frac{0.0438}{4} \right) = 4.4167\%.$$

From

$$R_2 = \frac{F_1 (T_2 - T_1) + R_1 T_1}{T_2}.$$

with  $F_1 = 4.2060\%$ ,  $T_1 = 300$ ,  $T_2 = 398$ , and  $R_1 = 4\%$ , the 398 day rate is

$$\frac{4.2060 \times 98 + 4 \times 300}{398} = 4.0507$$

The 489 day rate is obtained with  $F_1 = 4.4167\%$ ,  $T_1 = 398$ ,  $T_2 = 489$ , and  $R_1 = 4.0507\%$ :

$$\frac{4.4167 \times 91 + 4.0507 \times 398}{489} = 4.1188$$

We are assuming that the first futures rate applies to 98 days rather than the usual 91 days. The third futures quote is not needed.



## Exercise (7, Done)

On August 1 a portfolio manager has a bond portfolio worth \$10 million. The duration of the portfolio in October will be 7.1 years. The December Treasury bond futures price is currently 91-12 and the cheapest-to-deliver bond will have a duration of 8.8 years at maturity.

- a) How should the portfolio manager immunize the portfolio against changes in interest rates over the next two months?
- b) How can the portfolio manager change the duration of the portfolio to 3.0 years?

## Solution (7)

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## Exercise (7, Done)

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$$\frac{10,000,000 \cdot 7.1}{91,375 \cdot 8.8} =$$

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$$\frac{10,000,000}{91,375} \frac{7.1}{8.8} = 88.30$$

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*Rounding to the nearest whole number 88 contracts should be shorted.*

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*b) In a) the problem is designed to reduce the duration to zero. To reduce the duration from 7.1 to 3.0 instead of from 7.1 to 0, the treasurer should short*

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$$\frac{4.1}{7.1} \times 88.30 = 50.99$$

## Exercise (8)

The three-month Eurodollar futures price for a contract maturing in six years is quoted as 95.20. The standard deviation of the change in the short-term interest rate in one year is 1.1%.

Estimate the forward LIBOR interest rate for the period between 6.00 and 6.25 years in the future.

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$$\text{Forward Rate} = \text{Futures Rate} - \frac{\sigma^2}{2} T_1 T_2$$

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From

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with  $\sigma = 0.011$ ,  $T_1 =$

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with  $\sigma = 0.011$ ,  $T_1 = 6$ , and  $T_2 = 6.25$ , we have

$$\begin{aligned} \text{Forward Rate} &= \text{Futures Rate} - \frac{0.011^2}{2} \times 6 \times 6.25 \\ &= \text{Futures Rate} - \end{aligned}$$



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$$\begin{aligned}\text{Forward Rate} &= \text{Futures Rate} - \frac{0.011^2}{2} \times 6 \times 6.25 \\ &= \text{Futures Rate} - 0.002269\end{aligned}$$

## Solution (8)

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with continuous compounding. The forward rate is therefore



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$$4.84 - 0.23 =$$

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The convexity adjustment  $\frac{\sigma^2}{2} T_1 T_2$  is then about 23 basis points. The futures Rate is  $\frac{100-95.20}{100} = 4.8\%$  with quarterly compounding and an actual/360 day count. This becomes

$$4.8 \times \frac{365}{360} = 4.867\%$$

with an actual/actual day count.

It is

$$4 \ln \left( 1 + \frac{0.04867}{4} \right) = 4.84\%$$

with continuous compounding. The forward rate is therefore

$$4.84 - 0.23 = 4.61\%$$

with continuous compounding.