Exercises Chapter 3: Repeated games

Exercise 1: Price competition in Duopoly

Two firms are playing an infinitely-repeated prisoner's dilemma pricing game of the following form:

		Firm 2	
		Low	High
Firm 1	Low	(5,5)	(20,0)
	High	(0,20)	(10,10)

The firms simultaneously set prices at regular intervals. In the equilibrium of this game, each firm selects the low price. While the equilibrium results in profits of $5 for each firm, collusion can potentially result in payoffs of $10. The firms utilize trigger strategies in order to maintain the collusive outcome.

a) Suppose that both firms adopt the grim trigger strategy. They continue colluding until one of them cheats. Upon one of them defecting, they play the equilibrium strategy for the rest of the game.
What has to be true about the interest rate (r) for collusion to be sustainable?

b) Suppose that both firms adopt a tit-for-tat strategy. They initially collude. In future periods, a firm colludes if its competitor did in the previous period, and elects the lower price if its competitor cheated in the previous period.
What has to be true about the interest rate (r) for collusion to be sustainable?

Exercise 2: Bargaining

Two players 1 and 2 negotiate how they will distribute a cake of size 1. At time 0, player 1 makes an offer to player 2, denoted as x_0. If player 2 accepts, then player 1 obtains x_0 and player 2 obtains $(1-x_0)$. If Player 2 refuses, he made an offer, denotes as x_1, to player 1 at time 1. If the player 1 accepts, he obtains x_1 and player 2 gets $(1-x_1)$, otherwise, player 1 made a new offer to player 2 at date 2, and so on.

We assume that when a player is indifferent between accepting and rejecting an offer he chooses to accept. The discount factors for players 1 and 2 are δ_1 and δ_2, respectively. If both players accept the cutting $(x_t, 1-x_t)$ at date t, their respective payoffs are $\delta_1^t x_t$ and $\delta_2^t(1-x_t)$.

Show that when the players are allowed to bargain during a finite period of time T there is a unique subgame perfect Nash equilibrium. Find this equilibrium.

Exercise 3: Monopolistic competition

Three firms are in monopolistic competition for producing goods that are imperfect substitutes. They choose their prices simultaneously. Consumers' demand for the firm *i*, with *i=1,2, and 3*, writes as $q_i = 100 - 3p_i + \sum_{j \neq i} p_j$, where p_i denotes firm *i*'s price. We assume production costs are zero.

a) Solve the Nash equilibrium of the stage game. What are the associated profits?

b) Find the strategies and profits associated with the "cooperative" solution that would maximize the total profit.

c) Consider now the corresponding infinitely repeated game. Let δ_i denotes firm *i*'s discount factor. Define a trigger strategy that may sustain cooperation at equilibrium.

d) Show that there are values of δ_i (*i*=1,2,3) such that cooperating at every stage sustain a SPE. Give the corresponding strategies and value for δ_i (*i*=1,2,3).